The workshops at the MATRIX Mathematics of Risk 2017 program

20-25 November 2017

During that week, four five-hour workshops will be given. On Monday and Tuesday, two of them will be given and, after a day of rest on Wednesday, the remaining two will be given on Thursday and Friday. The table below shows the time slots when the lectures will be given. Please see below for the short courses' names and outlines.

The organisers reserve the right to change the timetable and/or courses offered should there emerge a necessity to do so.

	Mon 20/11	Tue 21/11	Wedn 22/11	Thur 23/11	Fri 24/11
10:00-11:00	**Kratz**	**Zhitlukhin**	No lectures	**Palmowski**	**Kabanov**
11:00-12:00	**Kratz**	**Zhitlukhin**		**Palmowski**	**Kabanov**
12:00-14:00	Lunch	Lunch	Lunch	Lunch	Lunch
14:00-15:00	**Zhitlukhin**	**Kratz**		**Kabanov**	**Palmowski**
15:00-15:30	Tea break	Tea break		Tea break	Tea break
15:30-16:30	**Zhitlukhin**	**Kratz**		**Kabanov**	**Palmowski**
16:30-17:30	**Zhitlukhin**	**Kratz**		**Kabanov**	**Palmowski**

Workshop 1: Marie Kratz [ESSEC Business School, CREAR risk research center, Paris]

Workshop name: Extreme Value Theory and its Applications to Insurance and Finance

Topics include:

- Overview of univariate EVT

- Presentation of a new self-calibrating method for modeling heavy tailed probability distributions

- A review of popular risk measures and their properties

- Risk aggregation of heavy tailed risks

- Statistical procedures for backtesting models

- Presentation and discussion of a multinomial test for risk measure

Outline:

We present an overview of Univariate Extreme Value Theory (EVT) providing standard tools to model the tails of distributions.

One of the main issues in the statistical literature of extremes concerns the tail index estimation, which governs the probability of extreme occurrences. This estimation relies heavily on the determination of a threshold above which a Generalized Pareto Distribution (GPD) can be fitted. Approaches to this estimation may be classified into two classes, one using standard Peak Over Threshold (POT) methods, in which the threshold to estimate the tail is chosen graphically according to the problem, the other using self-calibrating methods, where the threshold is algorithmically determined.

We introduce a new method developed with N. Debbabi and M. Mboup (2016) to fit the entire distribution, which finds important applications in risk management. Our approach belongs to the second class of self-calibrating method proposing a hybrid distribution for heavy tailed data modeling, which links a normal (or lognormal) distribution to a GPD via an exponential distribution that bridges the gap between mean and asymptotic behaviors. A new unsupervised algorithm is then developed for estimating the parameters of this model. The effectiveness of our self-calibrating method is studied in terms of goodness-of-fit on simulated data, and compared with other more standard EVT approaches. This method presents the marked advantage that it provides a good fit of the entire distribution rather than simply the tail, allowing for a full pricing of the risk when considering applications in risk management. We provide examples of applications in finance and insurance.

Then, after a brief review of popular risk measures and their properties, we discuss the evaluation of the distribution of aggregated heavy tails. It has direct implications on the estimation of risk measures and diversification benefit of a risk portfolio. We first present Normex a method to estimate the distribution of aggregated independent risks combining normal distribution and heavy tail corrections. We then tackle this issue according to the type of dependence between risks, after briefly introducing useful tools for measuring dependency.

Finally, after the modeling of a risk portfolio comes the question of model validation. We present standard statistical procedures for backtesting risk measures / models used in solvency regulation, followed by a new implicit backtest for Expected Shortfall, recently developed with Y. Lok and A. McNeil (2016). We suggest a simple multinomial approach, as an extension of the binomial approach routinely used by financial institutions. It turns out that this method gives very reasonable results, certainly much better than with the binomial backtest currently in use, helping to distinguish between models. In particular, the multinomial test is able to correctly discriminate models with fat-tails from model with light tails in presence of heavy tailed data.

Workshop 2: Mikhail Zhitlukhin [Steklov Mathematical Institute, Moscow]

Workshop Name: Financial measures of risk and performance

Outline:

In this course we will consider measures of financial risk and performance. The mathematical theory of risk measures was first presented in the seminal paper of Artzner et al. (1999) and since then has firmly taken its place in the modern finance due to the importance of question of risk management of financial institutions. The mathematical side of the theory is based on ideas from convex analysis and conjugate duality and uses insightful and important results of independent interest.

In the beginning of the course we will recall main constructions related to risk measures and well-known results about them. In the second part will be presented new objects which are tightly connected to risk measures (the fundamental risk quadrangle, performance measures, quasi-convex probabilities, etc.). The third part will concentrate on stochastic optimization problems with optimality criteria defined using risk measure and related functionals. We will consider both static and dynamic optimization problems.

An important role will be played by statistical aspects of measures of risk and performance, in particular questions of their estimation from past history of asset prices, and properties of corresponding estimators. Another important direction will be numerical methods for problems that we will consider: we will try not only to obtain theoretical solutions but also to develop efficient numerical methods for their computation.

Workshop 3: Zbigniew Palmowski [Wrocław University of Science and Technology]

Workshop Name: Ruin probabilities: exact and asymptotic results

Outline:

Ruin theory concerns the study of stochastic processes that represent the time evolution of the surplus of an insurance company. The initial goal of early researchers of the field, Lundberg (1903) and Cramér (1930), was to determine the probability for the surplus to become negative. In those pioneer works, the authors showed that the ruin probability decreases exponentially fast to zero with initial reserve tending to infinity when the net profit condition is satisfied and clam sizes are light-tailed.

During the lectures, we will explain when and why we can observe this phenomenon. We will also discuss the complimentary heavy-tailed case and explain what is the most likely way of getting ruined in this case.

During the lectures, we will show as well how to identify the exact expressions for (ultimate and finite time) ruin probabilities, or for more general so-called Gerber-Shiu functions. The main tool will be based on theory of ordinary differential equations and the Picard-Lefévre formula.

We will demonstrate the main techniques and results related with the exact and asymptotics of the ruin probabilities:

- ordinary differential equations for the Gerber-Shiu function, ballot theorem,

- Pollaczek-Khinchin formula, Lundberg bounds, change of measure, Wiener-Hopf factorization,

- the principle of one big jump and theory of scale functions of Lévy processes.

Finally, we will present some extensive statistical and numerical results and simulation methods of the ruin probability as well as the risk process itself.

Workshop 4: Yuri Kabanov [University of Franche-Comté, Besançon]

Workshop Name: Clearing in financial systems

Outline:

In 2001 Eisenberg and Noe suggested a clearing procedure in a simple static model describing a financial system composed of N banks. The assets of each bank are of two types: the cash (an external asset) and the credits provided to other banks of the system, called exposures; they are, in turn, the liabilities for its debtors. The clearing means simultaneous repaying all debts. Each bank returns to its counterparties the debts pro rata of their relative volume using its cash reserve and obtained repayments of the credited banks. The rule is: for each bank, either all debts are payed in full or the zero level of equity is attained; in the latter case the bank defaults. The totals reimbursed by the banks form an N-dimensional clearing vector. It was shown that clearing vectors must satisfy a nonlinear equation involving a stochastic matrix with rows formed by the fractions of the total liabilities of each bank to its creditors. The key observation is that this equation is a fixpoint problem for a monotone mapping f of a closed N-dimensional interval into itself. The existence of such points follows from the Knaster–Tarski theorem, a beautiful and simple result which proof needs only a few lines. The uniqueness of the clearing vector is a more delicate result. One can use the graph structure of the system built in the same way as is usually done in the theory of Markov chains. A more general model with crossholdings was developed simultaneously by Suzuki.

The ideas of the Eisenberg–Noe paper happened to be very fruitful and the model and methods used to its analysis were generalized in many directions having not only financial importance but posing an interesting mathematical questions in more sophisticated situations: on the uniqueness of clearing vector, on efficient numerical methods of its calculation, and so on. Nowadays the clearing problem became a section of the new mathematical discipline — theory of systemic risk.

The algorithm of calculating the largest clearing vector in a finite number of steps was investigated in a model with default losses by Rogers and Luitgard Veraart. Mathematically, the model is a minor generalization but it is interesting from the point of view of applications: it provides a natural framework to investigate problems of merging and rescue (bailout) of defaulting banks.

Elsinger considered a model combining crossholdings with seniorities of liabilities.

In the real world banks protect themselves from the default risk of their debtors using such financial instruments as credit default swaps (CDS), i.e. by buying from third parties contingent claims with pay-offs covering losses if the debtor is insolvent. Mathematically, this means that the liability matrices may depend on the clearing vector. Model of this type was analyzed by Fisher on the basis of ideas developed by Suzuki.

A specific class of models deals with situations where banks, besides the cash and liabilities, may have one or more illiquid assets whose selling might influence the market prices.

The plan of lectures:

- The Eisenberg-Noe-Suzuki model. Existence of the clearing vectors via fixpoint theorems. Uniqueness theorem.

- The Rogers-Veraart model. Calculation of the largest clearing vector.

- The Suzuki-Elsinger model with crossholdings.

- The Elsinger model with seniorities of liabilities.

- The Fisher model with CDS.

- Models with illiquid securities.