UNIVERSITY OF AMSTERDAM

Many-body strategies for multi-qubit gates

Kareljan Schoutens

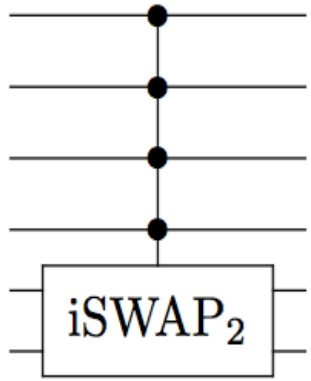
MATRIX workshop, July 13 2017

QuSoft

Delta Institute for Theoretical Physics

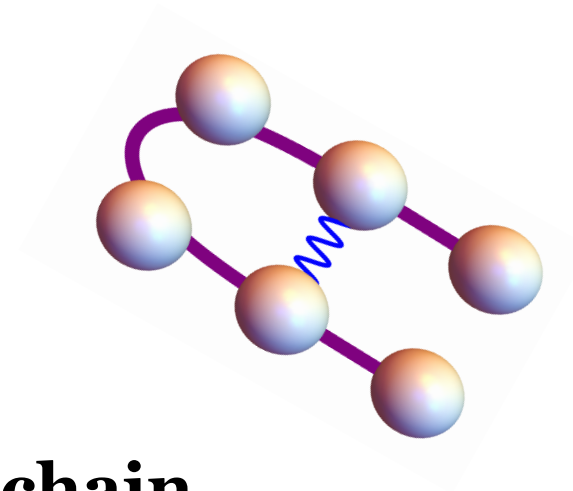
1-page summary

quantum circuits for quantum algorithms typically need **multi-qubit gates**: unitaries acting on more than 2 qubits

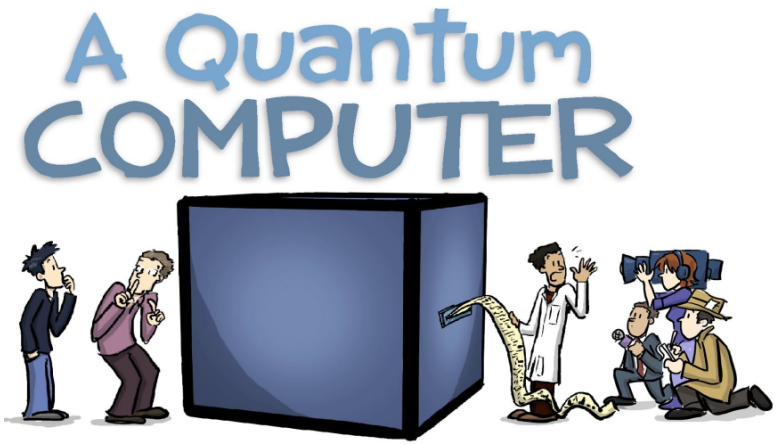


multi-qubit gates can be built from 1-qubit and 2-qubit gates, but such constructions can be cumbersome

we realize N-qubit gates via driven dynamics of $\boldsymbol{N}$ **coupled qubits**

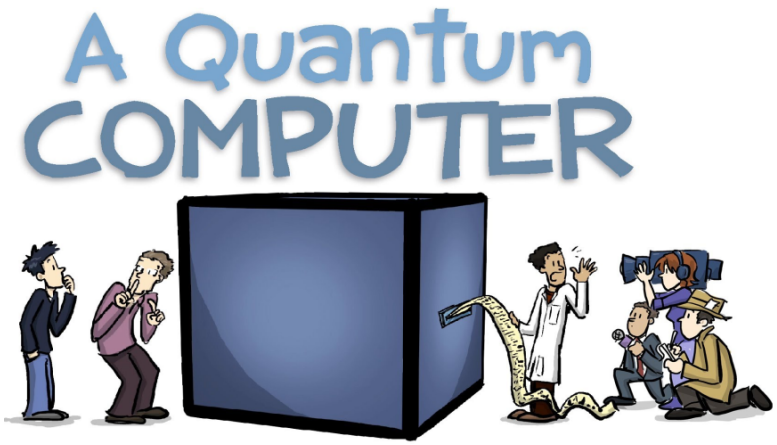


main mechanism is resonant coupling of eigenstates of **Krawtchouk qubit chain**



outline

- background and motivation
- many-body strategies for multi-qubit gates
- quantum control on the Krawtchouk chain



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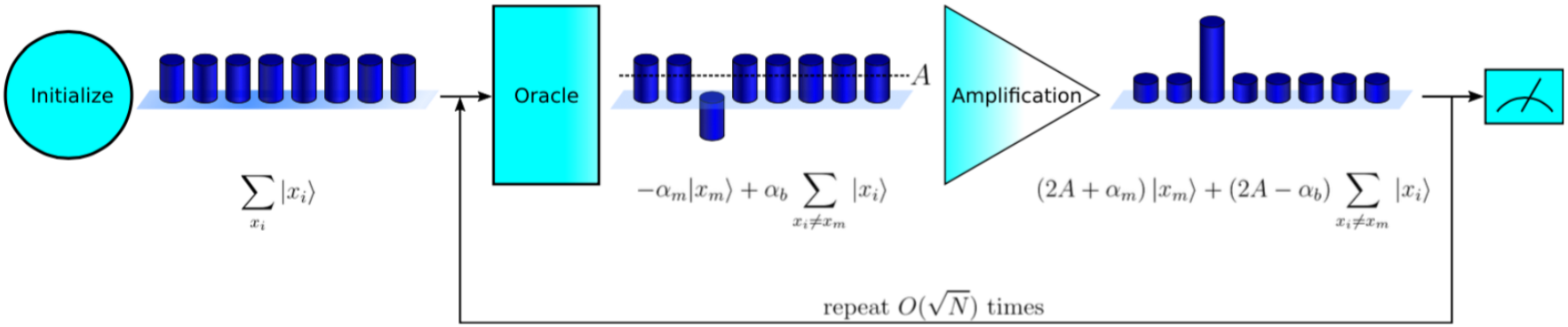
quantum algorithms

For specific problems quantum algorithms can be made to outperform classical computers by cunningly combining quantum parallelism with interference.

Grover search algorithm:

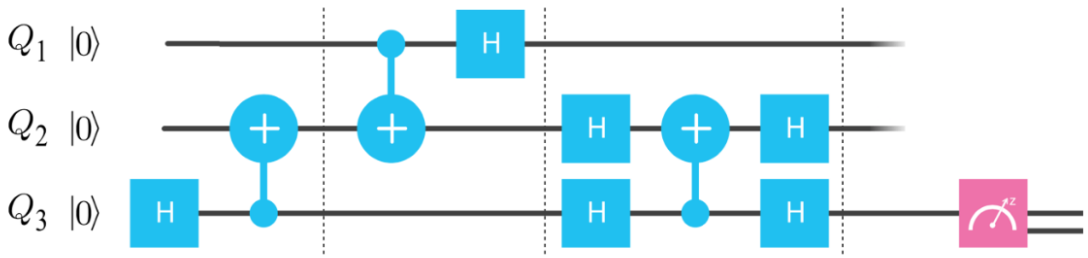
finding tagged element in size-N database in $O(\sqrt{N})$ steps

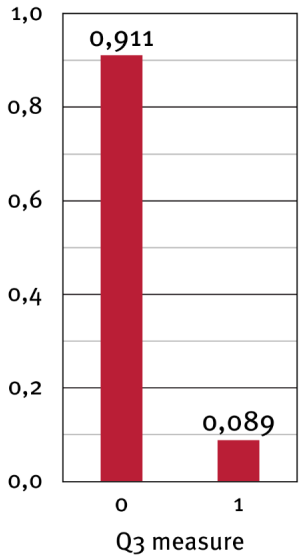


quantum circuit

3-step implementation of quantum algorithm on N-qubit quantum register

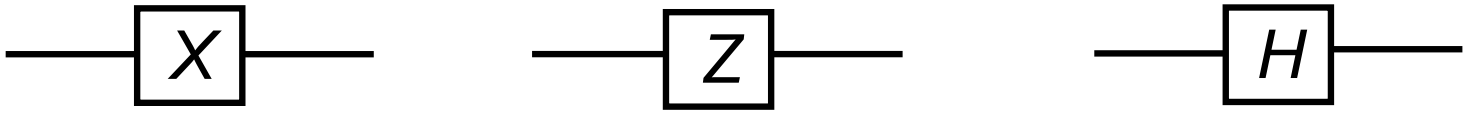
- **initialization**
- **unitary evolution** via quantum gates
- read-out through **measurement**





quantum gates

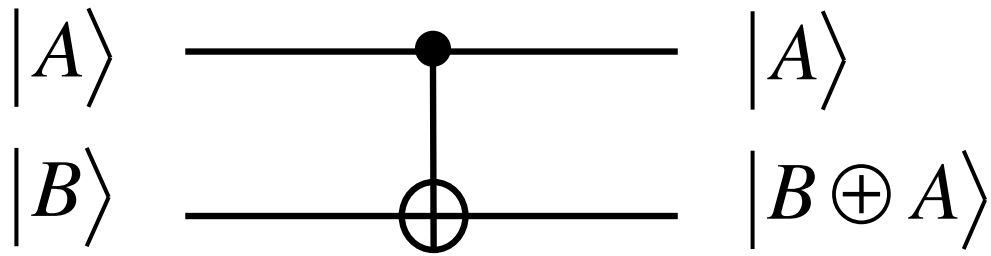
- **1-qubit gates:** *X, Z, H, ...*



$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

- **2-qubit gates:** CNOT, *XX(θ)*, SWAP, ...

CNOT:



universal gate sets

- **strong universality**
  all N-qubit unitaries can be built from CNOTs
  plus sufficiently many 1-qubit gates

- **weak universality**
  all N-qubit unitaries can be approximated to arbitrary
  precision using CNOTs plus suitable (finite) set
  of 1-qubit gates

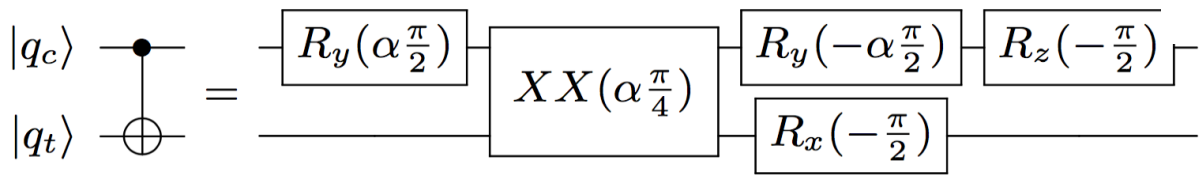
native gates and quantum compiling

- **native gate libraries**
  the 1-qubit and 2-qubit interactions that are natural for a given qubit platform lead to a `native gate library'.

- **quantum compiling**
  expressing universal gates in native gates

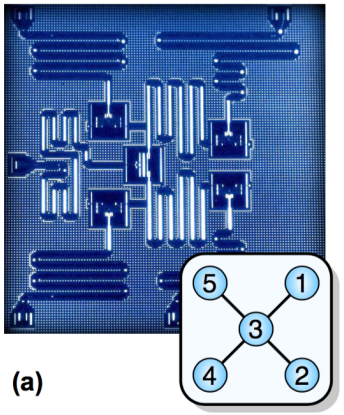
example: native gate library for trapped ions

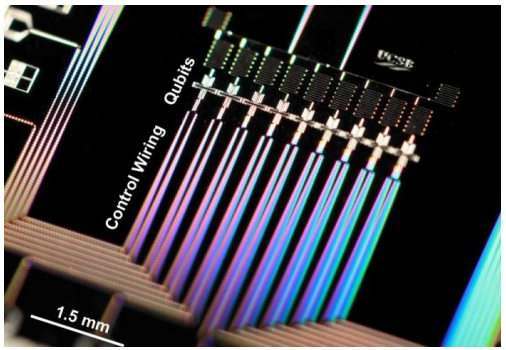
- all 1-qubit rotations $R_\alpha(\theta)$
- 2-qubit gates $X_i X_j\,(\theta)$

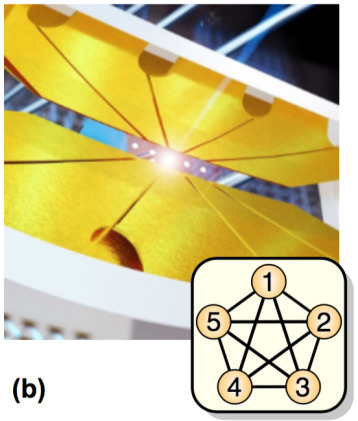


state of the art

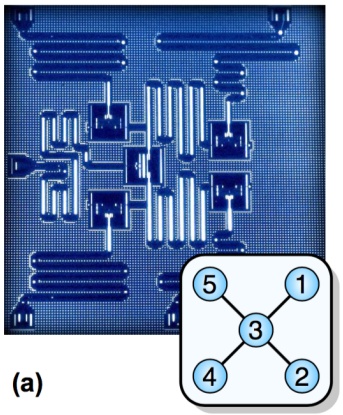
quantum hardware has progressed to the point that programmable qubit platforms with up to some 20 qubits are available → real-world testing of few-qubit quantum algorithms!







IBM Q
`Quantum Experience'



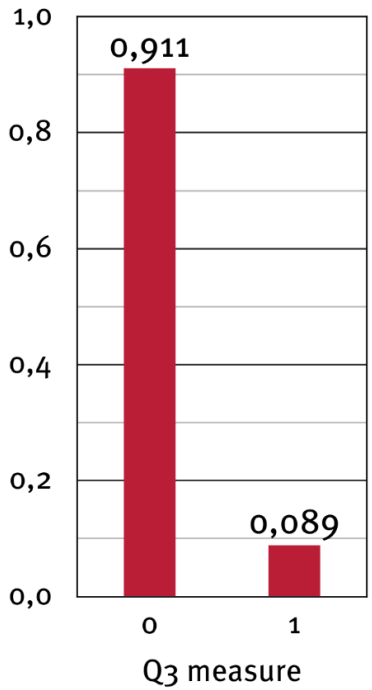
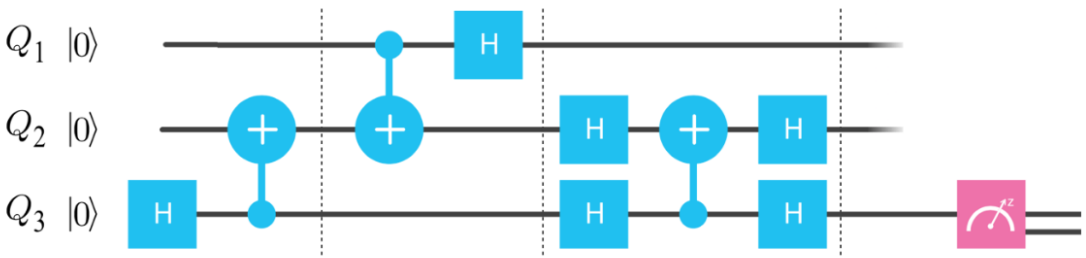


FIGURE C.3: The results of 8192 runs of the quantum circuit teleporting the state $|0\rangle$ shown in figure C.1.

Quantum teleportation: transfering qubit Q1 to Q3 at distant location



Bachelor thesis Jorran de Wit (2016)

Complete 3-Qubit Grover Search on a Programmable Quantum Computer

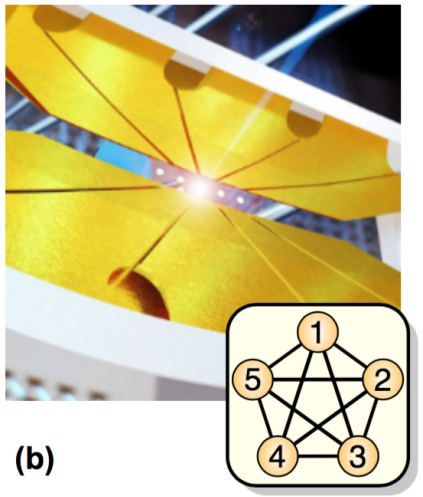
C. Figgatt,[1] D. Maslov,[2,1] K. A. Landsman,[1] N. M. Linke,[1] S. Debnath,[1] and C. Monroe[1,3]

[1] *Joint Quantum Institute, Department of Physics, and Joint Center for Quantum Information and Computer Science, University of Maryland, College Park, MD 20742, USA*

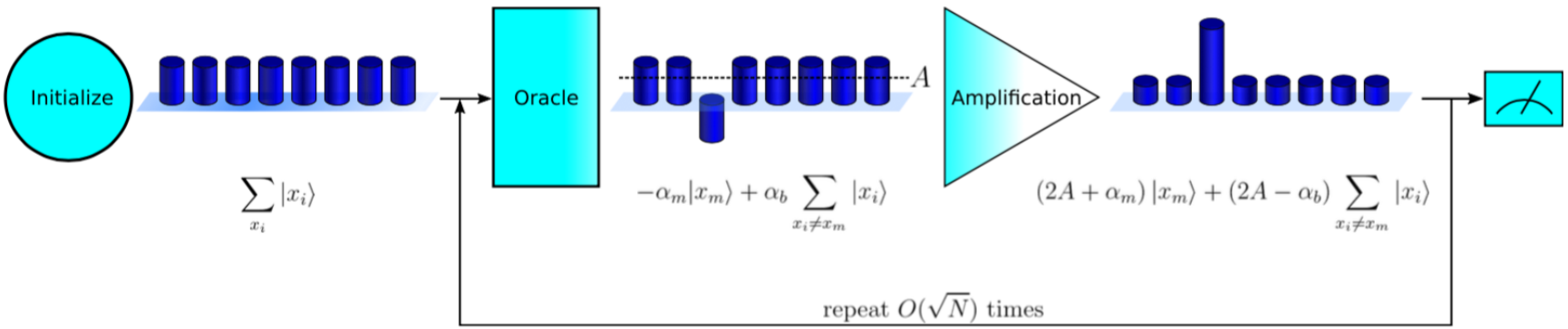
[2] *National Science Foundation, Arlington, VA 22230, USA*

[3] *IonQ Inc., College Park, MD 20742, USA*

(Dated: March 31, 2017)

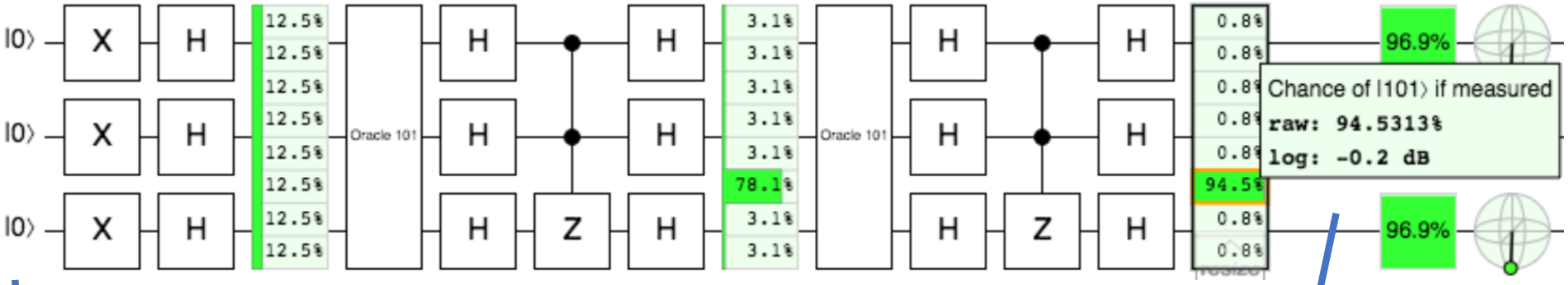


Grover search: finding tagged element in size-N database in $O(\sqrt{N})$ steps



3-qubit Grover search on Quirk:

finds 1 out of 8 elements in two steps



Oracle tagging the element |101>

Initializing the qubits to |0>

read-out gives tagged element |101> with 94.5% chance

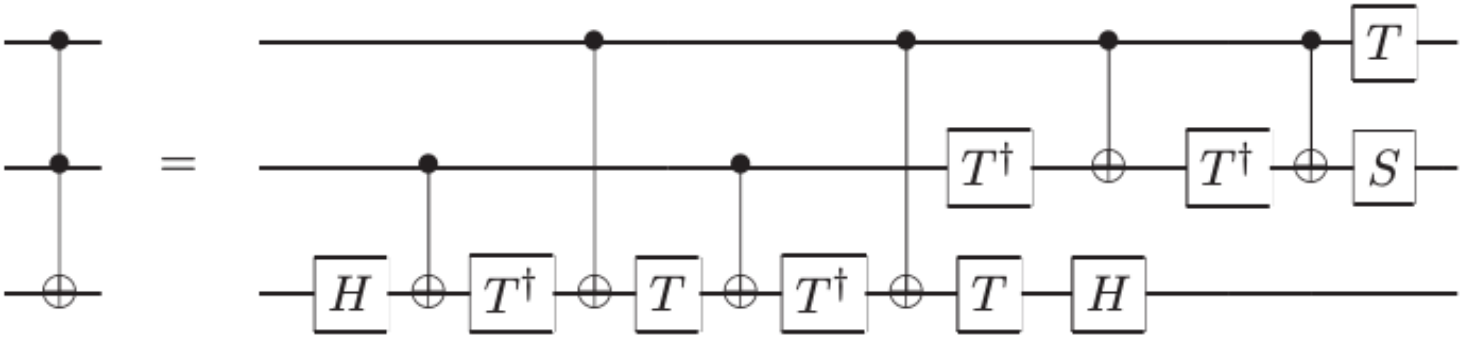
multi-qubit gates

- quantum algorithms such as Grover search use gates like

  CCNOT (Toffoli), CCZ, … , C^{N-1}NOT, C^{N-1}Z, etc

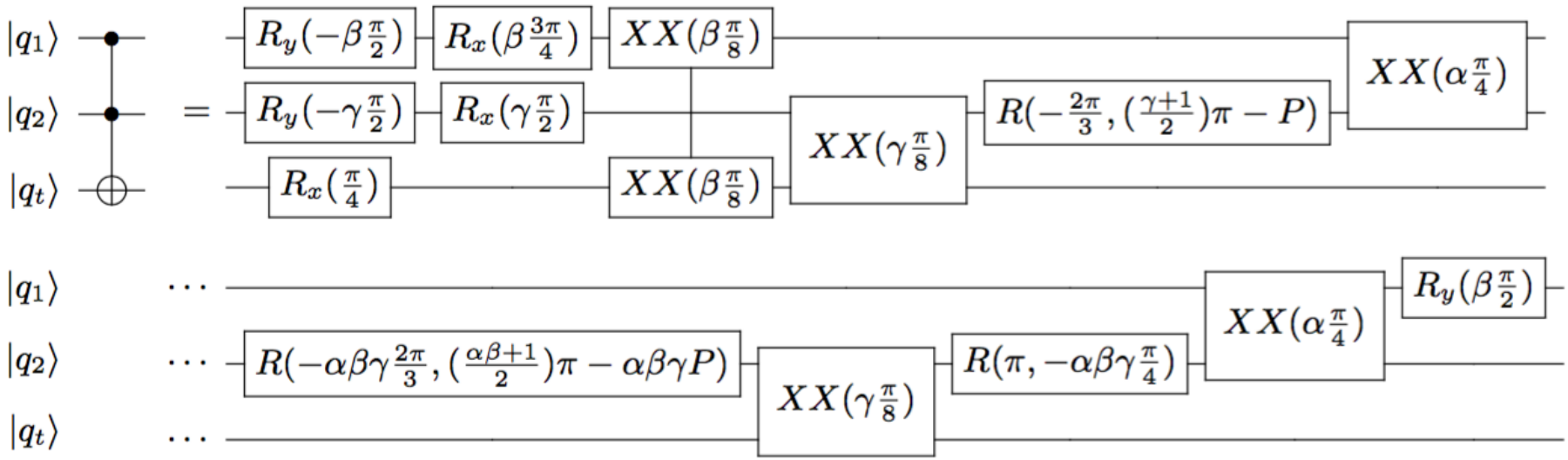
- building these from 1-qubit and 2-qubit gates requires lengthy circuits

multi-qubit gates



Toffoli-3 using standard Clifford + T gate library

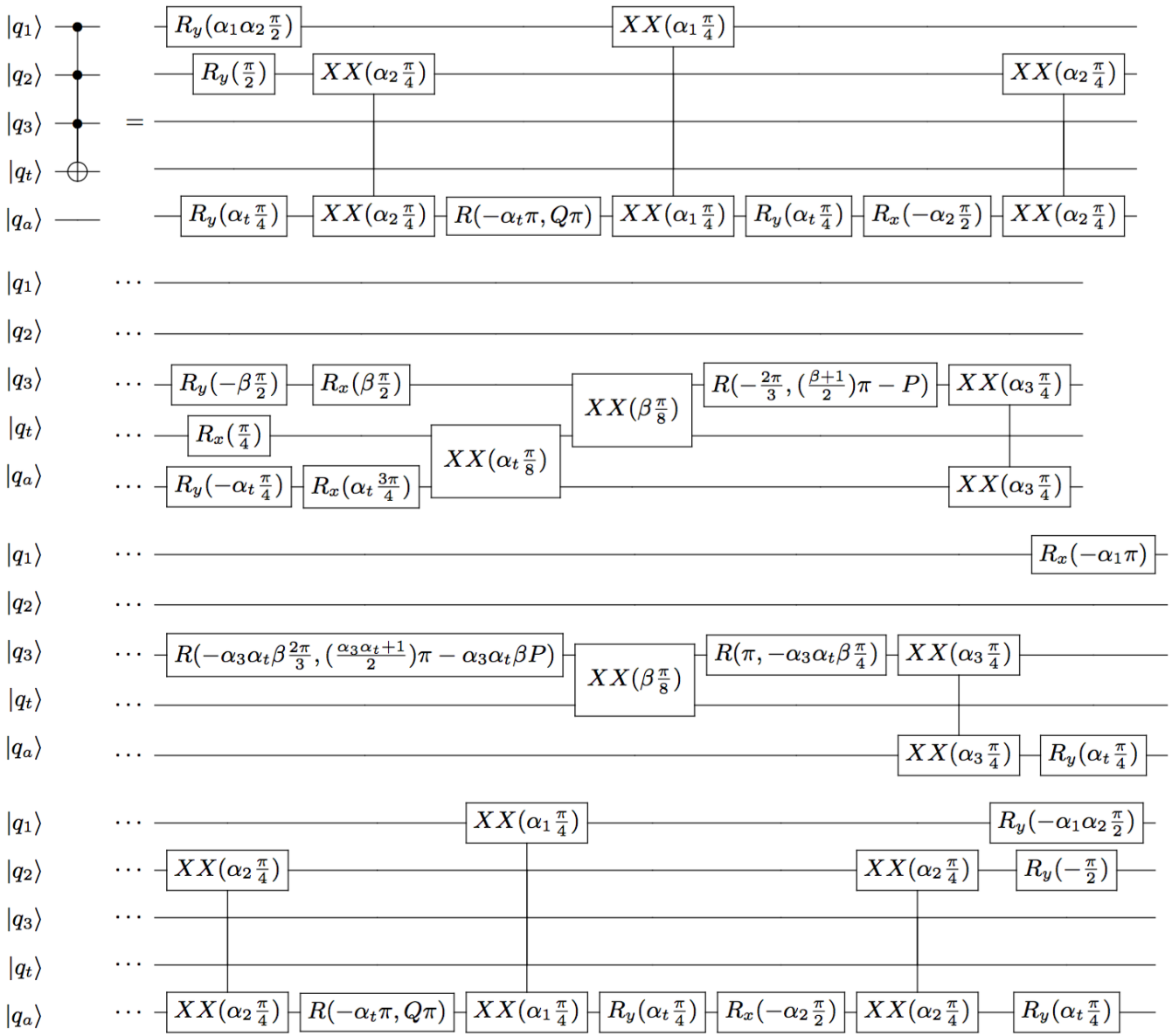
multi-qubit gates

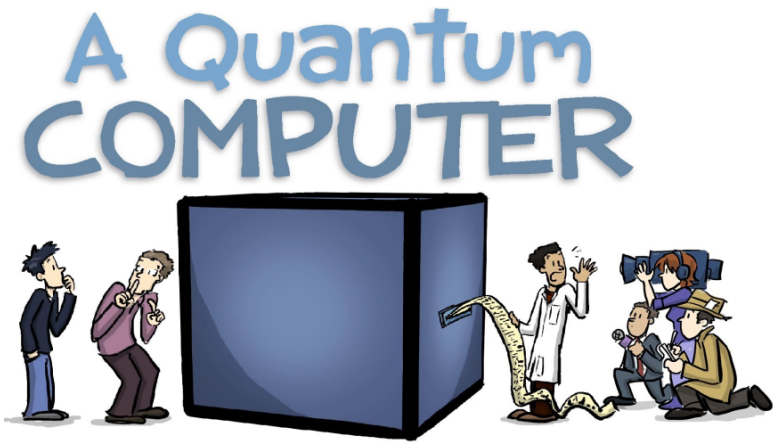


Toffoli-3 using XX/R gate library

multi-qubit gates

Toffoli-4 using XX/R gate library





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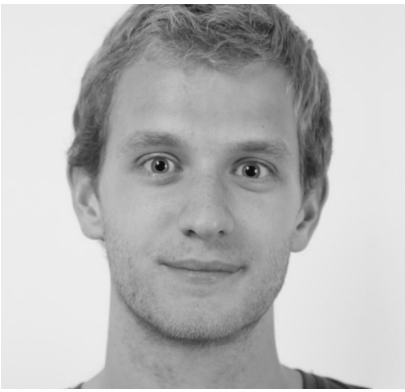
- quantum control on the Krawtchouk chain



Many-body strategies for multi-qubit gates - quantum control through Krawtchouk chain dynamics

Koen Groenland[1,2,3] and Kareljan Schoutens[1,2]

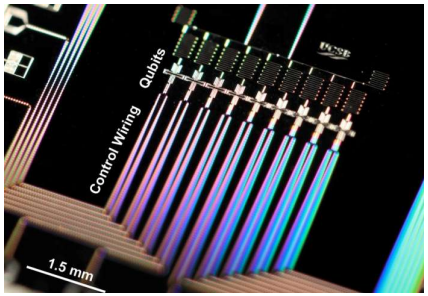
[1] *QuSoft, Science Park 123, 1098 XG Amsterdam, the Netherlands*
[2] *Inst. of Physics, Univ. of Amsterdam, Science Park 904, 1098 XH Amsterdam, the Netherlands*
[3] *CWI, Science Park 123, 1098 XG Amsterdam, the Netherlands*

(Dated: 17 July 2017)

We propose a strategy for engineering multi-qubit quantum gates. As a first step, it employs an *eigengate* to map states in the computational basis to eigenstates of a suitable many-body Hamiltonian. The second step employs resonant driving to enforce a transition between a single pair of eigenstates, leaving all others unchanged. The procedure is completed by mapping back to the computational basis. We demonstrate the strategy for the case of a linear array with an even number N of qubits, with specific $XX + YY$ couplings between nearest neighbors. For this so-called Krawtchouk chain, a 2-body driving term leads to the iSWAP_N gate, which can be reworked to an iSWAP_2 gate with $N-2$ controls or, using a single auxiliary qubit, to an $(N-1)$-Toffoli gate.

arXiv:1707.05144v1 [quant-ph] 17 Jul 2017

many-body strategy



idea

couple N qubits, leading to a many-body spectrum

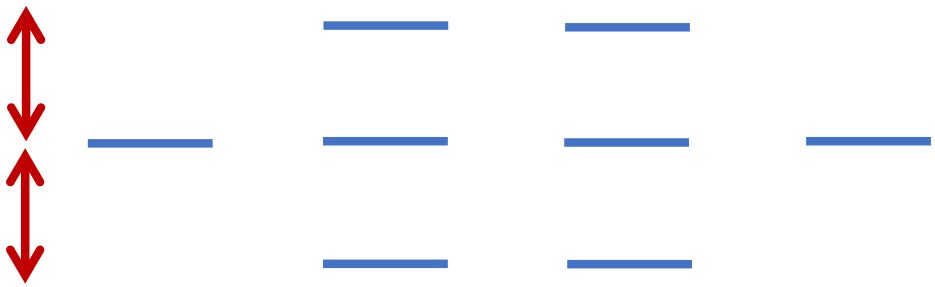
proposed protocol

- apply quantum circuit for *eigengate* to produce eigenstates from states in computational basis
- use resonant driving to selectively couple and interchange 2 out of 2^N eigenstates
- apply eigengate to return to computational basis

many-body strategy ...

protocol requires

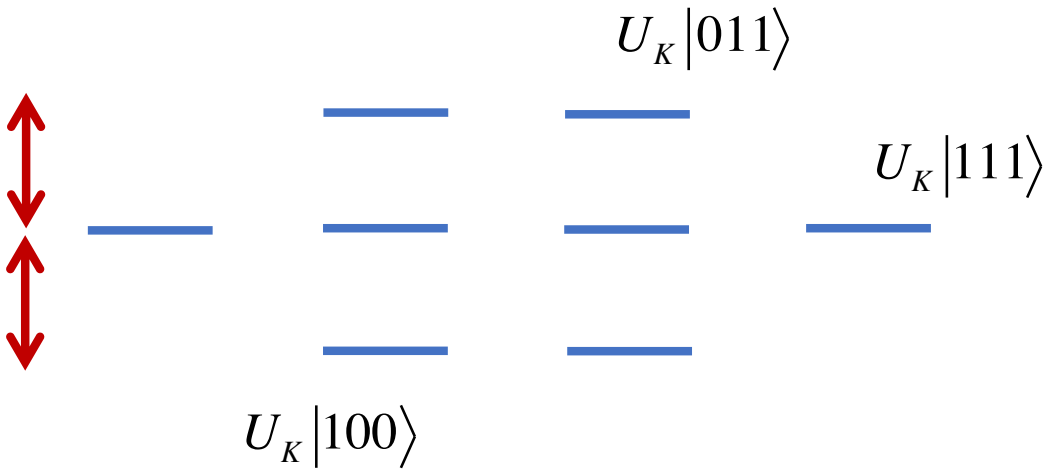
1. commensurate many-body spectrum



many-body strategy ...

protocol requires

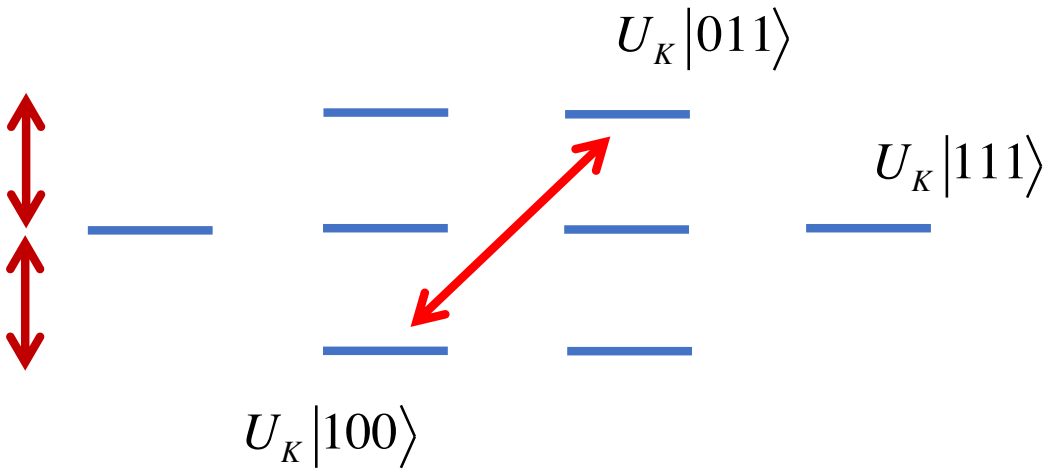
2. eigengate producing many-body eigenstates



many-body strategy...

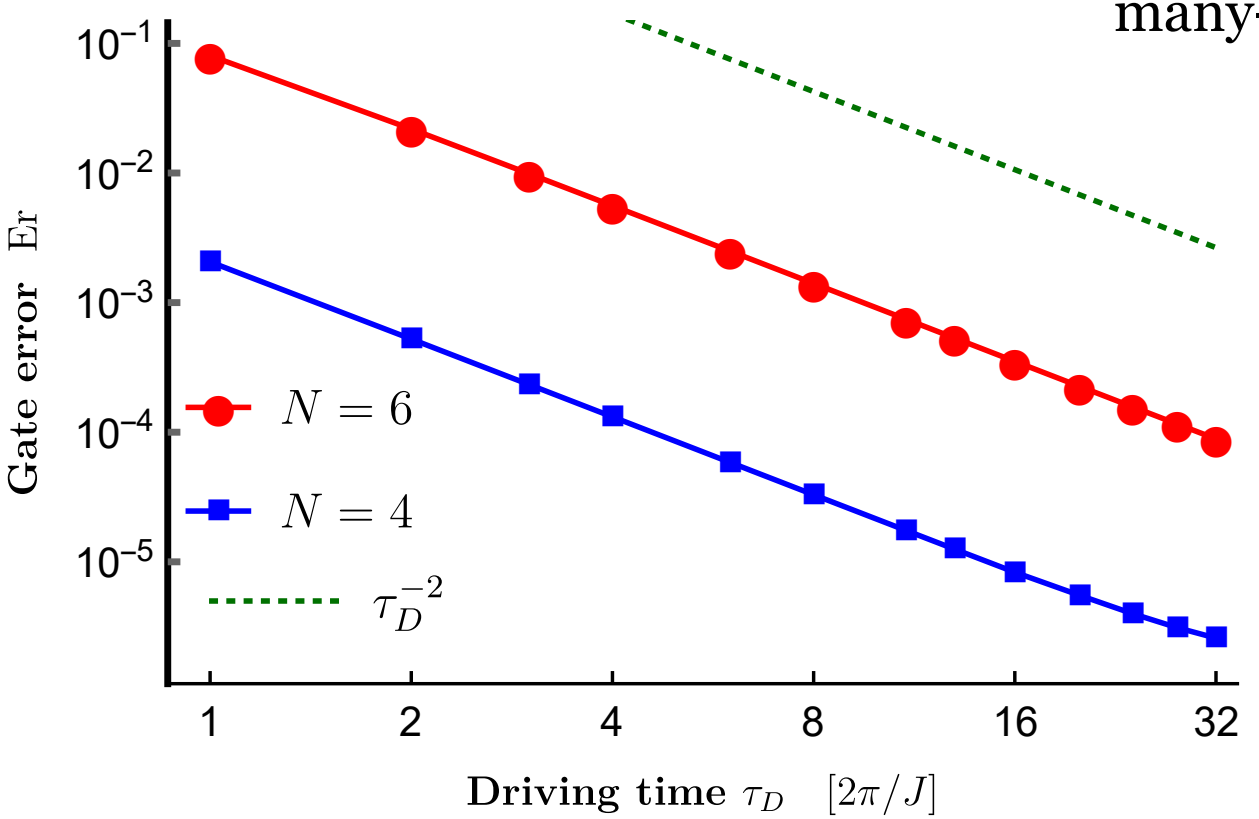
protocol requires

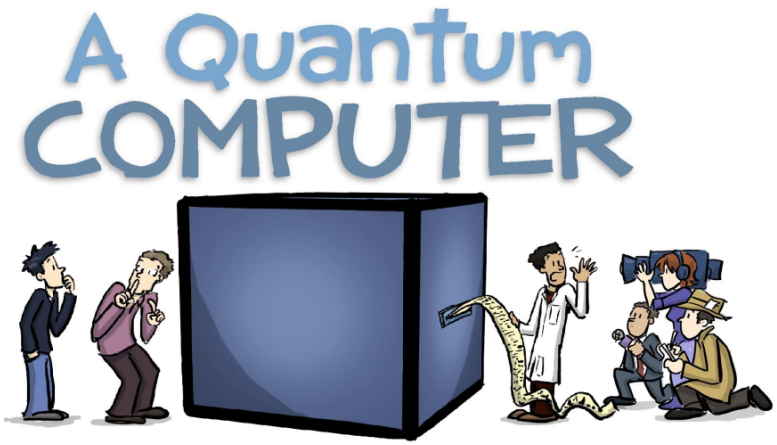
3. driving operator H_D



... for multi-qubit gates

iSWAP$_4$ and iSWAP$_6$ gates
realized through
many-body protocol





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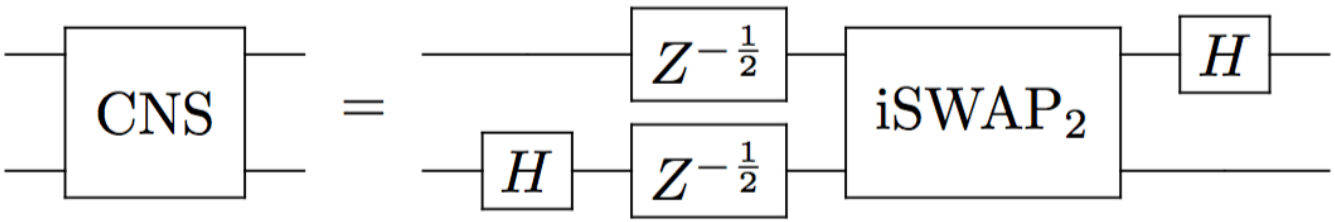
2-qubit XX+YY coupling

$$H^{(2)} = -\frac{J}{2}(X_1X_2 + Y_1Y_2)$$

- $t=\pi/J$ pulse of $H^{(2)}$ gives gate iSWAP$_2$,

  $|00\rangle \rightarrow |00\rangle$, $|01\rangle \rightarrow i\,|10\rangle$, $|10\rangle \rightarrow i\,|01\rangle$, $|11\rangle \rightarrow |11\rangle$

- combining iSWAP$_2$ with 1-qubit gates gives gate CNS, which is CNOT followed by SWAP



Krawtchouk chain (N=n+1)

$$H^K = -\frac{J}{2}\sum_{x=0}^{n}\sqrt{(x+1)(n-x)}\left[X_x X_{x+1} + Y_x Y_{x+1}\right]$$

- 1-body spectrum

$$\lambda_k = J(k - \frac{N-1}{2}), \qquad k = 0, 1, .., n$$

- eigenstates

$$\left|\{k\}\right\rangle_{H^K} = \sum_{x=0}^{n}\phi_{k,x}^{(n)}\left|\{x\}\right\rangle \qquad \phi_{k,x}^{(n)} = K_{k,x}^{(n)}\sqrt{\frac{\binom{n}{x}}{\binom{n}{k}2^n}}$$

with $K^{(n)}$ the **Krawtchouk polynomials**

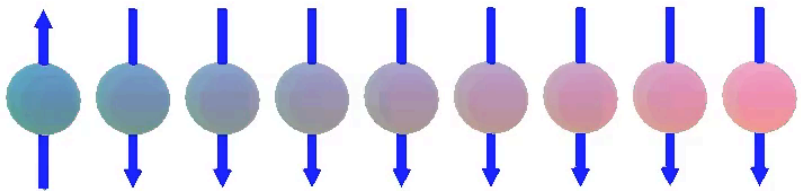
$$K_{k,x}^{(n)} = \sum_{j=0}^{k}(-1)^j\binom{x}{j}\binom{n-x}{k-j}$$

Krawtchouk chain

dynamics for Krawtchouk couplings known to be special

Christandl-Datta-Ekert-Landahl 2004

time evolution over time $t=\pi/(2J)$ gives Perfect State Transfer (PST) for state with single `particle' or `spin-flip'



animation:
Van der Jeugt

Krawtchouk chain ($N=n+1$)

$$H^K = -\frac{J}{2}\sum_{x=0}^{n}\sqrt{(x+1)(n-x)}\left[X_x X_{x+1} + Y_x Y_{x+1}\right]$$

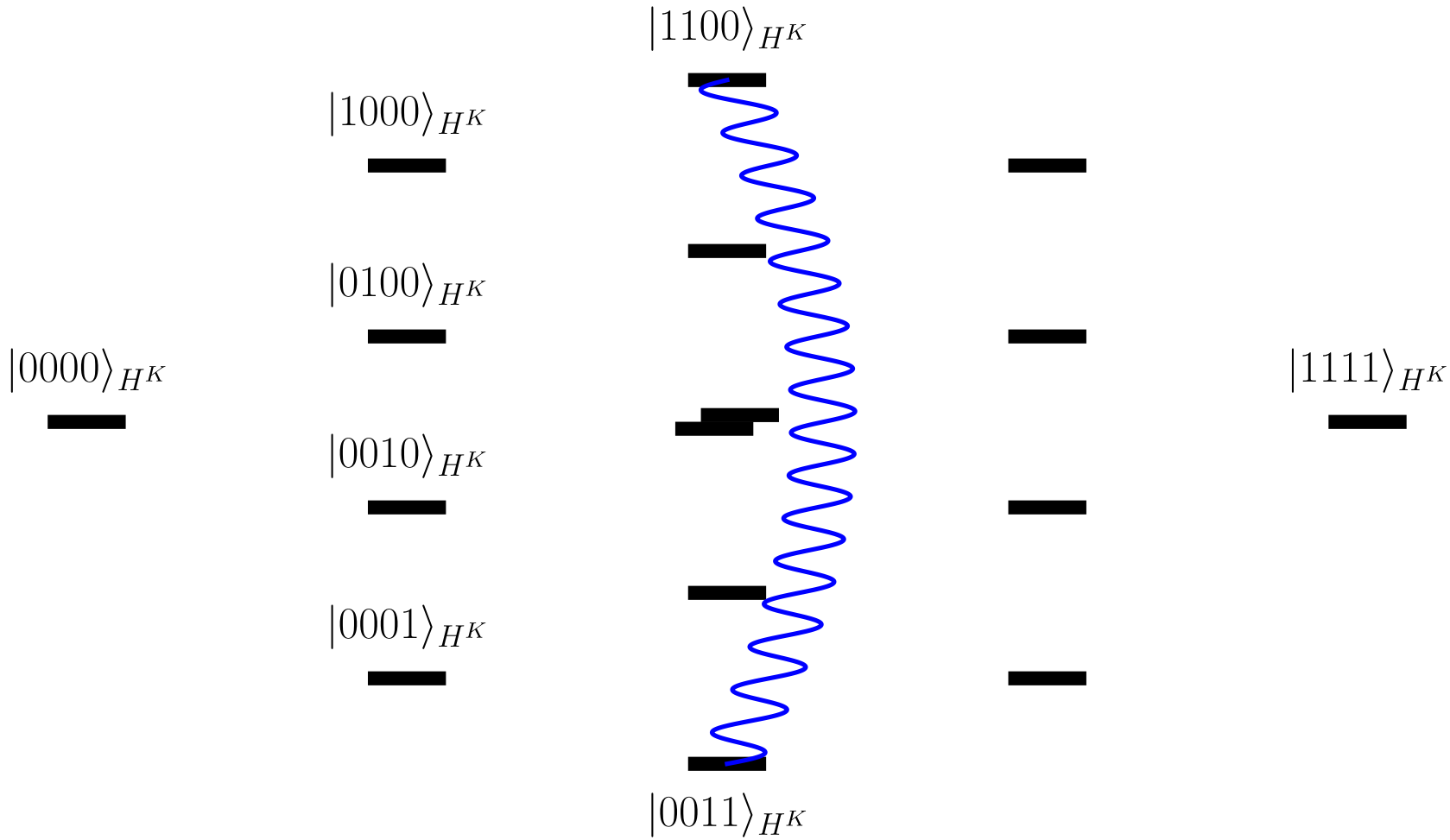
- **important clue:** mapping to free fermions through Jordan-Wigner transformation

$$\frac{1}{2}\left(X_j + iY_j\right) = \prod_{i=0}^{j-1}(1-2n_i)f_j \qquad \frac{1}{2}\left(X_j - iY_j\right) = \prod_{i=0}^{j-1}(1-2n_i)f_j^+$$

- many-body eigenstates built from fermionic eigenmodes

$$c_k^+ = \sum_{j=0}^{n}\phi_{k,j}^{(n)} f_j^+$$

Krawtchouk chain (N=4)



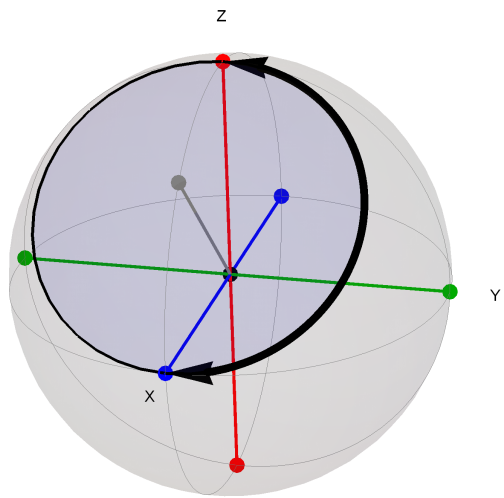
Krawtchouk eigengate

- exact *eigengate* for Krawtchouk chain eigenstates

$$U_K = \exp\left(-i\frac{\pi}{J}\frac{(H^K + H^Z}{\sqrt{2}}\right)$$

with

$$H^Z = \frac{J}{2}\sum_{x=0}^{n}(x - \frac{n}{2})(I - Z)_x$$



- important clue: Krawtchouk operators $L_X = H^K$ and $L_Z = H^Z$ satisfy angular momentum commutation relations
- use this to prove that

$$U_K H^Z = H^K U_K \quad \Rightarrow \quad U_K \left|s\right\rangle = \left|s\right\rangle_{H^K}$$

Krawtchouk eigengate, II

- equivalent expression

$$U_K = \exp\left(-i\frac{\pi}{2J}H^Z\right)\exp\left(-i\frac{\pi}{2J}H^K\right)\exp\left(-i\frac{\pi}{2J}H^Z\right)$$

- action on 1-particle states implies

$$\sum_{k=0}^{n}(-i)^k K_{x,k}^{(n)} K_{k,y}^{(n)} = i^{x+y-n/2} 2^{n/2} K_{x,y}^{(n)}$$

(agrees with Meixner's expansion formula)

Multi-qubit gate: iSWAP$_N$

- idea: for N even, driving term $H_D(t)$ that resonantly couples the

  highest energy state $U_K|00...01...11>$

  to the

  lowest energy state $U_K|11...10...00>$

- need to annihilate the $N/2$ fermionic modes with $\lambda_k>0$ and create the $N/2$ modes with $\lambda_k<0$

- can be done by the following 2-qubit operator

$$\sigma_j^- \sigma_{j+N/2}^+ = f_j^+ [1 - 2 f_{j+1}^+ f_{j+1}] ... [1 - 2 f_{j+N/2-1}^+ f_{j+N/2-1}] f_{j+N/2}$$

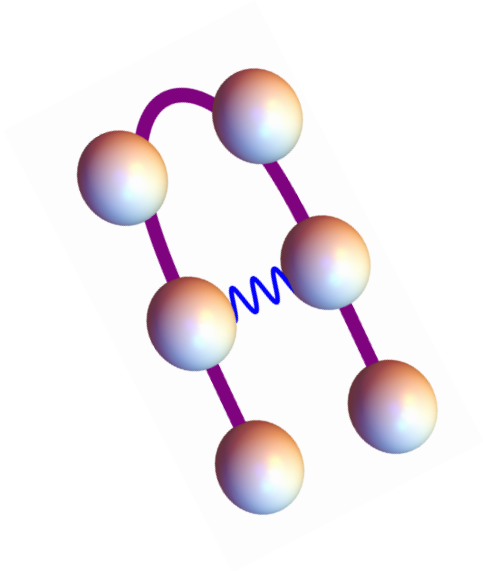
Multi-qubit gate: iSWAP$_N$

- for N=6: matrix element

$$\langle 111000 | U_K(\sigma_1^+\sigma_4^- - \sigma_4^+\sigma_1^-)U_K | 000111 \rangle = \frac{5}{32}$$

- resonant driving term

$$H_D^{(1,-)}(t) = i J_D \cos[9Jt][\sigma_1^+\sigma_4^- - \sigma_4^+\sigma_1^-]$$



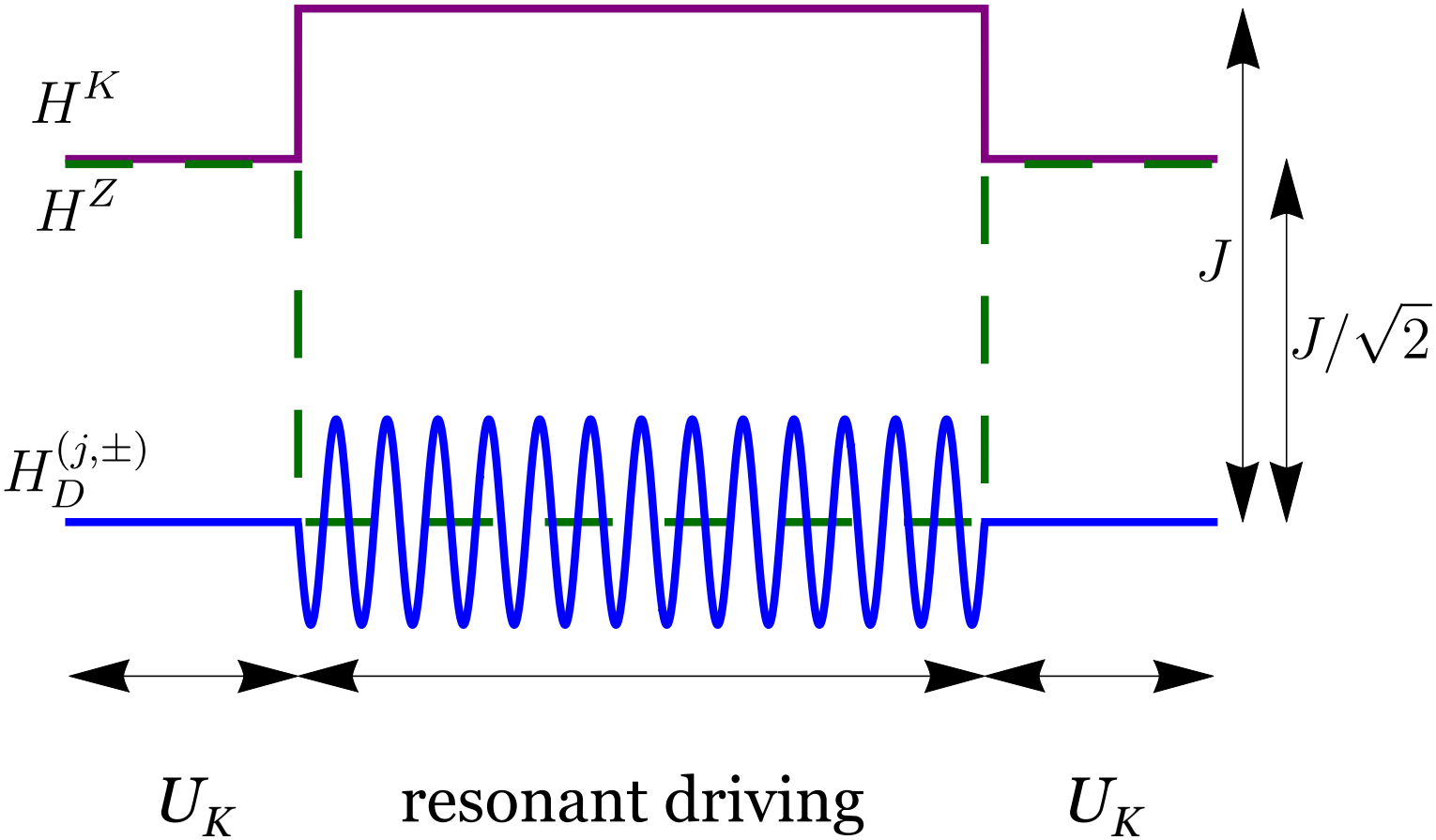
- conditions on driving time τ_D

$$\tau_D(5J_D/64) = \pi/2 \qquad \tau_D = M(2\pi/J)$$

so that (in leading order) |*000111*> and |*111000*> are interchanged and all dynamical phases return to 1

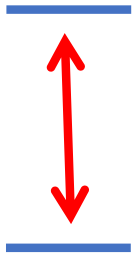
many-body protocol for iSWAP$_6$

$$|000111\rangle \to i|111000\rangle, \qquad |111000\rangle \to i|000111\rangle$$



resonant driving – fidelity

$$H_D(t) = \begin{pmatrix} E_1 & d\,\mathrm{e}^{i\omega t} \\ d\,\mathrm{e}^{-i\omega t} & E_2 \end{pmatrix} \qquad \Delta = \omega - (E_2 - E_1)$$

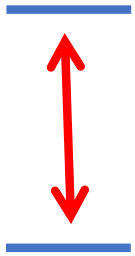


on resonance: $\Delta = 0$

$$v_1(t) = e^{-iE_1 t}\cos(d\,t), \quad v_2(t) = -ie^{-iE_2 t}\sin(d\,t)$$

resonant driving – fidelity

$$H_D(t) = \begin{pmatrix} E_1 & d\,\mathrm{e}^{i\omega t} \\ d\,\mathrm{e}^{-i\omega t} & E_2 \end{pmatrix} \qquad \Delta = \omega - (E_2 - E_1)$$



off resonance: $\Delta \neq 0 \quad d << \Delta$

$$dt = \pi / 2 \qquad t = 2\pi M \qquad \Delta, (E_2 - E_1) \text{ integer}$$

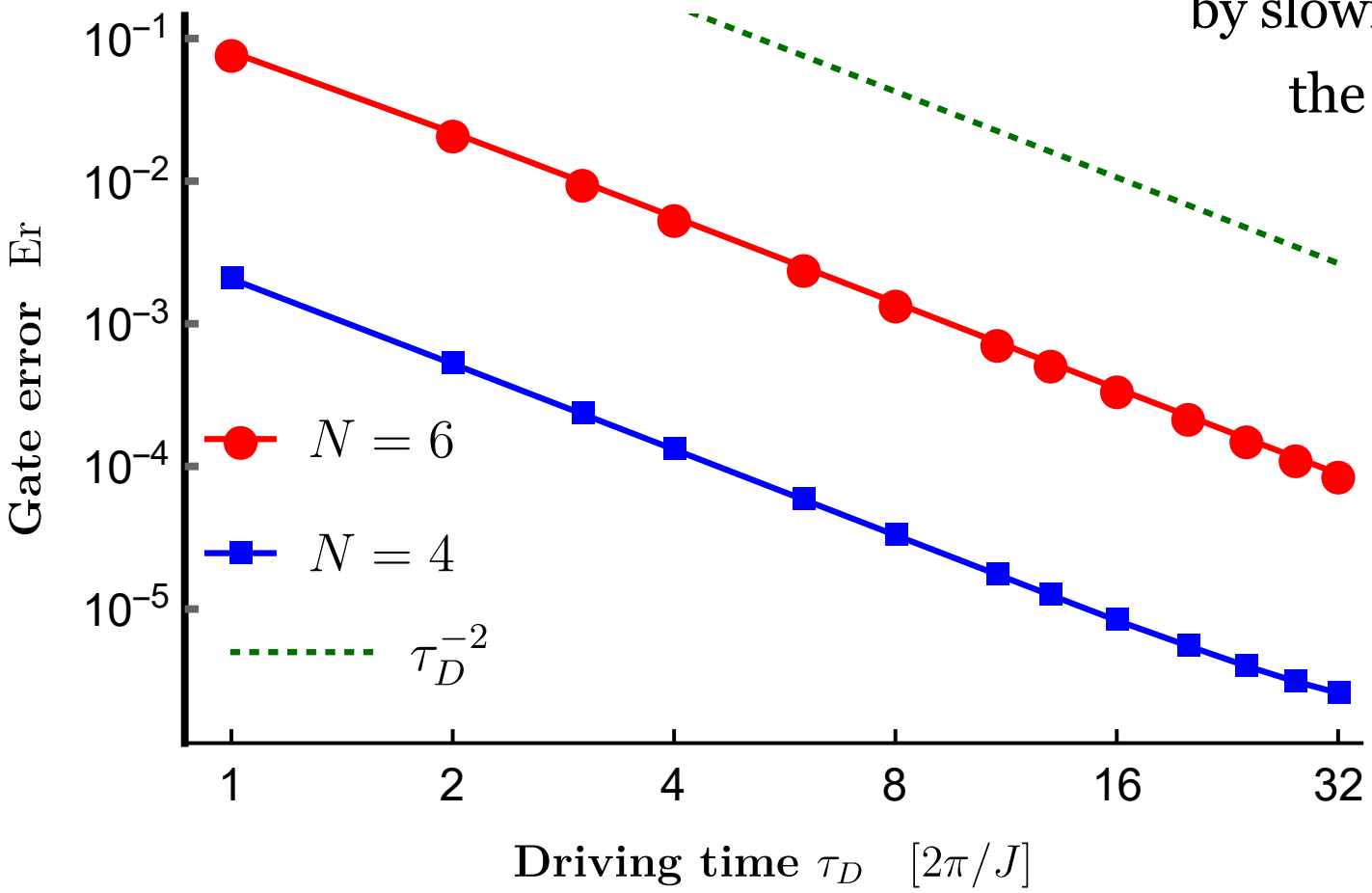
$$U_D(t) = \begin{pmatrix} \exp(-\frac{\pi i}{2}\frac{d}{\Delta}) & -\pi i(\frac{d}{\Delta})^2 \\ -\pi i(\frac{d}{\Delta})^2 & \exp(\frac{\pi i}{2}\frac{d}{\Delta}) \end{pmatrix}$$

$$\mathrm{Er} = 1 - \frac{1}{2}\left|Tr\left[U_D\right]\right|$$

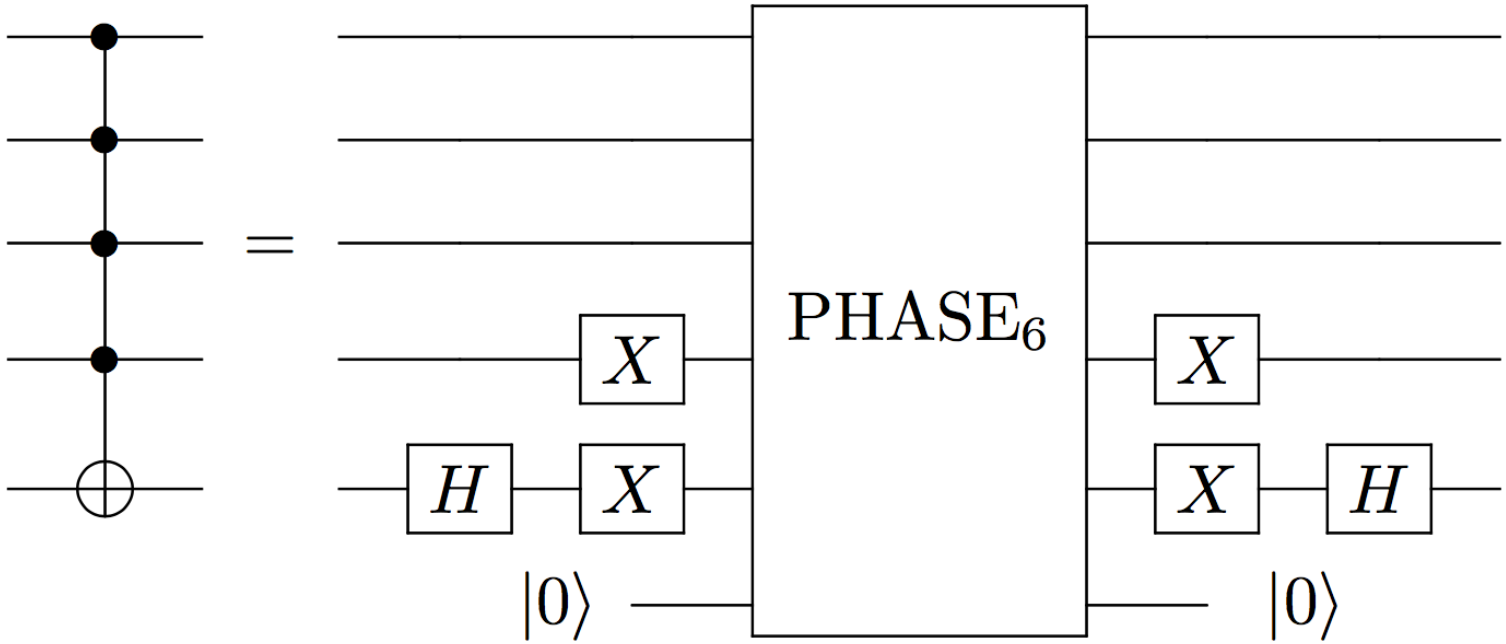
$$\approx \frac{1}{2}(\frac{\pi}{2})^2(\frac{d}{\Delta})^2 \propto \frac{1}{t^2}$$

gate fidelities for iSWAP$_4$ and iSWAP$_6$

gate fidelity enhanced by slowing down the resonant driving

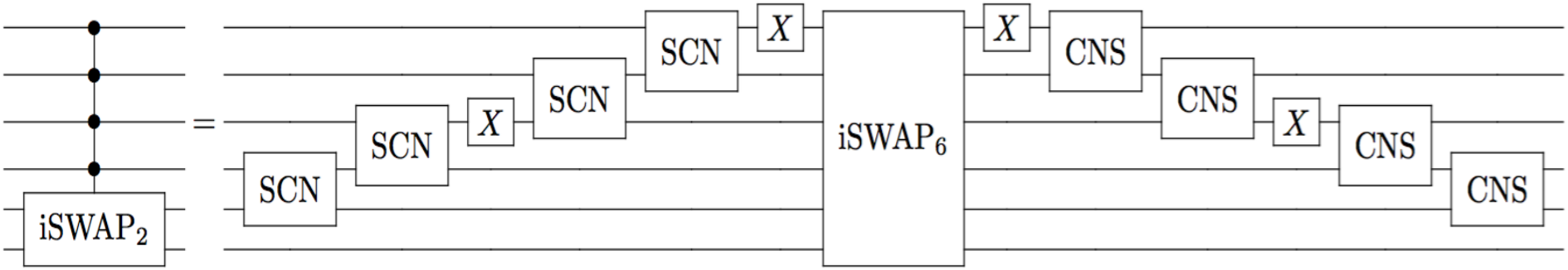


multi-qubit gates ...



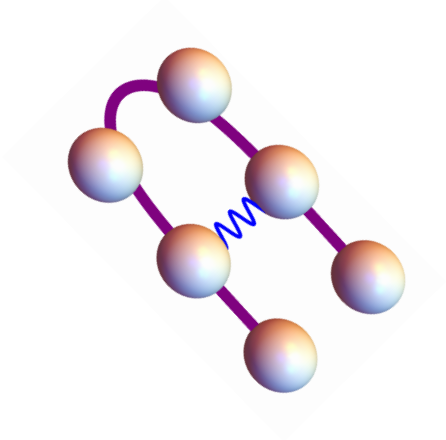
Toffoli-5 using double strength iSWAP$_6$ gate called PHASE$_6$

multi-qubit gates ...

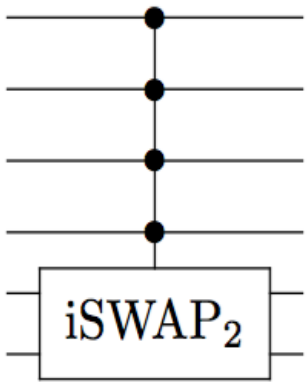


iSWAP$_2$ with 4 controls using iSWAP$_6$ gate

done & to be done



- ✓ exact eigengates giving fast quantum circuits for Krawtchouk eigenstates
- ✓ resonant driving targeting 2 out of 2^N states
- ✓ iSWAP_N reworked into multi-qubit gate with N-1 or N-2 controls.



- improve the resonant driving part (pulse shaping, correct for Lamb shifts)
- sensitivity to noise?
- window in N where protocol can be realistic?

- other incarnations of the strategy