Quenches in quantum field theory

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Talk presented at workshop on

"*Integrability in Low-Dimensional Quantum Systems*"

MATRIX Mathematical Research Institute

Melbourne/Creswick, 26 June - 21 July 2017

Outline

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Introduction

What is a quantum quench?

$$H_0 \underset{t=0}{\longrightarrow} H$$

Start evolution from ground state $|\Psi(0)\rangle$ of H_0

$$|\Psi(0)\rangle = \sum_n C_n |n\rangle$$

$$H|n\rangle = E_n|n\rangle$$

$$\langle\Psi(t)|\mathcal{O}|\Psi(t)\rangle = \sum_{n,m} C_n^* C_m e^{-i(E_m-E_n)t} \langle n|\mathcal{O}|m\rangle$$

If it approaches a stationary state

$$\langle\Psi(t)|\mathcal{O}|\Psi(t)\rangle \to \text{Tr } \rho_D \mathcal{O}$$

$$\rho_D = \sum_n |C_n|^2 \, |n\rangle\langle n| \quad \text{diagonal ensemble}$$

Global quantum quench:
H_0 and H are local, translationally invariant Hamiltonians.

Introduction

Why study quantum quenches?

1. Do quantum systems equilibrate and under what conditions?

$$\rho_D \sim \frac{1}{Z} \begin{cases} e^{-\beta H} & H \text{ non-integrable} \\ e^{-\sum_i \beta_i Q_i} & H \text{ integrable} \end{cases}$$

   What is the nature of steady state (Gibbs/generalised Gibbs)?

2. How does relaxation happen?
   - Weak/strong thermalisation
   - Relaxation time-scales
3. Universal features of out-equilibrium time-evolution?
   - e.g. light-cone evolution of entanglement and correlations Lieb-Robinson bounds
4. Consequences of integrability breaking
   - quantum equivalent of KAM theorem
   - prethermalisation
   - nature of cross-over between integrable and non-integrable behaviour

Introduction

QFT: universal description of long-distance behaviour

$\Rightarrow$ natural: quenches in statistical systems $\longrightarrow$ quenches in QFT

but they are also interesting in their own right.

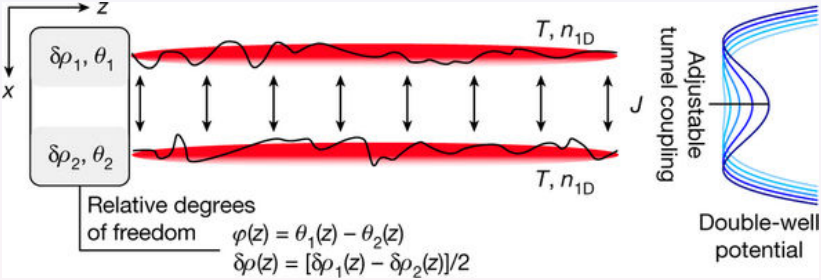
1. Issue of scales
   - sudden quench: short time scale τ
   - QFT: high energy cut-off Λ

   Validity: naively only for slow quenches (ramps) $\tau \gg \Lambda^{-1}$

2. Integrable quenches: what does it mean at all for a quench to be integrable?

3. Integrability breaking
   - prethermalisation?
   - perturbative/non-perturbative phenomena?

Experimental motivation



T. Schweigler et al., *Nature* **545** (2017) 323–326.

Ultracold gas of $^{87}_{37}$Rb atoms, confined to 2x1D: $\omega_\perp/\omega_\parallel \sim 10^3$

Relative phase $\varphi(x)$ and particle density difference $\delta\rho(x)$ described by sine-Gordon QFT:

$$H_{sG} = \int dx \left[g\delta\rho^2 + \frac{\hbar^2 n_{1D}}{4m} (\partial_x \varphi)^2 - 2\hbar J n_{1D} \cos\varphi \right]$$

Outline

What is an integrable quench in QFT?

Evolution after a quantum quench with post-quench Hamiltonian H

$$|\Psi(0)\rangle = \sum_n C_n|n\rangle$$

$$H|n\rangle = E_n|n\rangle$$

$$\langle\Psi(t)|\mathcal{O}|\Psi(t)\rangle = \sum_{n,m} C_n^* C_m e^{-i(E_m-E_n)t}\langle n|\mathcal{O}|m\rangle$$

C_n: overlaps.

What is an integrable quench? [Delfino, 2014; Schuricht 2015]

- H is integrable
- But maybe something must also be true for the C_n?

Examine $|\Psi(0)\rangle$ in massive QFT $\to$ basis of asymptotic states:

$$H|\theta_1,\dots,\theta_n\rangle = \left(\sum_{k=1}^{n} m\cosh\theta_k\right)|\theta_1,\dots,\theta_n\rangle$$

$$|\Psi(0)\rangle = \sum_{N=0}^{\infty}\frac{1}{N!}\int\frac{d\theta_1}{2\pi}\dots\frac{d\theta_N}{2\pi}K_N(\theta_1,\dots,\theta_N)|\theta_1,\dots,\theta_n\rangle$$

What is an integrable quench in QFT?

$$
\begin{aligned}
H|\theta_1,\ldots,\theta_n\rangle &= \left(\sum_{k=1}^{n} m\cosh\theta_k\right)|\theta_1,\ldots,\theta_n\rangle \\
|\Psi(0)\rangle &= \sum_{N=0}^{\infty}\frac{1}{N!}\int\frac{d\theta_1}{2\pi}\ldots\frac{d\theta_N}{2\pi}K_N(\theta_1,\ldots,\theta_N)|\theta_1,\ldots,\theta_n\rangle \\
&= \sum_{N=0}^{\infty}\frac{1}{N!}\int\frac{d\theta_1}{2\pi}\ldots\frac{d\theta_N}{2\pi}K_N(\theta_1,\ldots,\theta_N)Z^\dagger(\theta_1)\ldots Z^\dagger(\theta_N)|0\rangle
\end{aligned}
$$

- What should be true for this state for the quench to be "integrable"?
- Can we determine the K_N?

In analogy with integrable boundaries
[Ghoshal & Zamolodchikov, 1993]
: a quench is integrable whenever H is integrable and

$$
|\Psi(0)\rangle = \mathcal{N}\exp\left(\int_0^\infty d\theta K(\theta)Z^\dagger(-\theta)Z^\dagger(\theta)\right)|0\rangle
$$

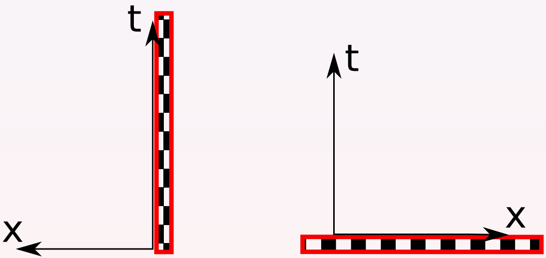
Boundary state approach to quenches

Quantum quenches in QFT [Calabrese & Cardy, 2006]

Case: post-quench Hamiltonian is CFT

$$|\Psi(0)\rangle = e^{-H\tau_0}|B\rangle$$

$|B\rangle$: conformally invariant boundary condition in crossed channel



Boundary condition   Crossed channel:boundary state

τ_0: extrapolation time – normalizability/finite energy density!

More general: involve all irrelevant operators $\tilde{\Phi}_k(x)$ [Cardy, 2015]

$$|\Psi(0)\rangle = e^{-\sum_k \tau_k \int dx \tilde{\Phi}_k(x)}|B\rangle$$

Squeezed initial state

Integrable quench:

$$|\Psi(0)\rangle = \mathcal{N}\exp\left(\int_0^\infty d\theta K(\theta) Z^\dagger(-\theta) Z^\dagger(\theta)\right)|0\rangle$$

$$K(\theta) = S(2\theta)K(-\theta) \text{ but: } K(\theta) \neq R(i\pi/2-\theta)$$

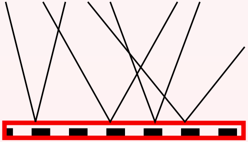
Extrapolation times: exponential suppression for high momenta

$$|\Psi(0)\rangle = \exp\left(-\sum_s \tau_s Q_s\right)|B\rangle$$

$$\Downarrow \quad Q_s = \int \frac{d\theta}{2\pi} q_s(\theta) Z^\dagger(\theta) Z(\theta)$$

$$K(\theta) = e^{-2E(\theta)\tau(\theta)} K_B(\theta)$$

$\tau(\theta)$: momentum-dependent extrapolation time



The importance of overlaps

Overlaps are inputs to many approaches to quenches

1. Thermodynamic Bethe Ansatz [Fioretto & Mussardo, 2009]
2. Quench action method [Caux & Essler, 2013]
3. Form factor methods [Bertini, Essler & Schuricht, 2014]
4. Semiclassical method [Kormos & Zaránd, 2015]

$\Rightarrow$ need to determine overlaps! But getting them is very difficult...

Lieb-Liniger, XXZ chain: for some initial states from Bethe Ansatz
[XXZ: Kozlowski and Pozsgay, 2012; Pozsgay, 2013;
De Nardis, Wouters, Brockmann & Caux, 2014
Piroli & Calabrese, 2014
LL: Nardis, Wouters, Brockmann & Caux, 2013]

What about field theory?

Mass quenches to sinh-Gordon theory

$$H = \int dx \left[\frac{1}{2}\pi^2 + \frac{1}{2}(\partial_x\phi)^2 + \frac{\mu^2}{g^2}\cosh g\phi(x)\right]$$
$$[\phi(t,x),\pi(t,y)] = i\delta(x-y)$$

Spectrum: single particle of mass m with S matrix

$$S(\theta,B) = \frac{\tanh\frac{1}{2}(\theta - i\frac{\pi B}{2})}{\tanh\frac{1}{2}(\theta + i\frac{\pi B}{2})} \qquad B(g) = \frac{2g^2}{8\pi + g^2}$$

Quench: free boson of mass m_0 to sinh-Gordon with coupling g and mass m

$$(m_0, g=0) \underset{t=0}{\longrightarrow} (m,g)$$

Time evolution:

$$\frac{d\mathcal{O}}{dt} = i[H,\mathcal{O}]$$

Quench: jump in $H \Rightarrow$ continuity in time-dependence of operators

$$\phi(x,t\to 0^-) = \phi(x,t\to 0^+) \quad \text{and} \quad \pi(x,t\to 0^-) = \pi(x,t\to 0^+)$$

Infinite number of infinite integral equations

Infinitely many equations: by taking all possible matrix elements

$$\langle \theta_1, \ldots, \theta_N | \left\{ \hat{\phi}(p) + \frac{1}{E_0(p)} \left[\hat{\phi}(p), H \right] \right\} | \Psi(0) \rangle = 0$$

and infinitely long integral equations by writing

$$\begin{aligned} |\Psi(0)\rangle &= \sum_{r=0}^{\infty} \frac{1}{r!} \int \prod_{j=1}^{r} \frac{d\theta_j}{2\pi} K_r(\theta_1, \ldots, \theta_r) |\theta_1, \ldots, \theta_r\rangle \\ & K_r(\ldots, \theta_i, \theta_{i+1}, \ldots) = K_r(\ldots, \theta_{i+1}, \theta_i, \ldots)\, S(\theta_{i+1} - \theta_i) \end{aligned}$$

Translational invariance:

$$K_r(\theta_1, \ldots, \theta_r) \propto \delta\Big(\sum_{j=1}^{r} m \sinh \theta_j\Big)$$

$\Rightarrow$ equations are only nontrivial for

$$p = -\sum_{j=1}^{N} m \sinh \theta_j$$

Extensivity for local charges

Cumulant form:

$$|\Psi\rangle = \exp\left(\sum_{r=1}^{\infty}\int \underbrace{\tilde{K}_r(\theta_1,\theta_2,\ldots,\theta_r)}_{\text{cumulants of } K_r}\prod_{i=1}^{r} Z^\dagger(\theta_i)d\theta_i\right)|0\rangle$$

Expectation value of local charges must be extensive:

$$\langle Q_s\rangle = \frac{\langle\Psi|Q_s|\Psi\rangle}{\langle\Psi|\Psi\rangle} \propto \text{volume} \qquad Q_s = \int d\theta\, q_s(\theta)Z^\dagger(\theta)Z(\theta)$$

Theorem: extensivity $\Rightarrow$ $\tilde{K}_r$ can only contain a single δ-function!
Corollary: assuming pair structure

$$K^{\Psi}_{2r}(\theta_1,\theta_2,\ldots,\theta_r) \quad\propto\quad \left(\prod_{i=1}^{r}\delta(\theta_{2i+1}+\theta_{2i})\ldots\right)_{\text{sym}}$$

extensivity implies

$$K_{2r} = 0 \qquad r > 1$$

$\Rightarrow$ we have an "integrable quench"

$$|\Psi(0)\rangle = \exp\left(\int_0^{\infty} K(\theta)Z^\dagger(-\theta)Z^\dagger(\theta)\right)|0\rangle$$

The Ansatz

Ansatz [Sotiriadis, GT and Mussardo, 2014]:

$$K(\theta) = K^{free}(k)K_D(\theta) = \frac{E_0(\theta) - E(\theta)}{E_0(\theta) + E(\theta)} K_D(\theta)$$

$$E(\theta) = m\cosh\theta \quad , \quad E_0(\theta) = \sqrt{m^2 \sinh^2\theta + m_0^2}$$

$$K_D(\theta) = i\tanh(\theta/2)\frac{\cosh(\theta/2 - i\pi B/8)\sinh(\theta/2 + i\pi(B+2)/8)}{\sinh(\theta/2 + i\pi B/8)\cosh(\theta/2 - i\pi(B+2)/8)}$$

Ghoshal-Zamolodchikov solution for Dirichlet BC $\varphi = 0$

Evidence:
numerically solves the first two members of the infinite hierarchy [GT, Horváth and Sotiriadis, 2016]

Limitation: no good theoretical argument for pair structure yet

- Expected to be good approximation for small quenches
- GZ solution: valid for infinitely large quench $m_0/m \gg 1$
- Heuristic arguments by analogy to integrable boundary states
- Numerical evidence from second member of hierarchy

Outline

Scaling Ising field theory

$$H_{ISC} = \sum_{i=1}^{N} \left(\sigma_i^x \sigma_{i+1}^x + h_z \sigma_i^z + h_x \sigma_i^x \right)$$

$\downarrow$ continuum limit

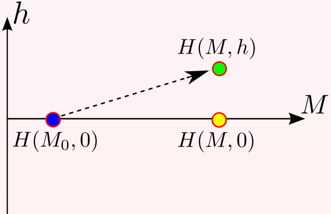
$$H_{IFT} = \frac{1}{2\pi} \int dx \left[\frac{i}{2} \psi(x) \partial_x \psi(x) - \frac{i}{2} \bar{\psi}(x) \partial_x \bar{\psi}(x) - iM \bar{\psi}(x) \psi(x) \right]$$
$$+ h \int dx \sigma(x)$$

Idea: use Hilbert space of free massive fermion in volume L

Truncated Free Fermionic Space Approach (TFFSA)

[Fonseca and Zamolodchikov, 2001]

FM/PM phase distinguished by FF Hilbert space content



Does this work at all?

Energy cutoff on fermionic space: Λ vs sudden quench $\tau^{-1} = \infty$!
Integrable case: $M_0 \to M$ $(h = 0)$

[Mestyán, Rakovszky, Collura, Kormos and GT, 2016]

Form factor methods
[Schuricht and Essler, 2012]

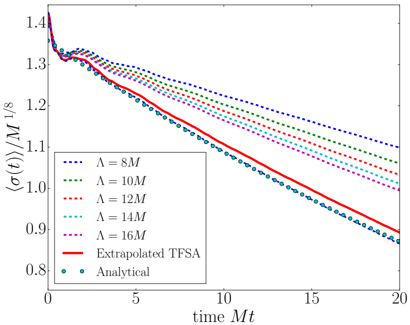
$\langle \sigma(t) \rangle = \bar{\sigma} e^{-t/\tau}$ for large t

$$\tau^{-1} = \frac{2M}{\pi} \int_0^\infty d\theta \, |K(\theta)|^2 \sinh\theta + O\left(K^6\right)$$

$$K(\theta) = \tan\left[\frac{1}{2}\arctan\left(\sinh\theta\right) - \frac{1}{2}\arctan\left(\frac{M}{M_0}\sinh\theta\right)\right]$$

$$\bar{\sigma} = \langle 0|\sigma(x)|0\rangle = \bar{s} M^{1/8}$$

$$\bar{s} = 2^{1/12} e^{-1/8} \mathcal{A}^{3/2}$$



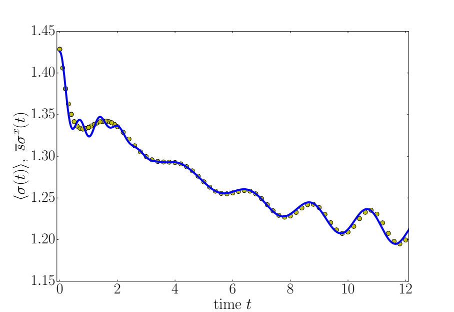
Ferromagnetic quench $M_0 = 1.5M$

Comparison to iTEBD

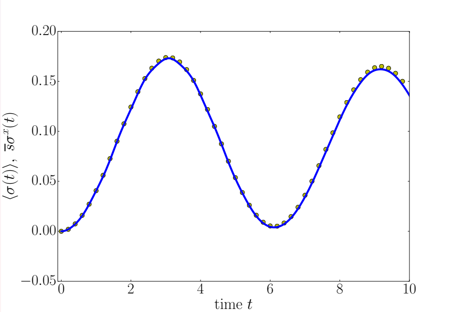
Non-integrable quenches: no analytic results, but one can use iTEBD = infinite volume Time Evolving Block Decimation

$$M = 2J\,|1 - h_z| \qquad a = 2/J$$

$$\sigma(na) = \bar{s}J^{1/8}\sigma_n^x \qquad \bar{h} = hM^{-15/8} = \frac{2^{-7/8}}{\bar{s}}\,(1 - h_z)^{-15/8}\,h_x$$



Ferromagnetic quench
$(1.5M, 0) \to (M, \bar{h} = 0.1)$



Paramagnetic quench
$(1.5M, 0) \to (M, \bar{h} = 0.05)$

Quenches with broken integrability

Ferromagnetic phase: confinement (no prethermalisation)
M. Kormos, M. Collura, GT and P. Calabrese, *Nat. Phys.* **13** (2017) 246-249.

Paramagnetic phase: oscillations, again no sign of prethermalisation

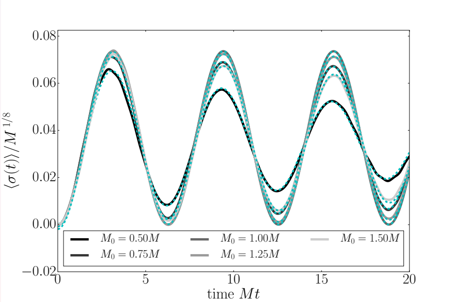
$$\langle\sigma(t)\rangle = Ae^{-t/\tau}(1-\cos\omega t)$$

Amplitude prediction
[Delfino, 2014; Delfino & Viti, 2017]

$$A = \frac{2h}{M^2}\left|F_{1,0}\right|^2$$
$$F_{1,0} = \langle A(0)|\sigma|0\rangle$$

2nd order FFPT [GT, 2009]

$$\omega = M\left(1+\delta\bar{h}^2\right)$$
$$\delta = 10.1593\ldots$$



Paramagnetic quench
$(M_0, 0) \to (M, \bar{h} = -0.01)$

Numerics: $\delta = 10.07\ldots$

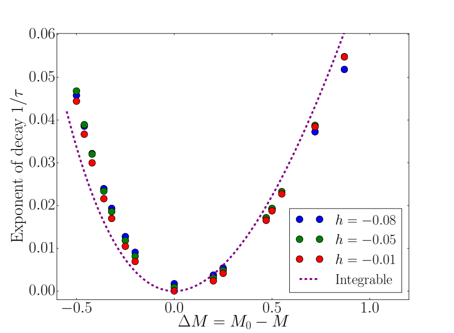
Quenches with broken integrability

Damping in paramagnetic phase:
for small h given by integrable result

Form factor methods
[Schuricht and Essler, 2012]

$$\tau^{-1} = \frac{2M}{\pi} \int_0^\infty d\theta \, |K(\theta)|^2 \sinh\theta + O\left(K^6\right)$$

$$K(\theta) = \tan\left[\frac{1}{2}\arctan\left(\sinh\theta\right) - \frac{1}{2}\arctan\left(\frac{M}{M_0}\sinh\theta\right)\right]$$



Paramagnetic quench
$(M_0, 0) \to (M, \bar{h})$

Outline

Overlaps in sine-Gordon theory

Sine-Gordon numerics: truncated conformal space approach

[TCSA: Yurov and Zamolodchikov, 1991; sG: Feverati, Ravanini and GT, 1998]

$$H = \underbrace{\int dx \left[\frac{1}{2} : (\partial_t \Phi)^2 + (\partial_x \Phi)^2 : \right]}_{\text{CFT}} \underbrace{- \frac{\lambda}{2} \int dx \left(V_1 + V_{-1} \right)}_{\text{perturbation}}$$

$$V_a =: e^{ia\beta\Phi} : \qquad , \qquad \Delta_a = \frac{a^2 \beta^2}{8\pi}$$

$$\lambda = \frac{2\Gamma(\Delta_1)}{\pi\Gamma(1-\Delta_1)} \left(\frac{\sqrt{\pi}\Gamma\left(\frac{1}{2-2\Delta_1}\right) M}{2\Gamma\left(\frac{\Delta_1}{2-2\Delta_1}\right)} \right)^{2-2\Delta_1} \qquad M\text{: soliton mass}$$

Breather overlap prediction: analytic continuation from sinhG

$$K_{B_1 B_1}(\vartheta) = \frac{E_0(\vartheta) - E(\vartheta)}{E_0(\vartheta) + E(\vartheta)} K_D(\vartheta) \qquad \xi = \frac{\beta^2}{8\pi - \beta^2}$$

$$K_D(\vartheta) = i \tanh\left(\frac{\vartheta}{2}\right) \frac{\cosh\left(\frac{\vartheta}{2} + \frac{i\pi\xi}{4}\right)}{\sinh\left(\frac{\vartheta}{2} - \frac{i\pi\xi}{4}\right)} \frac{\sinh\left(\frac{\vartheta}{2} + \frac{i\pi(1-\xi)}{4}\right)}{\cosh\left(\frac{\vartheta}{2} - \frac{i\pi(1-\xi)}{4}\right)}$$

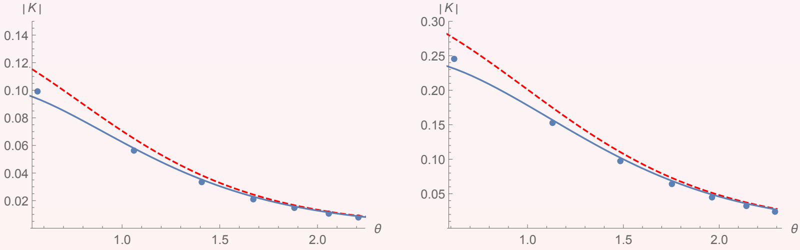
Overlaps from finite volume

$$|B\rangle = |0\rangle + \sum_{n=1}^{\infty} \int \prod_{i=1}^{n} \frac{d\vartheta_i}{2\pi} K_n(\vartheta_1, ...\vartheta_n)\delta\left(\sum_{i=1}^{n} m \sinh\vartheta_i\right) |\vartheta_1 ..., \vartheta_n\rangle$$

$\Downarrow N_n$: state density factor

$$|B\rangle_L = |0\rangle_L + \sum_{n=1}^{\infty} \sum_{I_1,...I_n}' N_n K_n(\vartheta_1^*, ...\vartheta_n^*)|I_1, ..., I_n\rangle_L$$

Instead of free to sG: sG-sG mass quench $M_0 \to M$



[D.X. Horváth and GT, 2017]

Why does TCSA work?

Recall cutoff-sudden quench problem!

Assume for simplicity a squeezed state form

$$|\Psi(0)\rangle = \mathcal{N} \exp\left(\int_0^\infty d\theta K(\theta) Z^\dagger(-\theta) Z^\dagger(\theta)\right)|0\rangle$$

Vacuum overlap

$$\log \mathcal{N} = -\frac{1}{2} L \int_0^\infty \frac{dq}{\pi} \log\left(1 + |K(q)|^2\right)$$

TCSA works until low-energy states dominate:

$$-\log \mathcal{N} \lesssim 1 \;:\; L < L_{\text{crit}}$$

But: QFT in volume L gives back $L = \infty$ well if $ML \gg 1$!

Remark: e.g. in our Ising calculations $ML_{\text{crit}} > 300$

Conclusion: TCSA works well for quenches producing low post-quench energy density.

Why does QFT work for sudden quenches?

Warning: the following is an intuitive argument!

Initial state normalizable ⇒ integral

$$\int_0^\infty dq\,|K(q)|^2$$

must converge at upper limit!
In fact, typically

$$|K(q)|^2 \propto \frac{1}{q^4} \qquad \text{for large q}$$

(free boson behaviour).
So high energy states are suppressed even for a sudden quench
⇒ field theory works if cut-off is sufficiently high.

However: power-like suppression!

⇒ as a consequence e.g. TCSA needs RG improvement!

Outline

Summary

1. Field theory can be used to model even sudden quenches of many-body systems.

2. Sudden quenches in QFT: a variety of available methods
   - Form factor expansions
   - Truncated Hamiltonian approaches (RG improved)
   - Semiclassical approach
   - Exact results for "integrable" quenches (squeezed initial states)

3. A few interesting problems
   - What is the physical condition for a quench to be integrable?
   - Can we get more exact information for them?
   - How to compute K for a given integrable quench?
   - Applications to experiments, integrability breaking etc.