Hypergeometric motives
and
Calabi–Yau differential equations

Week 1: January 9–13
Hypergeometric Motives and Finite Hypergeometric Functions

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00-9:00	Breakfast	Breakfast	Breakfast	Breakfast	Breakfast
9:30-10:30	IT Assistance (MATRIX house)	Ramakrishna	HGM seminar	Beukers	Beukers
10:30-11:00	Coffee Break	Coffee Break		Coffee Break	Coffee Break
11:00-12:00	Welcome Speech and Program Discussion	Collaboration	Free Time	Collaboration	Collaboration
12:00-14:00	Lunch Break	Lunch Break		Lunch Break	Lunch Break
14:00-15:00	Broadhurst	HGM seminar		HGM seminar	HGM seminar
15:00-15:30	Afternoon Break	Afternoon Break		Afternoon Break	Afternoon Break
15:30-17:00	Collaboration	Collaboration		Collaboration	Collaboration
17:00		Cheese and Wine			
19:00	Dinner	Dinner	Dinner	Welcome Dinner	Dinner

Frits Beukers *Finite hypergeometric sums and Dwork cohomology* (minicourse)
HGM (hypergeometric motives) seminar is run by **Fernando Rodriguez Villegas, David Roberts** and **Mark Watkins**

Week 2: January 16–20
Workshop

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00-9:00	Breakfast	Breakfast	Breakfast	Breakfast	Breakfast
9:30-10:30	Villegas	Salerno	Beukers	Voight	Delaygue
10:30-11:00	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
11:00-12:00	Broadhurst	Roberts	Osburn	Swisher	van Straten
12:00-14:00	Lunch Break	Lunch Break	Free Time	Lunch Break	Lunch Break
14:00-15:00	Sebbar	Wan		Watkins	Vlasenko
15:00-15:30	Afternoon Break	Afternoon Break		Afternoon Break	Afternoon Break
15:30-16:30	Yang	Tu		Frechette	Collaboration
16:30-17:30	Ramakrishna	Photo (16:45) Cheese and wine		Achinger	
17:30-19:00				Villegas	
19:00	Dinner	Dinner	Dinner	Dinner	Dinner

Piotr Achinger *Canonical liftings of ordinary Calabi–Yau varieties*

Classical Serre–Tate theory discusses deformations of ordinary abelian varieties. It implies that every such variety has a canonical lift to characteristic zero. Variants of this theory have been obtained for ordinary K3 surfaces by Nyygard and Ogus, and for ordinary curves by Mochizuki. I will report on a joint project in progress with Maciej Zdanowicz, in which we construct canonical liftings of ordinary Calabi–Yau varieties. Our version of Serre–Tate theory is 'weak' in the sense that it only yields liftings modulo p^2.

Frits Beukers *Explicit hypergeometric motives*

It is well-known that values of finite hypergeometric functions occur as summand of point counting formulas on varieties over finite fields. They arise as traces of Frobenius operators on some suitable cohomology spaces. In this talk we like make these things explicit and extend the definition of finite hypergeometric functions somewhat.

David Broadhurst *Walks at sunrise with Gauss, Bessel, Kloosterman, Calabi and Yau*

Returning walks on lattices and Feynman integrals from so-called sunrise diagrams provide fascinating links between the arithmetic-geometric mean of Gauss, moments of Bessel functions, L-series of modular forms, Kloosterman sums, logarithmic Mahler measures, Calabi–Yau differential equations, their mirror maps, Yukawa couplings and instanton numbers. I shall outline some of these connections.

Eric Delaygue *Arithmetic properties of hypergeometric mirror maps and Dwork congruences*

Mirror maps are power series which occur in Mirror Symmetry as the inverse for composition of $q(z) = \exp(f(z)/g(z))$, called local q-coordinates, where f and g are particular solutions of the Picard–Fuchs equation associated with certain one-parameter families of Calabi–Yau varieties. In several cases, it has been observed that such power series have integral Taylor coefficients at the origin. In the case of hypergeometric equations, I will discuss p-adic tools and techniques that enable one to prove a criterion for the integrality of the coefficients of mirror maps. This is a joint work with T. Rivoal and J. Roques.

Sharon Frechette *Finite-field Appell–Lauricella hypergeometric functions*

Using the dictionary between classical and finite field hypergeometric functions of one variable, recently developed by Fuselier, Long, Ramakrishna, Swisher and Tu, we define and develop a theory of multivariable finite-field hypergeometric functions. We establish transformations and symmetries of these multivariable functions, building analogues to classical results such as the cubic transformation for Appell hypergeometric functions shown by Koike and Shiga. We also explore the geometry of finite-field Appell hypergeometric functions through their relationship to the generalized family of Picard curves. This is joint work with Holly Swisher and Fang-Ting Tu.

Robert Osburn *Sequences, modular forms and cellular integrals*

It is well-known that the Apéry sequences which arise in the irrationality proofs for $\zeta(2)$ and $\zeta(3)$ satisfy many interesting arithmetic properties and are related to pth Fourier coefficients of modular forms. We discuss how these connections persist in the general context of sequences associated to Brown's cellular integrals.

Ravi Ramakrishna *Some supercongruences for truncated hypergeometric series*

We give a number of congruences between terminating hypergeometric series and quotients of p-adic gamma functions that are stronger than those one can expect to prove using commutative formal group laws. We prove them by using classical hypergeometric transformation formulae.

David Roberts *Hypergeometric motives and an unusual application of the Guinand–Weil–Mestre explicit formula*

We use the hypergeometric motive

$$M = H\left(\begin{matrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}, 1\right).$$

to illustrate the *Magma* hypergeometric motive package and how far it goes beyond classical settings. This motive M first appears in the cohomology of a dimension 15 variety. It has degree fourteen and naturally decomposes into a degree six and degree eight motive, $M = M_6 \oplus M_8$. We review the Guinand-Weil-Mestre explicit formula, and then use it to study the factorization $L(M, s) = L(M_6, s)L(M_8, s)$.

Adriana Salerno *Arithmetic Mirror Symmetry of $K3$ Surfaces and Hypergeometric Functions*

Mirror symmetry predicts surprising geometric correspondences between distinct families of algebraic varieties. In some cases, these correspondences have arithmetic consequences. Among the arithmetic correspondences predicted by mirror symmetry are correspondences between point counts over finite fields. In particular, we explore closed formulas for the point counts for our alternate mirror families of $K3$ surfaces, their relation to their Picard–Fuchs equations and hypergeometric functions.

This is joint work with: Charles Doran (University of Alberta, Canada), Tyler Kelly (University of Cambridge, UK), Steven Sperber (University of Minnesota, USA), John Voight (Dartmouth College, USA), and Ursula Whitcher (University of Wisconsin, Eau Claire, USA).

Abdellah Sebbar *Schwarzian differential equations and equivariant functions*

We study some Schwarz differential equations having some modular forms as coefficients. We are led to the notion of equivariant functions on the upper half-plane. These are meromorphic functions that commute with the action of a Fuchsian group. We use the properties of the Schwarz derivative to parameterize all the equivariant functions and give some applications. A generalization involving representations of the Fuchsian group is also discussed.

Duco van Straten *Calabi–Yau equations and L-functions*

Holly Swisher *Hypergeometric Functions over Finite Fields*

Based on the fundamental developments by many people including Evans, Greene, McCarthy, and others, we consider period functions for hypergeometric type algebraic varieties over finite fields and consequently study hypergeometric functions over finite fields in a manner that is parallel to the classical hypergeometric functions. We present a systematic approach for translating certain classical hypergeometric series transformations to the finite field setting via an explicit dictionary. Some specific examples will be discussed. This is joint work with Jenny Fuselier, Ling Long, Ravi Ramakrishna, and Fang-Ting Tu.

Fang-Ting Tu *Supercongruences Occurred to Rigid Hypergeometric Type Calabi–Yau Threefolds*

Due to Gouvea and Yui, every rigid Calabi–Yau manifold over $\mathbb{Q}$ is modular. We are interested in the rigid Calabi–Yau manifolds defined over $\mathbb{Q}$ whose Picard–Fuchs equations are corresponding to the functions $F(a, b, 1)$, where $F(a, b, z)$ is the ${}_4F_3$-hypergeometric function with argument z and parameters $1/a, 1-1/a, 1/b, 1-1/b, 1, 1, 1$ with $a, b = 2, 3, 4, 6$. Rodriguez Villegas has conjectured the explicit cuspidal Hecke eigenforms corresponding to these rigid Calabi–Yau manifolds. In this talk, we will discuss certain type supercongruences occurred to these rigid Calabi–Yau manifolds conjectured by Rodriguez Villegas.

Fernando Rodriguez Villegas *Motivic supercongruences* (Thursday)

Masha Vlasenko *On p-adic unit root formulas*

John Voight *Triangular modular curves*

The simplest kind of tesselation of the upper half-plane are given by triangles; the associated group of symmetries defines a Fuchsian group, called a triangle group. The most famous triangle group is $SL_2(\mathbb{Z})$, but in general triangle groups are not arithmetic. Nevertheless, there are naturally defined congruence subgroups of triangle groups, and the quotients by these subgroups we call "triangular modular curves" because they share many pleasing properties in common with the usual modular curves: most relevant for this workshop, they parametrize hypergeometric abelian varieties with level structure. In this talk, we give an expository account of the theory, reporting on joint work with Pete L. Clark where we exhibit the first arithmetic properties of these curves as well as joint work with Robert Kucharczyk where we present a notion of "canonical model" for them.

James Wan *Ramanujan-type series for* $1/\pi$

We will look at some new Ramanujan-type series for $1/\pi$, the general form of which is

$$\sum_{n=0}^{\infty} H_n(a_0 n + b_0) z_0^n = \frac{1}{\pi},$$

where a_0, b_0, z_0 are algebraic numbers and H_n is an arithmetic sequence (for instance, a product of binomial coefficients). These series are intimately connected with hypergeometric functions.

Mark Watkins *Jacobi sum motives and Grossencharacters*

We define Jacobi sum motives following Weil, and implement his 1952 paper to recognize them via the associated Grossencharacter. We then describe how to use this to compute tame prime Euler factor information for hypergeometric motives. This is joint work with David Roberts.

Yifan Yang *Special values of hypergeometric functions and periods of CM elliptic curves*

Let X be the Atkin–Lehner quotient of the Shimura curve $X_0^6(1)$ associated to a maximal order in an indefinite quaternion algebra of discriminant 6 over $\mathbb{Q}$. By realizing modular forms on X in two ways, one in terms of hypergeometric functions and the other in terms of Borcherds forms, and using Schofers formula for values of Borcherds forms at CM-points, we obtain special values of certain hypergeometric functions in terms of periods of elliptic curves over $\overline{\mathbb{Q}}$ with complex multiplication.

Week 3: January 23–27
Arithmetic and Combinatorial Properties of Periods of Calabi–Yau manifolds

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00-9:00	Breakfast	Breakfast	Breakfast	Breakfast	Breakfast
9:30-10:30	Discussion	Sebbar	van Straten	van Straten	Broadhurst
10:30-11:00	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
11:00-12:00	Zudilin	Collaboration	Vlasenko	Salerno	Delaygue
12:00-14:30	Lunch Break	Lunch Break	Free Time	Lunch Break	Farewell Lunch
14:30-15:30	Delaygue	Warnaar		Osburn	Departure
15:30-16:00	Afternoon Break	Afternoon Break		Afternoon Break	
16:00-17:00	Voight	Collaboration		Collaboration	
19:00	Dinner	Dinner	Dinner	Dinner	

David Broadhurst *20 Bessels (in progress)*

Eric Delaygue *Arithmetic properties of CY equations* (Monday)
and *Dwork's congruences* (Friday)

Abdellah Sebbar *Classification of modular subgroups*

Robert Osburn *An example of motivic supercongruences*

Adriana Salerno *Multiple zeta values (after Écalle)*

Duco van Straten *More on CY differential equations* (minicourse)

Masha Vlasenko *Formal groups informally*

John Voight *Belyi maps*

Ole Warnaar *Hypergeometric integrals*

Wadim Zudilin *Supercongruence conjectures for* ${}_nF_{n-1}$