MEAN CURVATURE FLOW (MCF) WITH FREE BOUNDARIES

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I will give an introductory course in Mean Curvature Flow (MCF) with Free boundaries with a time frame of 5-7 hours (some details will be given and some will be left as exercises). This course is not intended as a comprehensive study of all results and references. Many will be omitted due to the time constraint.

I start with a brief introduction to MCF containing definitions, examples, homothetic solutions and graph solutions (following mainly the book of K. Ecker "Regularity Theory for MCF"). In parallel with this first part I will present a short introduction to the geometry of hypersurfaces. Then I give an overview of how different geometric quantities behave under the flow.

Next we discuss two classical results on Global Behaviour of the flow. Firstly we discuss the case of convex, embedded, compact hypersurfaces contracting to round points in finite time by Mean Curvature Flow (Huisken / Gage and Hamilton). These are Type I singularities.

The second result is the long time existence of graphical entire hypersurfaces under MCF (Ecker and Huisken). Related to these I also present the Local Smooth Extension of a solution (based on PDE techniques).

If time permits I will include discussions on the Monotonicity formula for MCF (Huisken).

In the second part of the course we will be focusing on the Free Boundary setting. We introduce the boundary value problem and give some examples.

Similarly to the compact setting we give theorems on global behaviour. Convex, embedded, compact hypersurfaces inside an n-sphere with free boundary on the n-sphere, or with boundary on a hyperplane, contract to hemispherical points in finite time by MCFwFB (Stahl). In the FB case a continuation criteria is also obtained (Stahl). Entire graphs in a half space exist for all times (W.). Two other long time existence results are obtained in the case of FB hypersurfaces outside spheres (W.-Wheeler) and inside a torus (Lambert).

A Free Boundary analogue of the Monotonicity Formula is also available (Buckland).

When continuation criteria fails, singularities appear. Time permitting, I will discuss some types and properties of these (Buckland, Edelen, Wheeler-W.).