From finite honest times to optional semimartingales of class-(Σ)

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Structure of the Talk

- Overview
- Stochastic calculus for làglàd processes.
- The additive and multiplicative decomposition.
- Optional Semimartingale of class-(Σ).

Overview

Notation and Assumptions

We work on the usual filtered probability space $(\Omega, \mathcal{A}, \mathbb{F}, \mathbb{P})$, where $\mathbb{F} := (\mathcal{F}_t)_{t \geq 0}$ denotes a filtration satisfying the usual conditions,

For any stochastic process X, we set

- $X_{0-} = 0$ and $X_\infty := \lim_{t \to \infty} X_t$.
- $\overline{X}_t := \sup_{s \leq t} X_s$.
- the left and right limit of X to be X_- and X_+.

Overview

Given a random time τ, we are interested in the supermartingales

$$Z_t := \mathbb{P}(\tau > t \,|\, \mathcal{F}_t)$$
$$\widetilde{Z}_t := \mathbb{P}(\tau \geq t \,|\, \mathcal{F}_t)$$

- Z is called the Azéma supermartingale or the survival process.
- Used in pricing of credit risk instruments.
- Arbitrage in insider trading models (Enlargement of filtration)

Remark

- *In most examples $\widetilde{Z} = Z$ (Brownian filtration).*
- *It appears that $\widetilde{Z}$ is easier to work with? but NOT càdlàg.*

Honest Times

Definition

A random time τ is said to be a honest time if there exist an optional set $\Gamma \in \mathcal{O}(\mathbb{F})$ such

$$\tau(\omega) = \sup(s : (s, \omega) \in \Gamma)$$

Lemma (Jeulin [J])

For a finite honest time τ, one can take $\Gamma = \{\widetilde{Z} = 1\}$. That is any finite honest time τ can be represented as

$$\tau = \sup\{s : \widetilde{Z}_s = 1\}$$

It is a last passage time!!

Example - Path Decomposition for Brownian Motion

- For $a > 0$, let $T_a^* = \inf\{s : |B_s| = a\}$ and set

$$\tau := \sup\{s \leq T_a^* : B_s = 0\} = \sup\{s : B_{s \wedge T_a^*} = 0\}$$

- The Azéma supermartingale $Z_t = \mathbb{P}(\tau > t \,|\, \mathcal{F}_t)$ is given by

$$Z_t = 1 - \frac{1}{a}|B_{t \wedge T_a^*}|$$

- In this example $\widetilde{Z} = Z$.

Path Decomposition for Brownian Motion

- Apply Itô-Tanaka's formula, we obtain

$$1 - Z_t = \left(\frac{1}{a} \int_0^t \operatorname{sign}(B_s) dB_{s \wedge T_a^*} - 1 \right) + \left(1 + \frac{1}{a} L^0_{t \wedge T_a^*}(B) \right)$$

where L^0 is the local time of B at zero.

- Apply Skorohod reflection lemma to the martingale

$$\frac{1}{a} \int_0^t \operatorname{sign}(B_s) dB_{s \wedge T_a^*} - 1$$

- By uniqueness, we obtain

$$1 + \frac{1}{a} L^0_{t \wedge T_a^*}(B) = 1 + \sup_{s \le t} \left(- \int_0^s \operatorname{sign}(B_s) dB_{s \wedge T_a^*} \right)$$

Path Decomposition of Brownian Motion

Important Properties

- The Doob-Meyer decomposition of Z is given by

$$Z_t = 1 - \frac{1}{a}\int_0^t \mathrm{sign}(B_s)dB_{s\wedge T_a^*} + \frac{1}{a}L^0_{t\wedge T_a^*}(B)$$

- From Skorokhod reflection lemma, we have $Z = 1 - m + \overline{m}$.
- $\overline{m}$ is **continuous.**
- $\overline{m}$ is **carried on** on the set $\{1 - Z = 0\}$.

Azéma supermartingale

Under certain assumptions ($\approx$ Brownian filtraiton) it was shown by Nikeghbali and Yor [NY]

Theorem

Given a finite honest time τ the additive and multiplicative decomposition of Z takes the form

$$Z = 1 - m + \overline{m} \tag{1}$$

$$Z = M/\overline{M} \tag{2}$$

- *the process m is a local martingale with continuous supremum $\overline{m}$*
- *the process M is a local martingale with continuous supremum $\overline{M}$ such that $\lim_{t\to\infty} M_t = 0$.*

Azéma supermartingale

- Kardaras [K], Acciaio and Penner [AP] have attempted at extending the result.
- Acciaio and Penner [AP] gave a counter example from Aksamit et al. [ACDJ] (Poisson filtration) showing that in general it is not possible to have representation (2).
- Song [S] studied the necessary and sufficient conditions for Azéma supermartingale of a random time to take the form (2).

Remark

Kardaras [K] and Acciaio and Penner [AP] looks at the special case

- τ *avoids all stopping times i.e* $\mathbb{P}(\tau = T) = 0$ *for all stopping time* T*.*
- τ *is the end of a predictable set (rather than an optional set).*

The Final Result

Theorem

Given a finite honest time τ the additive and multiplicative decomposition $\widetilde{Z}$ takes the form

$$\widetilde{Z} = 1 - n + \overline{n}$$

$$\widetilde{Z} = N/\overline{N}$$

- *the process n is an local optional supermartingale with continuous supremum $\overline{n}$.*
- *the process N is a local optional supermartingale with continuous supremum $\overline{N}$ and such that $\lim_{t\to\infty} N_t = 0$.*

Related Topic - Semimartingales of class-(Σ)

Definition (Yor, Nikeghbali, Nikeghbali and Yor, Cheridito et al. [Y1, N1, NY, CNP])

A semimartingale X with decomposition $X = M + A$ is of class-(Σ) if the process of finite variation A is continuous and is carried on the set $\{X = 0\}$.

Example

- Continuous martingales M.
- Draw-down process $\overline{M} - M$.
- $|M|$, M^+ and M^-.
- (loosely speaking) In the Brownian filtration the Azéma submartingale $(1 - Z)$ of a honest times is a submartingale of class-(Σ).

Semimartingales of class-(Σ)

Lemma (Madan-Roynette-Yor Formula)

Given the strike price $K > 0$ and the stock process S which is a continuous uniformly integrable martingale then

$$\mathbb{E}[(K - S_t)^+ \mathbb{1}_{\{\tau_t \leq u\}} \,|\, \mathcal{F}_u] = (K - S_u)^+$$

where $\tau_t = \sup\{u \leq t : (K - S_u)^+ = 0\}$.

- The process $(K - S)^+$ is a submartingale of class-(Σ).
- Class-(Σ) gives a martingale proof of the Madan-Roynette-Yor formula.

General theory of processes

Projections

Theorem

Let X be a measurable process, such that for all stopping time T, $X_T \mathbb{1}_{\{T<\infty\}}$ is integrable. Then there exists
(i) a unique optional process ${}^oX$ such that, for all stopping times T,

$$\mathbb{E}_{\mathbb{P}}\left(X_T \mathbb{1}_{\{T<\infty\}} \,\middle|\, \mathcal{F}_T\right) = ({}^oX)_T \mathbb{1}_{\{T<\infty\}}.$$

The process ${}^oX$ is called the $\mathbb{F}$-optional projection of X.
(ii) a unique predictable process ${}^pX$ such that, for all predictable stopping times T,

$$\mathbb{E}_{\mathbb{P}}\left(X_T \mathbb{1}_{\{T<\infty\}} \,\middle|\, \mathcal{F}_{T-}\right) = ({}^pX)_T \mathbb{1}_{\{T<\infty\}}.$$

The process ${}^pX$ is called the $\mathbb{F}$-predictable projection of X.

Dual Projections

Theorem

If A is a locally integrable increasing process
(i) then there exists a unique increasing optional process A^o such that, for any non-negative bounded process X,

$$\mathbb{E}_{\mathbb{P}}\left(\int_{[0,\infty)} {}^{o}X_s \, dA_s\right) = \mathbb{E}_{\mathbb{P}}\left(\int_{[0,\infty)} X_s \, dA^o_s\right).$$

The process A^o is called the dual predictable projection of A.
(ii) then there exists a unique increasing predictable process A^p such that, for any non-negative bounded process X,

$$\mathbb{E}_{\mathbb{P}}\left(\int_{[0,\infty)} {}^{p}X_s \, dA_s\right) = \mathbb{E}_{\mathbb{P}}\left(\int_{[0,\infty)} X_s \, dA^p_s\right).$$

The process A^p is called the dual predictable projection of A.

Important Results

Let A be an integrable increasing process then

- ${}^{o}A - A^{o}$ is a uniformly integrable martingale.
- ${}^{o}A - A^{p}$ is a uniformly integrable martingale.
- ${}^{o}(\Delta A) = \Delta A^{o}$, i.e the jumps of the dual optional projections is equal to the optional projection of the jumps.

Remark

Using the first and third property, we have

$$
\begin{aligned}
{}^{o}A &= {}^{o}A - A^{o} + A^{o} \\
{}^{o}(A_{-}) &= {}^{o}A - A^{o} + (A^{o})_{-}
\end{aligned}
$$

which gives the additive decomposition of the submartingales ${}^{o}(A)$ *and* ${}^{o}(A)_{-}$.

Stochastic Calculus for Optional Semimartingale (Gal'chuk)

Stochastic Calculus for Optional Semimartingales

- Introduced by Gal'chuk [G1, G2, G3] for filtered probability space which DO NOT satisfy the usual conditions.
- Theory of stochastic integration for martingale with finite left and right limits (optional martingale)
- Itô calculus for optional processes with finite left and right limits.

Remark

The problem at hand is much simpler since we assume that the filtered probability space satisfies the usual conditions and therefore all martingales are càdlàg.

Stochastic Calculus for Optional Semimartingales

Given any process X with finite left and right limits.

- We denote the left and right jumps by

$$\Delta X := X - X_{-} \quad \text{and} \quad \Delta^{+} X := X_{+} - X$$

- The process X can be decomposed into $X = X^r + X^g$

$$X^g_t := \sum_{0 \leq s < t} \Delta^{+} X_s \quad \text{and} \quad X^r_t := X_t - X^g_t$$

- The process X^g is càglàd and X^r is càdlàg.
- From standard càdlàg calculus

$$X = X^c + X^d + X^g$$

where $X^d_t := \sum_{0 < s \leq t} \Delta X_s$.

Stochastic Calculus for Optional Semimartingales

Definition

A stochastic process X is said to be an optional (super)martingale if

1. X is an optional process,
2. for any stopping time T, $X_T \mathbb{1}_{\{T<\infty\}}$ is integrable,
3. there exists an integrable r.v ζ such that for any stopping time T, $X_T = \mathbb{E}(\zeta|\mathcal{F}_T)$ (resp, $X_T \geq \mathbb{E}(\zeta|\mathcal{F}_T)$) a.s. on $\{T < \infty\}$.

Definition

A stochastic process X is called an optional local martingale, if there exists a sequence $(R_n, X^{(n)})_{n\in\mathbb{N}}$ where $X^{(n)}$ is an optional martingale for any n and $R_n \uparrow \infty$ such that and $X = X^{(n)}$ on $[\![0, R_n]\!]$ and the random variable X_{R_n+} is integrable for any $n \in \mathbb{N}$.

Stochastic Calculus for Optional Semimartingales

Definition

A process A is said to be strongly predictable if A is predictable and A_+ is optional.

Definition

A stochastic process X is called an (special) optional semimartingale if it can be written as

$$X = X_0 + M + A$$

where M is an optional local martingale and A is an adapted (strongly predictable) process of finite variation.

Theorem (Itô formula)

Let $X = (X^1, \dots, X^n)$ be an optional semimartingale with decomposition $X^k = X^k_0 + M^k + A^k$. Let $F(x) = F(x_1, \dots, x_n)$ be a continuously twice differentiable function on $\mathbb{R}^n$ then for $t \in \mathbb{R}_+$,

$$
\begin{aligned}
F(X_t) = F(X_0) &+ \sum_{k=1}^{n} \int_{]0,t]} D^k F(X_{s-}) d(A^{k,r} + M^{k,r})_s \\
&+ \frac{1}{2} \sum_{k,l}^{n} \int_{]0,t]} D^k D^l F(X_{s-}) d\langle M^{k,c}, M^{l,c} \rangle_s \\
&+ \sum_{0<s\leq t} \left[F(X_s) - F(X_{s-}) - \sum_{k=1}^{n} D^k F(X_{s-}) \Delta X^k_s \right] + G_t
\end{aligned}
$$

where D^k is the partial derivative with respect to the k-th coordinate and $M^r = M^c + M^d$.

Theorem

The process G is related to the right hand jumps and is given by

$$G_t = \sum_{k=1}^{n} \int_{[0,t[} D^k F(X_s) d(A^{k,g} + M^{k,g})_{s+}$$
$$+ \sum_{0 \le s < t} \left[F(X_{s+}) - F(X_s) - \sum_{k=1}^{n} D^k F(X_s) \Delta^+ X_s^k \right]$$

- The integral against the martingale $M^{k,g}$ is non-standard. Which is understood in the sense of the Gal'chuk integral.
- However we have assumed usual condition therefore $M^{k,g} = 0$.

Lemma (Integration by parts formula)

Suppose X and Y are optional semimartingales then

$$X_t Y_t = X_0 X_0 + \int_{]0,t]} X_{s-} dY^r_s + \int_{[0,t[} X_s dY^g_{s+}$$
$$+ \int_{]0,t]} Y_{s-} dX^r_s + \int_{[0,t[} Y_s dX^g_{s+}$$
$$+ \langle X^{m,c}, Y^{m,c} \rangle_t + \sum_{0 \leq s < t} \Delta^+ X_s \Delta^+ Y_s + \sum_{0 < s \leq t} \Delta X_s \Delta Y_s$$

Remark

One can define the quadratic variation as follows

$$[X, Y]_t = \langle X^{m,c}, Y^{m,c} \rangle_t + \sum_{0 \leq s < t} \Delta^+ X_s \Delta^+ Y_s + \sum_{0 < s \leq t} \Delta X_s \Delta Y_s$$

Theorem (Stochastic exponential)

Let X be an optional semimartingale. There exists a unique (to within indistinguishability) optional semimartingale S such that

$$S_t = S_0 + \int_{]0,t]} S_{s-} dX^r_s + \int_{[0,t[} S_s dX^g_{s+}$$

The process S is given by the formula

$$S_0 \exp\left\{X - \frac{1}{2}\langle X^c, X^c\rangle\right\} \prod_{0<s\leq\cdot} (1+\Delta X_s)e^{-\Delta X_s} \prod_{0<s<\cdot} (1+\Delta^+ X_s)e^{-\Delta^+ X_s}$$

Doob-Meyer Decomposition

Theorem

An optional supermartingale X admits a decomposition $X = M - A$, (if and only if X belongs to the class (DL))

- *where M is a local optional martingale*
- *A is an increasing strongly predictable locally integrable process with $A_0 = 0$*
- *This decomposition is unique to within indistinguishably.*

Additive and Multiplicative Representations

Optional supermartingales

Given an arbitrary random time τ, we apply the notation of optional and dual optional projection to the increasing process $A := \mathbb{1}_{[\![\tau,\infty[\![}$.

- the optional Doob-Meyer decomposition of $1 - \widetilde{Z} = {}^{o}(A_-)$ is

$$ {}^{o}(A_-) = ({}^{o}A - A^{o}) + A^{o}_{-}, $$

- we set $m := 1 - ({}^{o}A - A^{o})$,
- the additive decomposition of $\widetilde{Z}$ is given by

$$ \widetilde{Z} = m - A^{o}_{-} $$

- A^{o} is carried on the set $\{1 - \widetilde{Z} = 0\}$.

Additive Decomposition

- Decomposing A^o_- into

$$A^o_{t-} = A^{o,c}_t + \sum_{0 \le s < t} \Delta A^o_s$$

- The càdlàg part **is continuous** and is given by

$$A^{o,r}_t = A^{o,c}_t$$

- The càglàd parts are given by

$$A^{o,g}_t = \sum_{0 \le s < t} \Delta A^o_s$$

- This shows $\widetilde{Z} = m - A^{o,c} - A^{o,g}$.

Additive Decomposition

- We set $n := m - A^{o,g}$. Note that $A^{o,g} = A^{o,d}_{-}$
- Note that $\overline{n}$ is càdlàg, as $A^{o,g}$ is left continuous and increasing.
- Recall that

$$\tau = \sup\{s : \widetilde{Z}_s = 1\}$$

- We observe that

$$\begin{aligned}\{\widetilde{Z} = 1\} &= \{m - A^{o,g} - A^{o,c} = 1\} \\ &= \{n = 1 + A^{o,c}\} \subset \{n = \overline{n}\}\end{aligned}$$

- For the reverse inclusion, we show $\{n = \bar{n}\} \subset [\![0, \tau]\!]$ and use the fact that $\{\widetilde{Z} = 1\}$ is the largest optional set contained in $[\![0, \tau]\!]$.

Additive Decomposition

Remark

The equality $\{\widetilde{Z} = 1\} = \{n = \bar{n}\} = \{n = 1 + A^{o,c}\}$ *is very important.*

- *On the set* $\{\widetilde{Z} = 1\}$*, we have* $\bar{n} = 1 + A^{o,c}$*.*
- *On* $\{\widetilde{Z} < 1\}$ *the processes* $\bar{n}$ *and* $A^{o,c}$ *does not move.*
- *Same initial condition:* $\bar{n}_0 = 1 + A_0 = 1$*.*
- *one can show that* $\bar{n} = 1 + A^{o,c}$*.*

Theorem

Given a finite honest time τ *the additive and multiplicative decomposition of* Z *takes the form*

$$\widetilde{Z} = 1 - n + \bar{n} \tag{3}$$

the process n *is a local martingale with continuous supremum* $\bar{n}$

Multiplicative Decomposition

- Define the làglàd process Y by

$$Y = 1 + \int_{]0,t]} \widetilde{Z}_s^{-1} dA_s^{o,c} + \int_{[0,t[} \widetilde{Z}_{s+}^{-1} dA_{s+}^{o,g}$$

- We define $\widetilde{D}$ to be the optional stochastic exponential of Y.

$$\widetilde{D}_t = 1 + \int_{]0,t]} \widetilde{D}_{s-} dY_s^c + \int_{[0,t[} \widetilde{D}_s dY_{s+}^g$$

- The unique solution to the optional stochastic exponential is

$$\widetilde{D} = e^{Y^c} e^{Y^g} \prod_{0 \leq s < \cdot} (1 + \Delta^+ Y_s) e^{\Delta^+ Y_s}.$$

- $\widetilde{D}$ can be decomposed multiplicatively into $\widetilde{D} = D^c D^g$

$$D^c := e^{Y^c} \quad \text{and} \quad D^g := e^{Y^g} \prod_{0 \leq s < \cdot} (1 + \Delta^+ Y_s) e^{\Delta^+ Y_s}.$$

Lemma (Multiplicative decomposition of $\widetilde{Z}$)

The process $M := \widetilde{Z}\widetilde{D}$ *is an càdlàg local martingale where*

$$\widetilde{D}_t = 1 + \int_{]0,t]} \widetilde{D}_{s-}\widetilde{Z}_s^{-1} dA_s^{o,r} + \int_{[0,t[} \widetilde{D}_s \widetilde{Z}_{s+}^{-1} dA_{s+}^{o,g}$$

and $\lim_{t\to\infty} M = 0$.

For honest times both of the above integrals are well defined

- $\{\widetilde{Z}_+ > 0\} \subset [\![0, \tau [\![$.
- The support of A^o is contained in $\{\widetilde{Z} = 1\} \subset [\![0, \tau]\!]$.
- The proof follows from integration by parts formula.

Multiplicative Decomposition

- We set $N := M/D^g$.
- We multiply $\widetilde{Z}$ by $\widetilde{D}$ to obtain

$$\begin{aligned}\{\widetilde{Z} = 1\} &= \{M = \widetilde{D}\} \\ &= \{N = D^c\} \subset \{N = \overline{N}\}\end{aligned}$$

- To show the reverse inclusion, we use the fact that $\{\widetilde{Z} = 1\}$ is the largest optional set contained in $[\![0, \tau]\!]$

$$\{\widetilde{Z} = 1\} = \{N = \overline{N}\} = \{N = D^c\}$$

- $D^c = \overline{N}$.
- We see that $\widetilde{Z} D^c D^g = M \implies \widetilde{Z} D^c = N$.

Semimartingales of class-(Σ)

Semimartingales of class-(Σ)

Definition (Yor, Nikeghbali, Nikeghbali and Yor, Cheridito et al. [Y1, N1, NY, CNP])

A semimartingale X with decomposition $X = M + A$ is of class-(Σ) if the process of finite variation A is continuous and only increase on the set $\{X = 0\}$.

Example

- Continuous martingales M.
- Draw-down process $\overline{M} - M$.
- $|M|$, M^+ and M^-.
- The Azéma submartingale $(1 - Z)$ of a honest time which avoids all stopping times.

Definition

The Skorokhod reflection condition: Let X be an optional semimartingale $X = M + A$, where $A = A^r + A^g$. The process X is said to satisfy the Skorokhod reflection condition if

$$A^r = \int \mathbb{1}_{\{X_{s-}=0\}} dA^r_s \quad \text{and} \quad A^g = \int \mathbb{1}_{\{X_s=0\}} dA^g_{s+}$$

Definition

A optional semimartingale $X = M + A$ is said to be of class-(Σ) is

- X satisfies the Skorokhod reflection condition.
- $A^r = A^c$ is continuous.

• The process $1 - \widetilde{Z}$ is a positive optional submartingale of class-(Σ).

Lemma (Madan-Roynette-Yor Formula)

Given the strike price $K > 0$ and the stock process S which is a continuous uniformly integrable martingale then

$$\mathbb{E}[(K - S_t)^+ \mathbb{1}_{\{\tau_t \le u\}} \,|\, \mathcal{F}_u] = (K - S_u)^+$$

where $\tau_t = \sup\{u \le t : (K - S_u)^+ = 0\}$.

Theorem

Suppose X a optional semimartingale of class-(Σ) and X_+ is of class-(DL) then

$$\mathbb{E}[X_{t+} \mathbb{1}_{\{\tau_t \le u\}} \,|\, \mathcal{F}_u] = X_{u+}$$

where $\tau_t = \sup\{u \le t : X_u = 0\}$.

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