

Asymptotic Structure of Sums of Random Variables and Efficient Rare-event Estimation

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Joint work with: Patrick J. Laub ^{†,‡}

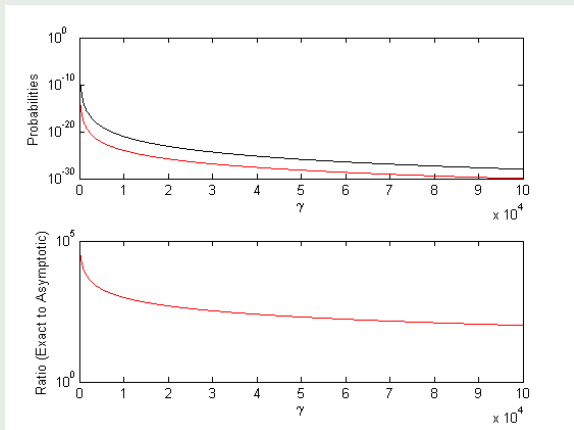
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Mathematics of Risk: MATRIX 2017
28th of November, 2017

Why not Just use Known Asymptotic?

Example ($f_i(x) = \lambda_i \alpha_i (1 + \lambda_i x)^{-(\alpha_i+1)}$, $i = 1, 2$.)

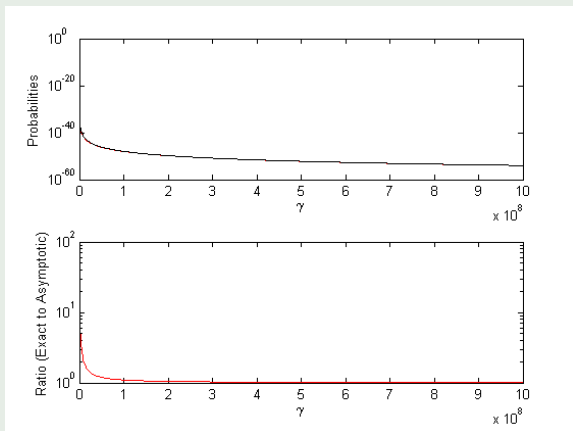
E.g. $\alpha_1 = 6$, $\lambda_1 = 1$, $\alpha_2 = 7$, and $\lambda_2 = 0.1$; $\widehat{\ell}_{\text{Asym}}(\gamma) = (1 + \gamma)^{-6}$.



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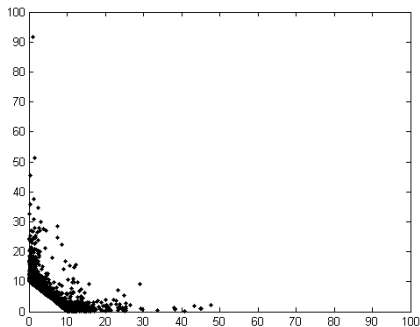
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Subexponential Tail Decay

Example

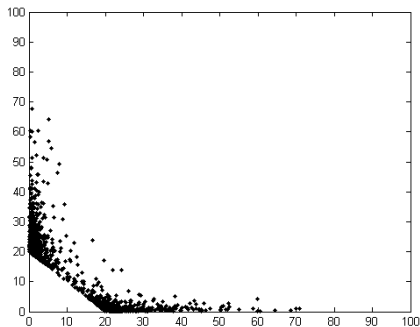
$$X_1, X_2 \sim \text{LogN}(0, 1): f(x) = (x \sqrt{2\pi})^{-1} e^{-\frac{\ln(x)^2}{2}}.$$



Subexponential Tail Decay

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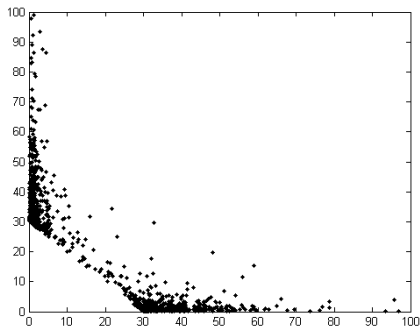
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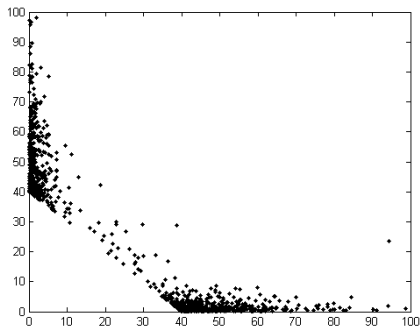
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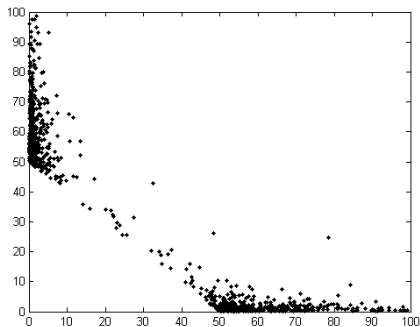
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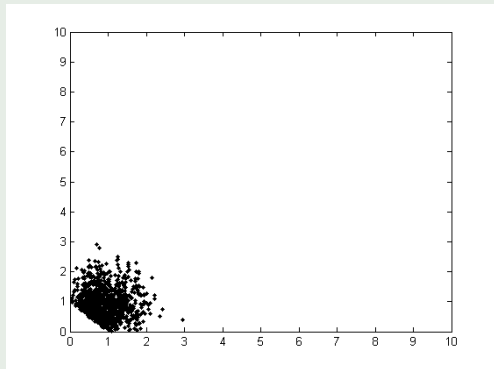
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Superexponential Tail Decay

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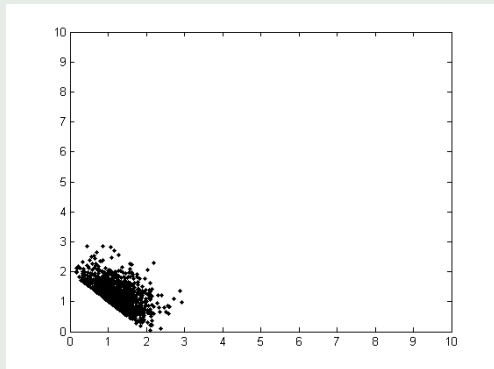
$X_1, X_2 \sim \text{Weib}(2, 1): f(x) = 2x e^{-x^2}$.



Superexponential Tail Decay

Example

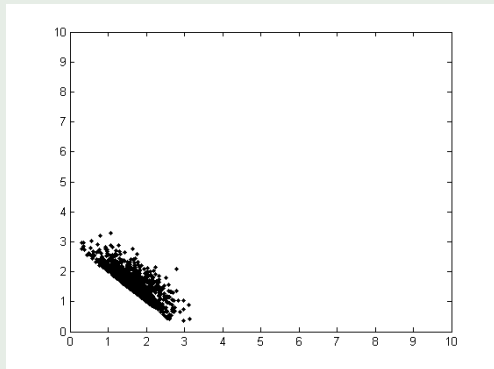
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Superexponential Tail Decay

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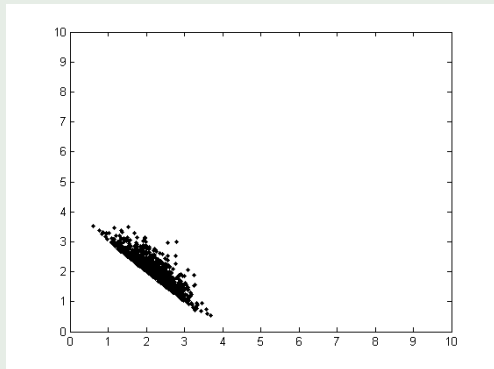
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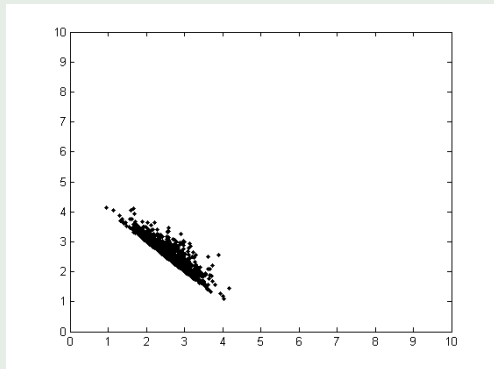
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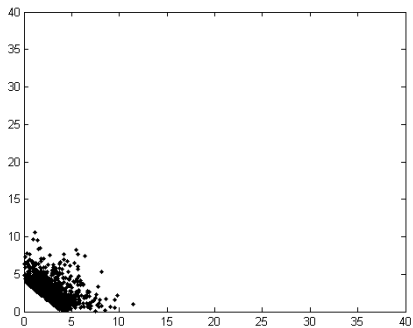
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Exponential Tail Decay

Example

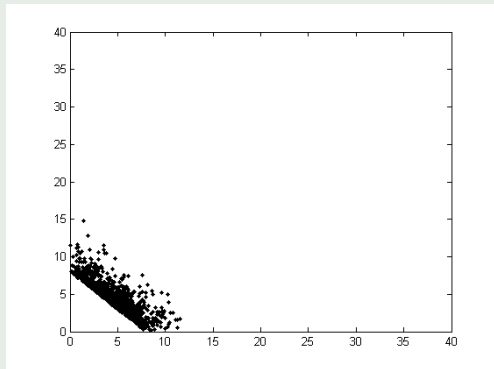
$X_1, X_2 \sim \text{Gamma}(2, 1): f(x) = x e^{-x}$.



Exponential Tail Decay

Example

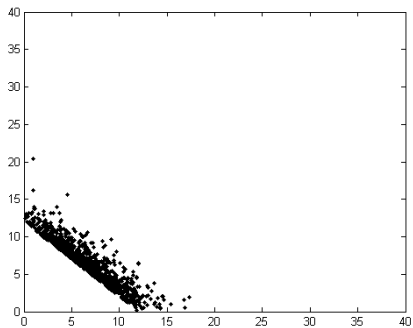
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Exponential Tail Decay

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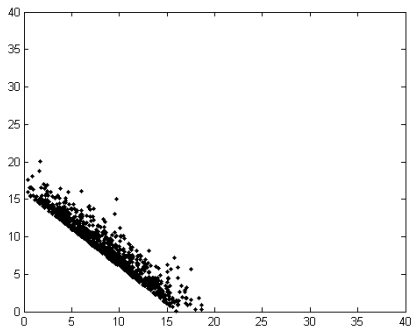
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Exponential Tail Decay

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