# A sequential test for the drift of a fractional Brownian motion

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## Main result (1)

Let $W_t$ be a standard Brownian motion, obtained by a certain transformation of $X_t$.

The optimal time to stop the observation is

$$\tau = \inf\{t \in [0,1] : |W_t + \tfrac{\mu}{\sigma}| \geqslant a(t)\}$$

where $a(t)$ is a positive function on $[0,1)$ with $a(1)=0$, such that:

- for $H \geqslant \frac{1}{2}$: $a(t)$ is continuous and strictly decreasing on $(0,1]$;
- for $H < \frac{1}{2}$: $a(t)$ is continuous and strictly decreasing on $[t_0,1]$, where $t_0 = \frac{1-2H}{4(1-H)}$.

## Main result (2)

For $t \in (t_0, 1]$ if $H < \frac{1}{2}$ and $t \in (0, 1]$ for $H \geqslant \frac{1}{2}$, the function $a(t)$ is the unique continuous non-increasing solution of the equation

$$F(t, a(t)) = \int_t^1 G(t, a(t), s, a(s))ds$$
$$a(1) = 0$$

where

$$F(t, x) = 2\sqrt{1-t} \cdot \varphi\Big(\frac{x}{\sqrt{1-t}}\Big) - 2x \cdot \Phi\Big(\frac{x}{\sqrt{1-t}}\Big)$$
$$G(t, x, s, y) = \frac{K_H s^{\beta-1}}{(1-s)^{\beta+1}}\bigg(\Phi\Big(\frac{y-x}{\sqrt{s-t}}\Big) - \Phi\Big(\frac{-y-x}{\sqrt{s-t}}\Big)\bigg)$$

with $K_H = 2(2-2H)^{\frac{2H-1}{2-2H}} \sigma^{\frac{H-2}{1-H}}$.

The boundary $a(t)$ satisfies the bound

$$a(t) \leqslant \frac{(1-t)^{\beta}}{2C_H t^{\beta-1}} \text{ for } t \in [t_0, 1)$$

where $\beta = \frac{1}{2-2H}$ and $C_H$ is a some constant.

## Stopping boundaries