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MATRIX Book Series

2016 MATRIX Annals

Editors

David R. Wood (Editor-in-Chief)

Jan de Gier

Cheryl E. Praeger

Terence Tao



MATRIX is Australia's international and residential mathematical research institute. It was established in 2015 and launched in 2016 as a joint partnership between Monash University and The University of Melbourne, with seed funding from the ARC Centre of Excellence for Mathematical and Statistical Frontiers. The purpose of MATRIX is to facilitate new collaborations and mathematical advances through intensive residential research programs, which are currently held in Creswick, a small town nestled in the beautiful forests of the Macedon Ranges, 130km west of Melbourne.

This book, the 2016 MATRIX Annals, is a scientific record of the five programs held at MATRIX in 2016:

- *Higher structures in geometry and physics*
- *Winter of disconnectedness*
- *Approximation and optimisation*
- *Refining C^* -algebraic invariants for dynamics using KK -theory*
- *Interactions between topological recursion, modularity, quantum invariants and low-dimensional topology*

The MATRIX Scientific Committee selected these programs based on scientific excellence and the participation rate of high-profile international participants. This committee consists of: Jan de Gier (Melbourne University, Chair), Ben Andrews (Australian National University), Darren Crowdy (Imperial College London), Hans De Sterck (Monash University), Alison Etheridge (University of Oxford), Gary Froyland (University of New South Wales), Liza Levina (University of Michigan), Kerrie Mengersen (Queensland University of Technology), Arun Ram (University of Melbourne), Joshua Ross (University of Adelaide), Terence Tao (University of California, Los Angeles), Ole Warnaar (University of Queensland), and David Wood (Monash University).

The selected programs involved organisers from a variety of Australian universities, including Federation, Melbourne, Monash, Newcastle, RMIT, Sydney, Swinburne and Wollongong, along with international organisers and participants. Each program lasted 1–4 weeks, and included ample unstructured time to encourage collaborative research. Some of the longer programs had an embedded conference or lecture series. All participants were encouraged to submit articles to the MATRIX Annals.

The articles were grouped into refereed contributions and other contributions. Refereed articles contain original results or reviews on a topic related to the MATRIX program. The other contributions are typically lecture notes based on talks or activities at MATRIX. A guest editor organised appropriate refereeing and ensured the scientific quality of submitted articles arising from each program. The Editors (Jan de Gier, Cheryl E. Praeger, Terence Tao and myself) finally evaluated and approved the papers.

Many thanks to the authors and to the guest editors for their wonderful work.

MATRIX is hosting eight programs in 2017, with more to come in 2018; see www.matrix-inst.org.au. Our goal is to facilitate collaboration between re-

searchers in universities and industry, and increase the international impact of Australian research in mathematical sciences.

David R. Wood
MATRIX Book Series Editor-in-Chief

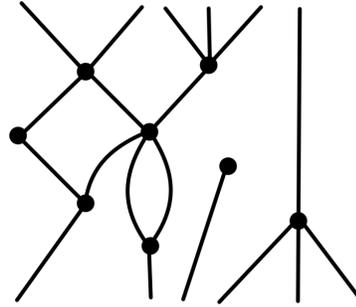
Higher Structures in Geometry and Physics

6–17 June 2016

Organisers

Marcy Robertson (Melbourne)

Philip Hackney (Macquarie)



The inaugural program at MATRIX took place June 6-17, 2016 and was entitled Higher Structures in Geometry and Physics. It was both a pleasure and a privilege to take part in this first ever program at MATRIX. The excellent working conditions, cozy environment, friendly staff, and energetic participants made this time both memorable and productive.

The scientific component of our program was comprised of a workshop with lecture series by several invited speakers, followed the subsequent week by a conference featuring talks on a range of related topics from speakers from around the globe. The two events were separated by a long weekend which gave the participants free time to discuss and collaborate. Within this volume is a collection of lecture notes and articles reflecting quite faithfully the ideas in the air during these two weeks.

Our title Higher Structures (not unlike the term down under) suggests a certain fixed perspective. For the participants in our program, this perspective comes from twentieth century examples of algebraic and categorical constructions associated to topological spaces, possibly with geometric structures and possibly taking motivation from physical examples. From this common frame of reference stems a range of new and rapidly developing directions, and activities such as this program at MATRIX play a vital role in weaving these threads into a collective understanding. The excitement of working in this rapidly developing field was felt during our time at MATRIX and I hope it comes across in this volume as well.

I would like to thank all of the authors who took the time to contribute to this volume. I would also like to thank the MATRIX staff and officials for hosting and facilitating this event and giving us the opportunity to share our work with this volume. Most importantly I would like to thank the organizers of our program Marcy and Philip for all of their hard work and for giving all of us participants this unique opportunity.

Ben Ward
Guest Editor



Participants

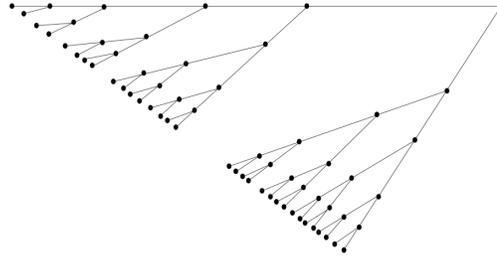
Ramon Abud Alcalá (Macquarie), Clark Barwick (Massachusetts Institute of Technology), Alexander Campbell (Macquarie), David Carchedi (George Mason), Gabriel C. Drummond-Cole (IBS Center for Geometry and Physics), Daniela Egas Santander (Freie Universität Berlin), Nora Ganter (Melbourne), Christian Haesemeyer (Melbourne), Philip Hackney (Osnabrueck), Ralph Kauffman (Purdue), Edoardo Lanari (Macquarie), Martin Markl (Czech Academy of Sciences), Branko Nikolic (Macquarie), Simona Paoli (Leicester), Sophia Raynor (Aberdeen), Emily Riehl (Johns Hopkins), David Roberts (Adelaide), Marcy Robertson (Melbourne), Chris Rogers (Louisiana), Martina Rovelli (EPFL), Matthew Spong (Melbourne), Michelle Strumila (Melbourne), TriThang Tran (Melbourne), Victor Turchin (Kansas State), Dominic Verity (Macquarie), Raymond Vozzo (Adelaide), Ben Ward (Stony Brook), Mark Weber (Macquarie), Felix Wierstra (Stockholm), Sinan Yalin (Copenhagen), Jun Yoshida (Tokyo), Dimitri Zaganidis (EPFL)

Winter of Disconnectedness

27 June – 8 July 2016

Organisers

Murray Elder (Newcastle)
 Jacqui Ramagge (Sydney)
 Colin Reid (Newcastle)
 Anne Thomas (Sydney)
 George Willis (Newcastle)



Our understanding of totally disconnected locally compact (t.d.l.c.) groups has been growing rapidly in recent years. These groups are of interest for general theoretical reasons, because half the task of describing the structure of general locally compact groups falls into the totally disconnected case, and also for purposes of specific applications, because of the significance that various classes of t.d.l.c. groups have in geometry, number theory and algebra. The workshop held at Creswick from June 27th – July 8th 2016 was the first part of a program in which leading researchers presented the most recent advances by giving short courses and individual lectures. Time was also set aside for collaboration between established researchers and students. Both general techniques and results relating to particular classes of t.d.l.c. groups were covered in the lectures.

Four courses of five lectures each were delivered at Creswick, as follows.

- Helge Glöckner (Paderborn): *Endomorphisms of Lie groups over local fields.*
- George Willis (Newcastle): *The scale, tidy subgroups and flat groups.*
- Anne Thomas (Sydney): *Automorphism groups of combinatorial structures.*
- Phillip Wesolek (Binghamton): *A survey of elementary totally disconnected locally compact groups.*

A second workshop was held in Newcastle at the end of July 2016 with the same structure.

Notes from the four courses just listed and from the courses

- Adrien Le Boudec (Louvain Le Neuve): *Groups of automorphisms and almost automorphisms of trees: subgroups and dynamics*
- Colin Reid (Newcastle): *Normal subgroup structure of totally disconnected locally compact groups*

delivered at the Newcastle workshop are published here. Although the distinction is not absolute, the notes by Reid, Wesolek and Willis cover general methods and those by Glöckner, Le Boudec and Thomas treat specific classes of t.d.l.c. groups. In several cases, they are based on notes taken by listeners at the workshops and we are grateful for their assistance. Moreover, the notes by Glöckner, which have

been refereed, are an expanded version of what was delivered in lectures and contain calculations and proofs of some results that have not previously been published.

We believe that these notes give the 2016 overview of the state of knowledge and of research directions on t.d.l.c. groups, and hope that they will also serve to introduce students and other researchers new to the field to this rapidly developing subject.

Dave Robertson
Guest Editor



Participants

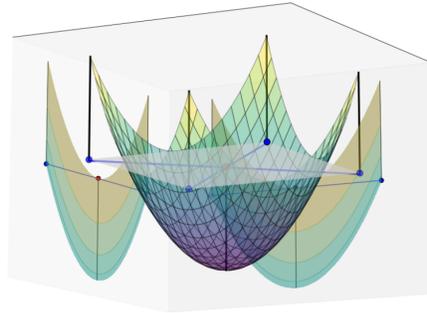
Benjamin Brawn (Newcastle), Timothy Bywaters (Sydney), Wee Chaimanowong (Melbourne), Murray Elder (Newcastle), Helge Glöckner (Paderborn), John Harrison (Baylor), Waltraud Lederle (ETH Zurich), Rupert McCallum (Tübingen), Sidney Morris (Federation), Uri Onn (ANU), C.R.E. Raja (Indian Statistical Institute, Bangalore), Jacqui Ramagge (Sydney), Colin Reid (Newcastle), David Robertson (Newcastle), Anurag Singh (Utah), Simon Smith (City University of New York), George Willis (Newcastle), Thomas Taylor (Newcastle), Anne Thomas (Sydney), Stephan Tornier (ETH Zurich), Tian Tsang (RMIT), Phillip Wesolek (Universite Catholique de Louvain)

Approximation and Optimisation

10–16 July 2016

Organisers

Vera Roshchina (RMIT)
 Nadezda Sukhorukova (Swinburne)
 Julien Ugon (Federation)
 Aris Daniilidis (Chile)
 Andrew Eberhard (RMIT)
 Alex Kruger (Federation)
 Zahra Roshanzamir (Swinburne)



There are many open problems in the field of approximation theory where tools from variational analysis and nonsmooth optimisation show promise. One of them is the Chebyshev (also known as uniform) approximation problem. Chebyshev's work is generally considered seminal in approximation theory, and remains influential to this day in this field of mathematics. His work also has many significant implications in optimisation, where uniform approximation of functions is considered an early example of an optimisation problem where the objective function is not differentiable. In fact, the problem of best polynomial approximation can be reformulated as an optimisation problem which provides very nice textbook examples for convex analysis.

The joint ancestry and connections between optimisation and approximation are still very apparent today. Many problems in approximation can be reformulated as optimisation problems. On the other hand, many optimisation methods require set and/or function approximation to work efficiently.

These connections were explored in the 50s, 60s and 70s. This was also the period when the area of nonsmooth analysis emerged. Convex and nonsmooth analysis techniques can be applied to obtain theoretical results and algorithms for solving approximation problems with nonsmooth objectives. These problems include Chebyshev approximation: univariate (polynomial and fixed knots polynomial spline) approximation and multivariate polynomial approximation. In 1972, P.-J. Laurent published his book, where he demonstrated interconnections between approximation and optimisation. In particular, he showed that many difficult (Chebyshev) approximation problems can be solved using optimisation techniques.

Despite this early work, historically the fields of optimisation and approximation have remained separate, each (re)developing their own methodology and terminology. There is a clear pattern though: advances in optimisation result in significant breakthroughs in approximation and, on the other hand, new approximation techniques and approaches advance the development of new optimisation methods and improve the performance of existing ones. Therefore, the lift off theme of the program was multivariate and polynomial spline approximation from the perspective of optimisation theory. We invited top researchers in approximation theory, polynomial and semialgebraic optimisation and variational analysis to find new ways to

attack the existing open problems and to establish major new research directions. A fresh look at approximation problems from the optimisation point of view is vital, since it enables us to solve approximation problems that cannot be solved without very advanced optimisation techniques (and vice versa), and discover beautiful interconnections between approximation and optimisation.

The morning sessions of the Program consisted of lectures:

- Approximation of set-valued functions (Nira Dyn, Tel-Aviv University);
- Algebraic, Convex analysis and Semi-Infinite programming approach to Chebyshev Approximation (Julien Ugon, Federation University Australia and Nadezda Sukhorukova, Swinburne University of Technology);
- The sparse grid combination technique and optimisation (Markus Hegland, Australian National University);
- Quasi-relative interior and optimisation (Constantin Zalinescu, Al. I. Cuza University).

The afternoons were dedicated to smaller groups discussions. These discussions were vital for the research papers submitted to this volume.

1. **Compact convex sets with prescribed facial dimensions, by Vera Roshchina, Tian Sang and David Yost.**

The rich soup of research ideas that was stirred up during the workshop helped us with a breakthrough in a seemingly unrelated research direction. During the workshop David Yost came up with a neat inductive idea that finished the proof of the dimensional sequence theorem that he and Vera Roshchina have been working on for a while. Later during the workshop the fractal ideas introduced by Markus Hegland motivated us to consider convex sets with fractal facial structure. Even more surprisingly we discovered that a beautiful example of such a set can be obtained from the spherical gasket studied by Tian Sang in her prior research on infinite Coxeter groups.

2. **Chebyshev multivariate polynomial approximation: alternance interpretation, by Nadezda Sukhorukova and Julien Ugon and David Yost.**

The notion of alternating sequence (alternance) is central for univariate Chebyshev approximation problems. How can we extend this notion to the case of multivariate approximation, where the sets are not totally ordered? In one of the papers, namely, Chebyshev multivariate polynomial approximation: alternance interpretation by Sukhorukova, Ugon and Yost, the authors work on this issue and propose possible solutions. In particular, a very elegant formulation for necessary and sufficient optimality conditions for multivariate Chebyshev approximation.

Julien Ugon and Nadezda Sukhorukova
Guest Editors



Participants

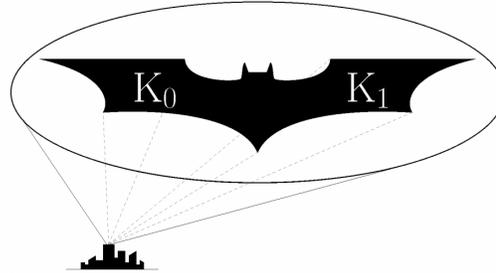
Alia Al nuaimat (Federation), Fusheng Bai (Chongqing Normal University), Yi Chen (Federation), Jeffrey Christiansen (RMIT), Brian Dandurand (RMIT), Reinier Diaz Millan (Federal Institute of Goias), Nira Dyn (Tel Aviv), Andrew Eberhard (RMIT), Gabriele Eichfelder (Technische Universitat Ilmenau), Markus Hegland (ANU), Alexander Kruger (Federation), Vivek Laha (Indian Institute of Technology, Patna), Jeffrey Linderoth (Wisconsin- Madison), Prabhu Manyem (Nanchang Institute of Technology), Faricio Oliveira (RMIT), Zahra Roshan Zamir (Swinburne), Vera Roshchina (RMIT), Tian Sang (RMIT), Jonathan Scanlan, Vinay Singh (National Institute of Technology, Mizoram), Nadezda Sukhorukova (Swinburne), Julien Ugon (Federation), Dean Webb (Federation), David Yost (Federation), Constantin Zalinescu (University Al. I. Cuza Iasi), Jiapu Zhang (Federation)

Refining C^* Algebraic Invariants for Dynamics using KK -Theory

18–29 July 2016

Organisers

Magnus Goffeng
(Chalmers / Gothenburg)
Adam Rennie (Wollongong)
Aidan Sims (Wollongong)



THE BOTT PROJECTION

This graduate school and workshop were motivated by intense recent interest and progress in the non-commutative geometry of dynamical systems.

The progress has been of several sorts. The assignment of C^* -algebras to dynamical systems is not new, but has become much more sophisticated in recent years. The K -groups of such dynamical C^* -algebras provide invariants of the original dynamical system, but are not always fine enough to capture the structural features of greatest interest. In response to this, precise characterisations of the relationships between more detailed K -theoretic invariants and equivalence classes of dynamical systems have recently been sharpened significantly.

The extra ingredient whose potential applications to such problems we hoped to highlight to attendees is recent progress in importing ideas from algebraic topology to dynamical-systems theory through the computability of the Kasparov product. The Kasparov product is a far-reaching generalisation of index theory, and provides an abstract composition rule for morphisms in the KK -category of C^* -algebras. The KK -category extends and refines the correspondence category of C^* -algebras; and a correspondence can be regarded as a generalised dynamical system and is closely related to the construction of dynamical C^* -algebras.

With this background in mind, we started the first week with three lecture series during the mornings, with informal Q&A and research in the afternoons. The three lecture courses were

- Robin Deeley: *Groupoids and C^* -algebras*;
- Bram Mesland: *Kasparov's KK -theory*;
- Adam Rennie & Aidan Sims: *Hilbert modules and Cuntz–Pimsner algebras*.

Groupoid C^* -algebras and Cuntz–Pimsner algebras are two of the most flexible and best developed frameworks for modelling dynamical systems using C^* -algebras. The basics of Kasparov's KK -theory and the recent advances in the computability of the product proved central to the progress seen during the workshop.

The lecture series by Deeley, Mesland, Rennie, and Sims set the stage for the second week, bringing attendees, particularly the significant student cohort, together

around a common language and a joint leitmotif. The talks ranged widely over the core topics, their applications, and neighbouring disciplines. The five papers which follow give a good indication of the breadth of the conference.

Deeley offers refinements of Putnam's homology theory for dynamical systems, and Ruiz et al provide strong invariants for a special class of dynamical systems. Bourne (with Schulz-Baldes) provides an application of KK -theoretic techniques to topological insulators. Goffeng and Mesland provide a detailed account of novel aspects of the non-commutative geometry of the Cuntz algebras, while Arici probes the non-commutative topology and geometry of quantum lens spaces using techniques which are perfectly in tune with the theme of the workshop.

Adam Rennie
Guest Editor



Participants

Zahra Afsar (Wollongong), Francesca Arici (Radboud University Nijmegen), Chris Bourne (Erlangen-Nurnberg), Guo Chuan Thiang (Adelaide), Robin Deeley (Hawaii), Anna Luise Duwenig (Victoria), James Fletcher (Wollongong), Iain Forsyth (Leibniz University Hannover), Elizabeth Anne Gillaspay (Universitet Manster), Magnus Goffeng (Chalmers Technology/Gothenburg), Peter Hochs (Adelaide), Marcelo Laca (Victoria), Lachlan MacDonald (Wollongong), Michael Mampusti (Wollongong), Bram Mesland (Leibniz University Hannover), Alexander Munday (Wollongong), Adam Rennie (Wollongong), Karen Rught Strung (Polish Academy of Sciences), Efren Ruiz (Hawaii at Hilo), Thomas Scheckter (UNSW), Aidan Sims (Wollongong), Hang Wang (Adelaide), Yasuo Watatani (Kyushu)

Interactions between Topological Recursion, Modularity, Quantum Invariants and Low-dimensional Topology

28 November – 23 December 2016

Organisers

Motohico Mulase (UC Davis)
Norman Do (Monash)
Neil Hoffman (Oklahoma State)
Craig Hodgson (Melbourne)
Paul Norbury (Melbourne)

$$\langle \text{Diagram 1} \rangle = (-A^2 - A^{-2}) \langle \text{Diagram 2} \rangle$$

$$= (-A^2 - A^{-2})(A^9 - A^5 - A - 2A^{-3} + A^{-7} - A^{-11})$$

This program contributed to the active international research effort under way at present to connect structures in mathematical physics with those in low-dimensional topology, buoyed by recent theoretical advances and a broad range of applications. It brought together people in cognate areas with common interests, including algebraic geometry, conformal field theory, knot theory, representation theory, quantum invariants and combinatorics.

The program was motivated by recent generalisations of the technique of topological recursion, as well as fundamental conjectures concerning invariants in low-dimensional topology. These include the AJ conjecture, the Jones slope conjecture of Garoufalidis, and the underlying topological significance of the 3D-index. Algorithmic techniques to compute these and related invariants also featured in the program.

The program began with a week of short courses, comprising three lectures each, on the following topics.

- *Conformal field theory* (Katrin Wendland, Freiburg)
- *Hyperbolic knot theory* (Jessica Purcell, Monash)
- *Quantum invariants* (Roland van der Veen, Leiden)
- *Topological recursion* (Norman Do, Monash)

The program for the middle week was largely informal and reserved for research collaborations. The final week was dedicated to an international conference, which gathered together leading experts in the areas of mathematical physics, topological recursion, quantum invariants, and low-dimensional topology to address recent advances and explore new connections between these fields.

The program attracted a total of 51 attendees. During the conference, there were 33 talks, of which 23 were delivered by international visitors. Among these talks were the following.

- Jørgen Andersen: Verlinde formula for Higgs bundles
- Feng Luo: Discrete uniformization for polyhedral surfaces and its convergence
- Rinat Kashaev: Pachner moves and Hopf algebras
- Scott Morrison: Modular data for Drinfeld doubles

- Hyam Rubinstein and Craig Hodgson: Counting genus two surfaces in 3-manifolds
- Gaëtan Borot: Initial conditions for topological recursion
- Tudor Dimofte: Counting vortices in the 3d index
- George Shabat: Counting Belyi pairs over finite fields
- Leonid Chekhov: Abstract topological recursion and Givental decomposition
- Piotr Sułkowski: Knots and BPS / Super-quantum curves

The articles in this proceedings represent different aspects of the program. Kashaev's contribution describes a topological quantum field theory in four dimensions. Licata-Mathews and Spreer-Tillmann describe topological and geometric results for three-manifolds. Shabat describes first steps towards generalising Belyi maps to finite fields. Roland van der Veen kindly contributed notes from his short course on quantum invariants of knots.

Norman Do, Neil Hoffman, Paul Norbury
Guest Editors



Participants

Jørgen Andersen (Aarhus), Vladimir Bazhanov (ANU), Gaetan Borot (Max Planck), Benjamin Burton (Queensland), Alex Casella (Sydney), Wee Chaimanowong (Melbourne), Abhijit Champanerkar (CUNY), Anupam Chaudhuri (Monash), Leonid Chekhov (Steklov), Blake Dadd (Melbourne), Tudor Dimofte (California, Davis), Norman Do (Monash), Petr Dunin-Barkovskiy (Moscow), Omar Foda (Melbourne), Evgenii Fominykh, Sophie Ham, Robert Cyrus Haraway III (Sydney), Craig Hodgson (Melbourne), Neil Hoffman (Oklahoma), Joshua Howie (Monash), Adele Jackson (ANU), Max Jolley (Monash), Rinat Kashaev (Geneva), Seonhwa Kim (IBS), Ilya Kofman (CUNY), Reinier Kramer (Amsterdam), Andrew James Krickler (NTU), Priya Kshirsagar (UC Davis), Alice Kwon (CUNY), Tung Le (Monash), Oliver Leigh (British Columbia), Danilo Lewanski (Amsterdam), Joan Elizabeth Licata (ANU), Beibei Liu (UC Davis), Feng Luo (Rutgers), Joseph Lynch (Melbourne), Alessandro Malusa (Aarhus), Clément Maria (Queensland), Daniel Mathews (Monash), Sergei Matveev (Chelyabinsk), Todor Milanov Kavli (IPMU), Scott Morrison (ANU), Motohico Mulase (UC Davis), Paul Norbury (Melbourne), Nicolas Orantin (EPFL), Erik William Pettersson (RMIT), Aleksandr Popolitov (Amsterdam), Jessica Purcell

(Monash), Robert Quigley-McBride, Hyam Rubinstein (Melbourne), Axel Saenz Rodriguez (Virginia), Sjabbo Schaveling, Henry Segerman (Oklahoma), Georgy Shabat (Independent), Rafael Marian Siejakowski (NTU Singapore), Ruifang Song (UC Davis), Piotr Sulkowski (Warsaw & Caltech), Dominic James Tate (Sydney), Stephan Tillmann (Sydney), Roland van der Veen (Leiden), Paul Wedrich (Imperial), Katrin Wendland (Friburg), Campbell Wheeler (Melbourne), Adam Wood (Melbourne), Tianyu Yang (Melbourne)