

# InterCriteria Analysis Approach as a Tool for Promising Decision Making in Physiological Rhythms

Krassimir Atanassov and Tania Pencheva

**Abstract** Recently developed InterCriteria Analysis (ICrA) approach has being intensively gained popularity as quite promising approach to support decision making process in biomedical informatics studies, and in particular – in physiological rhythms. ICrA has been elaborated to discern possible similarities in the behaviour of pairs of criteria when multiple objects are considered. The approach is based on the theories of intuitionistic fuzzy sets and index matrices. Up to now, ICrA has been successfully applied in economics, different industry fields, ecology, artificial intelligence, e-learning, etc. ICrA has been demonstrated as promising tool also in studies related to medicine and bioinformatics, which are in the focus of this investigation.

## 1 Introduction

The idea of InterCriteria Analysis (ICrA) has been originally developed in the period 2014-2015 [4, 5]. In the coming years, the interest to the concept increased significantly. The approach has become a subject of theoretical studies as well as of applications in various fields, e.g. industry, economics, education, medical and biotechnological processes, artificial intelligence, including neural networks, expert systems, bio-inspired and metaheuristics algorithms, etc. Recently, a survey on theory and applications of ICrA approach [7] has been spread to the scientific community.

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The concept of ICrA is based on two mathematical formalisms, developed by Atanassov, namely the theories of Index Matrices (IM) [1, 3] and Intuitionistic Fuzzy Sets (IFSs) [2], thus relying on methodology different from the classical correlation analysis.

ICrA has been designed as a novel method for detecting the levels of pairwise correlations for a set of criteria and a set of objects (measurements or evaluations) against these criteria. The ultimate goal of ICrA is to detect if some of the criteria exhibit high enough correlations with others, so that skipping them from the further decision making process would not affect the whole process [8]. The motivation behind this method is for an eventual elimination of some of the criteria, when measurement against these comes at a higher cost, consumes more time or other resources, or is considered undesirable in any other reason. Selecting these high enough correlations requires either an expert decision or an algorithm for the precise establishment of the thresholds, beyond which the top-correlating criteria are selected in order to yield certain problem-specific conclusions.

## 2 Brief description of ICrA

Here we will briefly repeat the theoretical framework of the proposed approach, firstly proposed in [5]. The approach employs an index matrix  $M$  of  $m$  rows  $\{O_1, \dots, O_m\}$  and  $n$  columns  $\{C_1, \dots, C_n\}$ , where for every  $i, j, k, l$  ( $1 \leq i \leq j \leq m, 1 \leq k \leq l \leq n$ ),  $O_i$  is an evaluated object,  $C_k$  is an evaluation criterion, and  $e_{O_i C_k}$  is the evaluation of the  $i$ -th object against the  $k$ -th criterion, defined as a real number or another object that is comparable according to relation  $R$  with all the rest elements of the index matrix  $M$ .

$$M = \begin{array}{c|cccccc} & C_1 & \dots & C_k & \dots & C_l & \dots & C_n \\ \hline O_1 & e_{O_1 C_1} & \dots & e_{O_1 C_k} & \dots & e_{O_1 C_l} & \dots & e_{O_1 C_n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ O_i & e_{O_i C_1} & \dots & e_{O_i C_k} & \dots & e_{O_i C_l} & \dots & e_{O_i C_n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ O_j & e_{O_j C_1} & \dots & e_{O_j C_k} & \dots & e_{O_j C_l} & \dots & e_{O_j C_n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ O_m & e_{O_m C_1} & \dots & e_{O_m C_k} & \dots & e_{O_m C_l} & \dots & e_{O_m C_n} \end{array} \quad (1)$$

From the above requirement for comparability follows the relation  $R(e_{O_i C_k}, e_{O_j C_l})$  for each  $i, j, k$ . The relation  $R$  has dual relation  $\bar{R}$ , which is true in the cases when relation  $R$  is false, and vice versa.

For the needs of this decision making method, pairwise comparisons between every two different criteria are made along all evaluated objects. During the comparison, it is maintained one counter of the number of times when the relation  $R$  holds, and another counter for the dual relation.

Let  $S_{k,l}^\mu$  be the number of cases in which the relations  $R(e_{O_i C_k}, e_{O_j C_k})$  and  $R(e_{O_i C_l}, e_{O_j C_l})$  are simultaneously satisfied. Let also  $S_{k,l}^\nu$  be the number of cases in which the relations  $R(e_{O_i C_k}, e_{O_j C_k})$  and its dual  $\bar{R}(e_{O_i C_l}, e_{O_j C_l})$  are simultaneously satisfied. As the total number of pairwise comparisons between the object is  $m(m-1)/2$ , it is seen that there hold the inequalities:

$$0 \leq S_{k,l}^\mu + S_{k,l}^\nu \leq m(m-1)/2 \quad (2)$$

For every  $k, l$ , such that  $1 \leq k \leq l \leq m$ , and for  $m \geq 2$  the following two numbers are defined:

$$\mu_{C_k, C_l} = 2 \frac{S_{k,l}^\mu}{m(m-1)}, \nu_{C_k, C_l} = 2 \frac{S_{k,l}^\nu}{m(m-1)}. \quad (3)$$

In the terms of ICrA,  $\mu_{C_k, C_l}$  is a *degree of agreement*, while  $\nu_{C_k, C_l}$  – a *degree of disagreement*. Obviously, both  $\mu_{C_k, C_l}$  and  $\nu_{C_k, C_l}$ , are numbers in the  $[0, 1]$ -interval, and their sum is also a number in this interval. What is complement to their sum to 1 is the number  $\pi_{C_k, C_l}$ , which corresponds to a *degree of uncertainty*.

The pair, constructed from these two numbers, plays the role of the intuitionistic fuzzy evaluation of the relations that can be established between any two criteria  $C_k$  and  $C_l$ . In this way, the index matrix  $M$  that relates evaluated objects with evaluating criteria can be transformed to another index matrix  $M^*$  that gives the relations among the criteria:

$$M^* = \begin{array}{c|cccc} & C_1 & C_2 & \dots & C_n \\ \hline C_1 & \langle 1, 0 \rangle & \langle \mu_{C_1, C_2}, \nu_{C_1, C_2} \rangle & \dots & \langle \mu_{C_1, C_n}, \nu_{C_1, C_n} \rangle \\ C_2 & \langle \mu_{C_2, C_1}, \nu_{C_2, C_1} \rangle & \langle 1, 0 \rangle & \dots & \langle \mu_{C_2, C_n}, \nu_{C_2, C_n} \rangle \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_n & \langle \mu_{C_n, C_1}, \nu_{C_n, C_1} \rangle & \langle \mu_{C_n, C_2}, \nu_{C_n, C_2} \rangle & \dots & \langle 1, 0 \rangle \end{array}, \quad (4)$$

From practical considerations, it has been more flexible to work with two index matrices  $M^\mu$  and  $M^\nu$ , rather than with the index matrix  $M^*$  of intuitionistic fuzzy pairs (IFPs).

The final step of the algorithm is to determine the degrees of correlation between the criteria. Let  $\alpha, \beta \in [0, 1]$  be the threshold values (which are of the user's choice), against which we compare the values of  $\mu_{C_k, C_l}$  and  $\nu_{C_k, C_l}$ . We call that criteria  $C_k$  and  $C_l$  are in:

- *positive consonance*, if  $\mu_{C_k, C_l} > \alpha$  and  $\nu_{C_k, C_l} < \beta$ ;
- *negative consonance*, if  $\mu_{C_k, C_l} < \beta$  and  $\nu_{C_k, C_l} > \alpha$ ;
- *dissonance*, otherwise.

In a completely identical way, it is possible (though not always meaningful) to build a matrix giving the correlations between the objects. The only difference is that the input index matrix  $M$  has to be transposed, and the resultant matrix, e.g.,  $M^{**}$ , is with dimensions  $m \times m$ .

### 3 ICrADa software package description

Here we provide a quick overview of ICrADa – the software implementing the ICrA approach [9]. It is written in the Java programming language and requires the installation of Java Virtual Machine. This makes it usable under Linux and Windows environment. ICrADa is freely available for use and its latest version ICrADa v2.3 can be downloaded from <http://intercriteria.net/software/> (Last access August 24, 2020).

In order to easily load data from other software products, the capability to load *csv* (comma separated values) files with headers (row and column) which are taken as names for objects and criteria, was added to the software. This allows loading of tables from *MS Excel/LibreOffice Calc*.

The user interface consists of a left panel for the input data, the central panel for the result of ICrA in a coloured table view, and the rightmost panel showing the graphical interpretation of the results.

For better visualization of the results, table cell colours were added, according to the following rules, depending on the user defined  $\alpha$  and  $\beta$  thresholds:

- The results are displayed in dark-green colour in case of *positive consonance*;
- The results are displayed in light-blue colour in case of *negative consonance*;
- Otherwise, in case of *dissonance* – violet colour.

The default values used by the software ICrADa are  $\alpha = 0.75$  and  $\beta = 0.25$ , respectively.

ICrADa saves a draft automatically each 15 minutes and on program exit in order to prevent accidental loss or overwriting of data.

The features outlined above allow for better automation in working process with program and additional improvements in that regard are also planned in the future.

### 4 ICrA applications in biomedical research and physiological rhythms

Going slightly beyond physiological rhythms, ICrA approach has encountered numerous applications aiming to support decision making in different areas, connected to medical investigations and bioinformatics. In [11], ICrA is applied on a dataset of thermo-dynamic parameters derived from thermograms of blood plasma proteome of patients with colorectal cancer recorded by differential scanning calorimetry. The goal of the study was to establish interdependences between the derived calorimetric parameters that were not inferred so far from the calorimetric data and to discuss their importance for the clinical application of differential scanning calorimetry.

In [10], ICrA, combined with Pearson's and Spearman's correlation analysis, is applied to a large dataset of calorimetric and biochemical parameters derived for the serum proteome of patients diagnosed with multiple myeloma. As a result, intercriteria dependences have been identified that are general for the various types

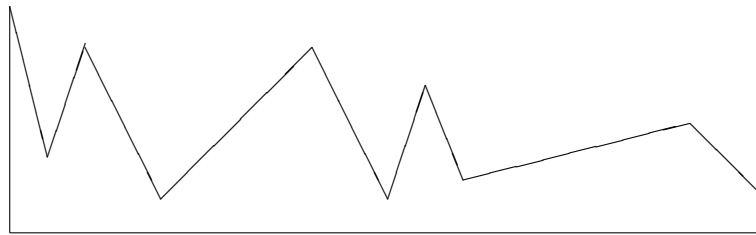
of multiple myeloma and thus can be regarded as a characteristic of this largely heterogeneous disease: strong contribution of the monoclonal protein concentration to the excess heat capacity of the immunoglobulins-assigned thermal transition; shift of the albumin assigned calorimetric transition to allocation where it overlaps with the globulins assigned transition and strong shift of the globulins assigned transition temperature attributable to monoclonal proteins conformational changes.

In [12], ICrA is applied to real data connected with health-related quality of life (HrQoL). The EQ-5D-3L questionnaire for measuring HrQoL for a representative sample of 1050 residents of Burgas (the fourth-largest in Bulgaria) is used. The data was analyzed to identify the best correlations between the indicators, to discover dependent and independent indicators and the relationships between them. The comparison can help to describe the behavior of the used indicators and their assessment. The increase of the coefficient of consonance and the entry in the zone of strong positive consonance means strong correlation between the respective pair of criteria, which may justify the removal of one of the criteria in the pair on the basis that its informational values is lesser. Removal of indicators leads to simplification of the process of evaluation.

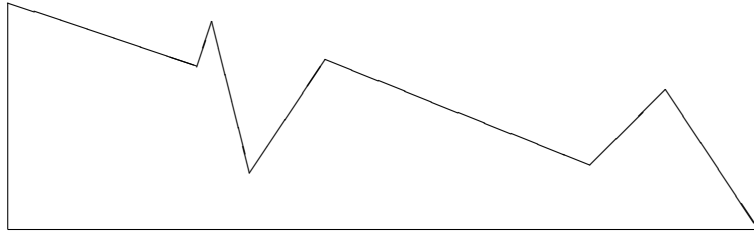
In [13, 14] a dataset of Behtere's disease patients is analyzed applying ICrA, aiming at approbation of this novel approach to medical data with the goal to discover correlations between important health indicators based on available patients' data. The selected set of health indicators comprises: physical functioning; role functioning based by physical conditional; bodily pain; general health status; vitality; social functioning; role functioning based by emotional conditional; and mental health. Results obtained confirm once again that the health condition depends on the emotional condition and determines the social functioning of the patients under observation.

When looking for possible application of ICrA toward the physiological rhythms, in this investigation a novel idea of ICrA application in a totally new direction is proposed here, namely to adapt ICrA assessments in a way to compare two curves, which – in particular case, might represent physiological rhythms.

Let us have two lines  $L_1$  (see Fig. 1) and  $L_2$  (see Fig. 2).

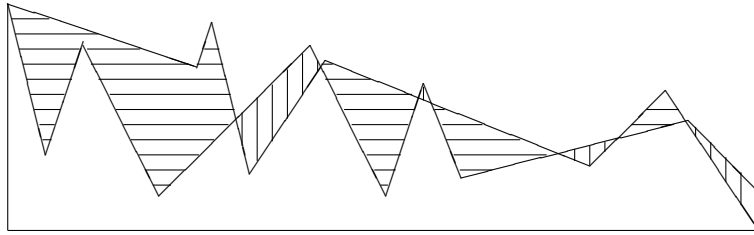


**Fig. 1** Line  $L_1$



**Fig. 2** Line  $L_2$

Let  $M$ ,  $N$  and  $P$  be the degrees of coincidence, of difference and of uncertainty of between both lines. Then the three degrees can be evaluated as it is shown on Fig. 3, where the area of the part of the figure that is in white corresponds to  $M$ , the area of the part marked with horizontal lines corresponds to  $N$  and the part marked with vertical lines corresponds to  $P$  (see Fig. 3).



**Fig. 3** An example for evaluation of degrees of coincidence, of difference and of uncertainty between lines  $L_1$  and  $L_2$

Further, we can assess the nearness between the two lines based on the intuitionistic evaluations  $\mu$  and  $\nu$ , where

$$\mu = \frac{M}{M+N+P}, \nu = \frac{N}{M+N+P}. \quad (5)$$

In fact, this is an IFP with a degree of uncertainty  $\pi = 1 - \mu - \nu = \frac{P}{M+N+P}$ .

Thereby, in case of  $n$  lines, we can search the nearness between them using ICrA and working with IFPs.

This idea might be further used in analyses of different types of physiological rhythms, including ECGs, as well as to even harder from a mathematical point of view electromyography (EMG) signals.

Some first steps have been done in the ICrA application in analysing the features in a database of electrocardiography signal (ECGs). The investigation is carried out over the training set of the Computing in Cardiology Challenge 2017 Database from PhysioNet (<https://physionet.org>). Some very promising results have been obtained, but in this investigation the researchers were faced to some limits of ICrAData, namely working with a big data. This research is in a fast progress now, both on the software improvement, as well as in data analysis.

## 5 Conclusions

In all applications so far, ICrA shows prerequisites to assist in decision making processes in order to guide the selection of the most appropriate choice among many. In this investigation, ICrA has been demonstrated as a quite promising tool to assist decision making in such challenging field as physiological rhythms. The novel idea for a comparison of curves presenting physiological rhythms based on ICrA approach has been introduced, which may be in benefit in other fields of research as well.

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