Mixing, tunnelling and the direction of time in the context of Reichenbach's principles

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Abstract This work reviews the understanding of the direction of time introduced by Hans Reichenbach, including the fundamental relation of the perceived flow of time to the second law of thermodynamics (i.e. the Boltzmann time hypothesis), and the principle of parallelism of entropy increase. An example of a mixing process with quantum effects, which is advanced here in conjunction with Reichenbach's ideas, indicates the existence of a physical mechanism that reflects global conditions prevailing in the universe and enacts the direction of time locally (i.e. the "time primer"). Generally, this mechanism, whose effects are often enacted by presuming antecedent causality, remains unknown at present. The possibility of experimental detection of the time primer is also discussed: if the time primer is CPT-invariant, its detection may be possible in high-energy experiments under the current level of technology.

Key words: the direction of time, the second law of thermodynamics, mixing, decoherence, quantum tunnelling, the time primer

> It appears that mixing processes, in the most general sense of the term, are the instruments which indicate a direction of time
>
> Hans Reichenbach

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1 Introduction

Discussing time is always difficult since the notion of time is deeply embedded into both our language and our intuition. Many key words in English (e.g. "then","follows", "since") and most other languages and cultures imply both a logical link and a temporal arrangement. The perceived flow of time and conceptual inferences are almost indistinguishable, or at least they are not properly distinguished by most languages we use. Immanuel Kant [1] wrote in 1781:

> Time is a necessary representation that grounds all intuitions. In regard to appearances in general one cannot remove time, though one can very well take the appearances away from time. Time is therefore given a priori.

One one hand this intuition assists us in everyday life and in the formulation of scientific theories not related to the nature of time. One the other hand, this intuition needs to be subordinated to rational thought when the nature of time is discussed, and this can be difficult. It is worthwhile to note that the conventional intuitive interpretation of the flow of time is the most common interpretation, but certainly not the only one possible: there are indigenous tribes living in the North-Western part of Queensland, who intuitively perceive time as being directed from East to West.

The perceived flow of time is thought to reflect causality — the fundamental directional connection between events unfolding in time, as well as the possibility of explaining observed phenomena in terms of more basic principles. The two sides of causality, related to 1) atemporal logical statements of a generic nature (e.g. objects fall because of the action of gravity) and 2) directional dependence between specific consecutive events (the vase is shattered because it was pushed from the table), may be interpreted synergistically [2] or be clearly distinguished [3]. It is the second interpretation, which is often referred to as *antecedent causality*, that we are interested most in this work. In the 1st half of the 20th century, there was a common belief that the directional properties of the perceived flow of time can be explained in terms of more objective casual relations that are postulated a priori as one of the fundamental intrinsic properties of nature. This belief had to face mounting difficulties in defining causality, and largely evaporated toward the end of the 20 century. As early as in 1914, Bertrand Russell [4] noted that

> The view that the law of causality itself is a priori cannot, I think, be maintained by anyone who realises what a complicated principle it is.

The conceptual understanding of causality has grown to accommodate randomness, counterfactual logic, etc. but, overall, our interpretation of causality remains largely intuitive and rather short of being the basis of rational thought. Antecedent causality is now explained in terms of physical laws rather than placed at the foundation of these laws. Dowe [5] defines the direction of casual action in terms of physical laws that possess temporal asymmetry: either the second law of thermodynamics or CP violations in the quantum world. Tying causality to the second law of thermodynamics in one form or another has become the central element of conventional thinking about the problem ([6–9]). The strongest form of the link between

the direction of time and the second law of thermodynamics is given by the Boltzmann time hypothesis, which proclaims that the arrow of time and the second law are two sides of the same physical effect [10–13]. Hawking [12] explains this: "the second law of thermodynamics is really a tautology", since the direction of our perceived flow of time is, in fact, determined by the second law. The physical side of the direction of time is covered in a number of principal publications [14–17].

The second half of the 20th century is marked by two seminal, yet very different, books that endeavour to bridge philosophical and physical arguments about the direction of time [11, 18]. The book by Huw Price is well written and delivers its message

> I have been trying to correct a variety of common mistakes and misconceptions about time in contemporary physics — mistakes and misconceptions whose origins lie in the distorting influence of our own ordinary temporal perspective, and especially of the time asymmetry of that perspective

in a clear and articulate form. The other book is the last book written by Hans Reichenbach. He was not able to complete his work and the book was published by Mrs. Reichenbach in 1956, after her husband's death in 1953. The book tends to mix philosophical and physical arguments in a way that might be confusing for both philosophers and physicists, yet Reichenbach's book is probably the greatest book about time ever written. According to his wife, Reichenbach considered his last book to be the culmination of his contribution to philosophy. The *Boltzmann time hypothesis*, the *principle of parallelism of entropy increase* and *the principle of the common cause* are, perhaps, the most important contributions presented in the book. While the Boltzmann time hypothesis gradually became accepted by many philosophers and physicists, the principle of parallelism of entropy increase is still a subject of debates [7, 8, 19–22].

The present work is, of course, not intended to review all issues related to the arrow of time and causality within a short article. Conceptual issues are discussed only in the context of selected examples that can illustrate physical statements in a concise and transparent manner. Without attempting to overview or replace the comprehensive publications cited above, this work focuses on select few problems. Section 2 briefly overviews the understanding of the directionality of time suggested by Reichenbach. Section 3 analyses an example of a mixing process and demonstrates the significance of time priming pointing to existence of unknown physical mechanisms of very small magnitude associated with the direction of time. Section 4 discusses a wider scope of issues focusing on the possibility of experimental evaluation of these mechanisms. The Appendix considers the example of Section 3 and involves evaluation of a quantum system in thermodynamic conditions when decoherence or recoherence are present.

2 The direction of time and the second law

Our experience of time is very directional — we remember the past but cannot possibly remember the future and our photographs always show us younger than we are now. If we see dents on bumpers of two cars that are standing next to each other, we conclude that these cars have just collided and, certainly, not that they are going to collide in the future. At an intuitive level, we characterise these directional properties of time as "time flow" but, according to the fundamental Boltzmann time hypotheses, these properties of time reflect the objective reality and directional nature of the second law of thermodynamics. Unlike most physical theories (e.g. classical and quantum mechanics, relativity and electromagnetism) which are time-symmetric, this law is time-directional, stating that, in an isolated system, entropy must increase (or stay the same) forward in time. Following Reichenbach, the Boltzmann time hypotheses is explained below by using a gedanken experiment called "footsteps on a beach".
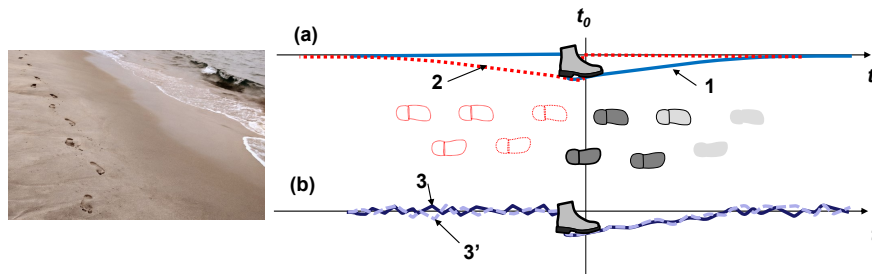

Fig. 1 Footprints on a beach: a) effect of the second law of thermodynamics and b) effect of random disturbances. Curves: 1-realistic; 2-violating the second law; 3,3'- realistic disturbed by wind.

2.1 Why don't we remember the future?

The sand on a beach is always levelled by wind and water – this is the state of maximal entropy where all specific information is destroyed. We might try to change this by stepping on the sand and leaving footprints. These footprints, however, cannot stay forever and will soon disappear. This process, shown by line 1 in Figure 1, is perfectly consistent with the second law of thermodynamics. Another possibility is shown by line 2 — footsteps gradually appear and then are removed by a walking man. The second scenario is not realistic as it contradicts the second law of thermodynamics: footsteps cannot appear forward in time under the influence of random factors such as wind and waves. To be more precise, this can, in principle, happen,

but the probability of such event is so small so that it can be safely neglected. The second law of thermodynamics is a probabilistic law — it predicts the behaviour of entropy not with absolute certainty but with overwhelming probability.

If we see footsteps on the beach, do they mean that someone walked on the beach in the past (line 1) or that someone will walk on the beach in the future (line 2)? According to the second law, footsteps cannot possibly appear without a reason (i.e. a man walking) in the past but do not need a reason to disappear. In the same way marks, photos, memories, scratches of car paint, etc. reflect past events but tell us nothing about future events. This conclusion is obvious but its link to the second law of thermodynamics is not trivial.

The Boltzmann time hypothesis has not been accepted universally. Karl Popper, one of the most distinguished philosophers of the 20th century, argued that the Boltzmann time hypothesis cannot be true due to thermodynamic fluctuations and that Boltzmann would not suggest his hypothesis if he knew more about these fluctuations [23]. Popper's remarks are usually accurate, sharp and impressively prescient, but this statement seems rather controversial. First, Boltzmann was well aware of thermodynamic fluctuations and even interpreted (for the sake of illustration) his imaginary world of reversed time as a gigantic galactic fluctuation [10]. Second, exactly the same fluctuation argument was later used not against but in support of connection between the arrow of time and the second law of thermodynamics [5]. The flow of time is a powerful illusion; it is very useful in real life and even in scientific applications, but, as noted by Price [18], it can easily produce a distorted view when issues related to the direction of time are discussed. Although details of specific opinions may vary, most philosophers and physicists tend to accept the existence of deep underlying link between the perceived direction of time and the action of the second law of thermodynamics [5, 10–12, 18, 24].

2.2 Parallelism of entropy increase

The importance of this principle was stressed by Reichenbach, who considered the main system to be divided into branch systems (i.e semi-independent subsystems branching from the main system) and suggested that "*in the vast majority of branch systems, the directions toward higher entropy are parallel to one another and to that of the main system*". Since "the main system" can be deemed to encompass the whole universe, its direction toward higher entropy is the temporal direction of overall entropy increase in the universe. This principle does not preclude occasional fluctuations that might slightly decrease local entropy and, therefore, it is not clear to what extent this principle represents an independent statement. For example, Boltzmann believed that local entropy trends simply reflect global increase of entropy in the observable part of the universe, while Reichenbach insisted that parallelism of entropy increase is an independent principle, which, generally, cannot be derived from the global temporal conditions imposed on the universe: despite the presence of fluctuations, entropy increases in branch systems are more consistent

than it can be inferred from the global entropy increase. Since a microstate of each branch system can be characterised by a point in a phase space of very large dimensions, the state of maximal entropy corresponds to the uniform distribution of such points over all possible microstates. Reichenbach interprets increase of entropy as a *generalised mixing* process, which is associated with diffusion of particles or points towards being distributed over larger volumes in the physical and/or phase spaces. This interpretation of the entropic directionality as a trend to expand distributions in phase spaces of large dimensions is often used by physicists [15]. The principle of parallelism of entropy increase is presented and discussed in a few publications, most notably in books by Davies [19] and Sklar [20].

While association of causality with the second law is now widely acknowledged, the physical origins of the second law remain unclear. The second law is fundamental but largely empirical: it declares that entropy increases forward in time but does not explain why. Since all major physical laws and theories are time symmetric, the most common explanation is that the temporal asymmetry of the second law is due to asymmetric temporal boundary conditions imposed on the universe (these conditions can be referred to as the past hypothesis or low-entropy Big Bang). Albert [7, 8] believes that this explanation is perfectly sufficient but, according to Reichenbach, the principle of parallelism of entropy increase is needed (in addition to the commonly presumed low-entropy conditions in the early universe) to explain the observed consistency of the second law [11]. Winsberg [21] agrees with Reichenbach, while North [22] supports Albert. As discussed further in Section 3, there are reasonable arguments on both sides of this debate but, overall, it seems rather unlikely that the second law can be replaced by a combination time-symmetric physical laws and time-assymetric temporal boundary (i.e. initial and final) conditions.

The principle of parallelism of entropy increase allows us to apply entropic considerations to relatively small thermodynamic systems or even to non-thermodynamic macroscopic objects. We often imply this principle when we commingle macroscopic and microscopic considerations. For example, one can associate an entropy change to random reshuffling of playing cards, although this change is insignificant compared to changes in thermodynamic entropy — the latter is larger by a factor of $\sim 1/k_B$, where k_B is the Boltzmann constant. While applying entropic considerations to macroscopic objects mostly produces reasonable outcomes and good intuitive illustrations of thermodynamic principles, such applications are less rigorous compared to the very high level of statistical certainty associated with the laws involving the thermodynamic entropy. Macroscopic interpretations of entropy are subject to conditions that are difficult to stipulate in a rigorous and universal manner and, therefore, may produce incorrect inferences if taken out of context. Reichenbach notes that we can put cards back into their original order if we need to, but we cannot possibly reorder molecules exactly into their original positions. The grains of sand from the example shown in Figure 1 may be very small, but they are macroscopic objects.
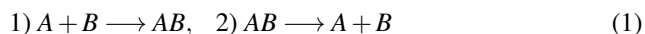
2.3 The principle of the common cause

Reichenbach states this principle as "if an improbable coincidence has occurred, there must exist a common cause"; this cause should be in the past as common effects in the future cannot cause improbable coincidences. The term "improbable coincidence" for two events A and B, refers to the simultaneous occurrence of A and B in excess of $P(A)P(B)$ — the probability if they were independent events. Price refers to this property as the principle of the independence of incoming influences (PI^3) — indeed incoming influences (i.e. those that do not have a common cause) must be statistically independent. This principle is intuitively obvious but, again, the essence of the Boltzmann time hypothesis is that this effect is, in fact, a consequence of the second law. Figure 1b illustrates this point. Consider a model when wind and waves naturally impose some degree of roughness on the sand level. The lines 3 and 3' shown in this figure correspond to the effect of wind and waves causing the surface at two selected points to fluctuate at random. These points level out only if only someone steps on them. Levelling, however, does not last for long, since wind and waves gradually introduce new disturbances, which erase the footprints. The usual state of the surface is rough and influences of events cannot propagate back in time (since this propagation specified by curve 2 contradicts the second law) — these conditions require that dependences are induced by past events.

It is probably true that Reichenbach's treatment of mutual causes and mutual effects in his last book presents a combination of physical and philosophical arguments, intermixing them to extent that may become puzzling for both physicists [12] and philosophers [25]. Perhaps applying these ideas to conventional elements of statistical physics can provide a more transparent illustration. In the next subsection, we give an example of chemical kinetics that illustrates Reichenbach's key point — the link between the principle of the common cause and the second law of thermodynamics.

2.4 Chemical kinetics and causality

Consider the following reactions

$$1)\, A + B \longrightarrow AB, \quad 2)\, AB \longrightarrow A + B \tag{1}$$

which are assumed not to have any heat effect. As illustrated in Figure 2, these reactions can be interpreted as open (left) and closed (right) casual forks analysed by Reichenbach, who denoted AB by C (cause) or E (effect). Events A, B, AB respectively denote appearance of molecules A, B, AB in a volume V, which is much smaller than V_t — the total volume under consideration. In the first reaction, A and B are causes that have a common effect AB, while in the second reaction, A and B are effects that have a common cause AB. Hence, according to the principle of the common cause $P(A+B) = P(A)P(B)$ for the first reaction but not for the second.

Here, $P(A+B)$ is the probability of simultaneous presence of A and B in the volume V. If $P(A+B)$ is significantly larger than $P(A)P(B)$, then, in accordance with the third principle of Reichenbach, there must be a common cause — the second reaction in (1).

Considering that A and B are independent causes of the first reaction and AB is the cause of the second reaction, the overall reaction rates of the first and second reactions can be expressed by

$$W_1 = V_t K \frac{N_A}{V_t} \frac{N_B}{V_t}, \ W_2 = V_t K \frac{N_{AB}}{V_t} \tag{2}$$

where $P(X) = N_X V / V_t$ for any $X = A, B, AB$, N_X is the total number of molecules X in the volume V_t and K is the reaction rate constant. Note that kinetic equation

$$\frac{dN_A}{dt} = \frac{dN_B}{dt} = -\frac{dN_{AB}}{dt} = W_2 - W_1 \tag{3}$$

implies that the entropy defined as

$$S = N_A \ln\left(e\frac{V_t}{N_A}\right) + N_B \ln\left(e\frac{V_t}{N_B}\right) + N_{AB} \ln\left(e\frac{V_t}{N_{AB}}\right) \tag{4}$$

cannot decrease; i.e.

$$\frac{dS}{dt} = \frac{dN_A}{dt} \ln\left(\frac{V_t N_{AB}}{N_A N_B}\right) = K\left(N_{AB} - \frac{N_A N_B}{V_t}\right) \ln\left(\frac{V_t N_{AB}}{N_A N_B}\right) \geq 0 \tag{5}$$

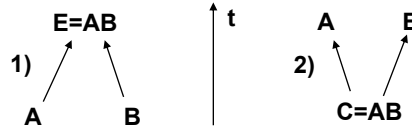in accordance with the second law of thermodynamics.

Fig. 2 Chemical reations shown in the form of casual forks.

We may try alternative anticasual arrangements when causes are located in the future and effects are in the past. According to the anticasual assumptions, the first reaction is caused by AB while the second reaction is caused by two independent events A and B. This means that the overall reaction rates are now

$$W_1 = V_t K \frac{N_{AB}}{V_t}, \ W_2 = V_t K \frac{N_A}{V_t} \frac{N_B}{V_t} \tag{6}$$

so that the entropy change rate is given by

$$\frac{dS}{dt} = \frac{dN_A}{dt} \ln\left(\frac{V_t N_{AB}}{N_A N_B}\right) = K\left(\frac{N_A N_B}{V_t} - N_{AB}\right) \ln\left(\frac{V_t N_{AB}}{N_A N_B}\right) \leq 0 \qquad (7)$$

This illustrates that casual or anticasual assumptions imply the following trends for the entropy: increasing in time for the former and decreasing in time for the latter. Of course, only the casual case corresponds to the real world.

If quantum effects are to be considered (it is arguable that interactions of atoms are determined by quantum effects), then the casual case (2)-(5) corresponds to persistent decoherence of the molecules before and after the reaction, while the anticasual case (6)-(7) corresponds to persistent recoherence [26]. There is a physical connection between causality and the temporal direction of decoherence [26, 27]. The second law of thermodynamics is a macroscopic law, but it is enacted by microscopic irreversible processes — quantum decoherences and collapses [15–18]. (We tend to use these the terms "decoherences" and "collapse" interchangeably, as there is a significant overlap between implications of these terms — see Appendix of Ref. [13] for details.)

3 Why mixing is time-directional?

Despite temporal symmetry of the overwhelming majority of the physical laws, entropy tends to increase or stay the same with a high degree of certainty for any thermodynamic system, small or large. The temporal boundary conditions imposed on the universe (e.g. a low-entropy Big Bang) must play a key role in this trend — these conditions are often sufficient to explain many effects associated with directionality of time even if physical laws are deemed to be completely time-symmetric. Indeed, if the universe has a very strong overall trend to increase the entropy and the universe is divided into semi-autonomous subsystems (branches according to Reichenbach), then increase of entropy must be more likely than decrease of entropy in these subsystems. While the low-entropy initial conditions imposed on the universe are important and instrumental in explaining entropy increase for many physical phenomena, this does not mean that all observed physical effects can be directly explained by imposing these conditions while assuming that all physical laws are strictly time-symmetric. Therefore, the principle of parallelism of entropy increase is indicative of some fundamental properties of the universe that need to be understood and examined further.

These points are illustrated here by analysing time-directional properties of mixing. We consider diffusion of N_t molecules (called particles) of a substance in a gas. The number N_t is relatively small so that molecules do not interact with each other; the admixture remains passive and does not affect major thermodynamic quantities such as pressure and density, although N_t is large enough in absolute terms to produce reliable statistical quantities that can be observed macroscopically as concentrations.

3.1 Importance of the initial conditions

The particles (molecules) $j = 1, ..., N_t$ are released at the same location $x_j = x_0$ at $t = t_1$ and diffuse forward in time $t > t_1$. The particle trajectories $x_j(t)$ represent Brownian motion, while the average concentration of particles $f(x,t)$ satisfies the diffusion equation

$$\frac{\partial f}{\partial t} = D \frac{\partial^2 f}{\partial x^2} \tag{8}$$

Note that particle trajectories are time-symmetric — that is we cannot distinguish trajectories running forward in time from those running backward in time. The direction of diffusion is determined by the initial conditions $x_j = x_0$ at $t = t_1$. If we impose final conditions at $t = t_2 > t_1$ (for example, this can be done by considering the following process $x'_j(t) = x_j(t) - x_j(t_2) + x_0$, which satisfies $x'_j = x_0$ at $t = t_2$), then the concentration of trajectories $x'_j(t)$ would be characterised by diffusion equation (8) but with a negative diffusion coefficient $D' = -D$; i.e. this is diffusion occurring backward in time[1].

   This seem to favour temporal boundary conditions as a driving force behind irreversibility. The processes described by the diffusion equation with positive and negative diffusion coefficients are radically different. The direction of the diffusion is determined not by the random variations of particle positions, which do not have a time arrow, but by imposing the initial or the final conditions. The influence of initial or final conditions, however, disappears in the equilibrium state $f = \text{const}$ of fully mixed components (assuming that the diffusion takes place in a finite volume). Indeed, once the steady-state is achieved, say within the interval $t_1^\circ < t < t_2^\circ$ where $t_1 < t_1^\circ < t_2^\circ < t_2$, it is impossible to tell the direction of the diffusion process, even if the most detailed current characteristics of trajectories are monitored — information about initial or final conditions has been lost. Setting initial conditions at $t = t_1$ cannot be distinguished from setting the final conditions at $t = t_2$ by observing equilibrium solution at $t_1^\circ < t < t_2^\circ$. Equilibria achieve maximal entropy and destroy information.

   The example of this subsection reflects the *lattice of mixture model* examined by Reichenbach [11]. We see that, within limitations of this model, the overall initial conditions imposed on the whole system are sufficient to ensure directionality of mixing processes in every macroscopic subsystem. The evolution of the universe can be interpreted as a generalised mixing process where particles diffuse to occupy a larger and larger number of microstates. Since the universe was presumably formed with low-entropy initial conditions and has not achieved its equilibrium state, this consideration provides a justification for generally preferring initial conditions to final conditions in today's environment. It might seem that Reichenbach's principle of parallelism of entropy increase is excessive — the low-entropy initial condition

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[1] Note that this reversal is different from the reversal of the Kolmogorov backward equation and time reversal of Markov diffusion processes preserving $f(x,t)$ — see ref. [28]. It is also possible to use both conditions at $t = t_1$ and $t = t_2$, leading to so called Brownian bridge, but this case is not considered here.

imposed on the universe ensures both overall entropy increase and, as long as overall equilibrium is not reached, proper directionality of various local thermodynamical processes. While under some idealised conditions, global entropy increase induces entropy increases in local processes, there are important details that are missing in this inference. The lattice of mixture model reveals some useful properties but, nevertheless, is a significant oversimplification of the physical reality.
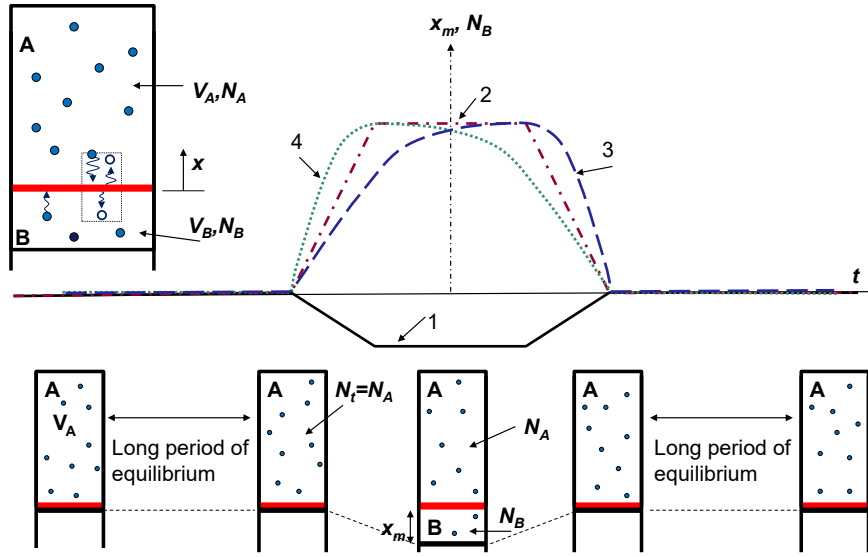

Fig. 3 Experiment with admixture passing through semi-permeable membrane. Curves: 1 – position of the piston $x_m(t)$; 2 – equilibrium $N_B(t)$; 3 – $N_B(t)$ for $C = +1$; 4 – $N_B(t)$ for $C = -1$.

3.2 Why is the principle of parallelism of entropy increase essential?

At this point we consider a modified experiment, which is illustrated in Figure 3. A cylinder having a finite volume V_t contains N_t particles of the passive admixture (as considered previously) and is kept in a state of thermodynamic equilibrium for a long time. The cylinder is located in a remote part of the universe away from any possible influences of the matter that populates the universe. The piston remains at $x = 0$ for a long time so that $V_A = V_t$ and $V_B = 0$, then moves down and up in a time-symmetric manner so that $V_B > 0$ as shown in Figure 3 and then, again, remains at $x = 0$ for a very long time so that $V_B = 0$. In addition to admixture molecules, the cylinder may also be filled by a gas to ensure that the system under

consideration is thermodynamic. The volumes A and B are divided by a very thin semi-permeable membrane that is fully permeable for the gas (if gas is present) and only partially permeable for the molecules of admixture, so that these molecules can occasionally tunnel through the membrane. When considered from a quantum-mechanical perspective, the membrane is interpreted as a potential barrier that can be tunneled through, while the rest of the walls are formed by impervious barriers of a high potential. We note that such experiments are not only conceptually possible but, due to recent technological advances [29], also practically feasible. Obviously, $N_A + N_B = N_t = \text{const}$ and $V_A = \text{const}$. The number of particles N_t is sufficiently large to ensure that N_A and N_B are macroscopic parameters, which can be measured by classical instruments.

For simplicity of evaluation, the probability of successful tunnelling of admixture molecules through the membrane is assumed to be small so that the concentrations of particles remain uniform in volumes A and B (although not necessarily the same on both sides of the membrane — see Figure 3). Since particles do not interact, they can be considered autonomously from one another. The concentrations of particles on both sides of the membrane are determined by quantum tunnelling through the membrane. Classical statistics is assumed so that most of the quantum states are vacant: all states have the same probability of occupation determined by the concentrations of the particles: N_A/V_A on one side and N_B/V_B on the other.

As the particles tunnel through the membrane, they must decohere since, otherwise they would be simultaneously present in volumes A and B, be governed by unitary evolutions and not subject to the laws of statistical physics (see Ref. [30]). We, however, do not have any experimental evidence that this can happen when an object is progressively screened from the direct influence of the initial and final conditions imposed on the universe. If decoherence is terminated, we would effectively obtain a less cruel version of Schrödinger's cat — a substance whose particles are not located in volumes A or B but are in superposition states between these volumes (strictly speaking, N_A and N_B are not classically defined in this case). After decoherence and collapse of the wave function, particles appear either on one side of the membrane or the other with some classical probability. As we do not wish to discriminate the direction of time a priori, we must admit that the particles can decohere or recohere (i.e. decohere backward in time), as discussed in the Appendix. The concentration of particles is governed by the equation (see Appendix and Refs. [26, 27])

$$\frac{dN_B}{dt} = -\frac{dN_A}{dt} = CK\left(\frac{N_A}{V_A} - \frac{N_B}{V_B}\right) \tag{9}$$

where K is the rate constant for transition through the membrane, which, as shown in the Appendix, must be the same for transitions from A to B and from B to A. The constant $C = +1$ corresponds to predominant decoherence and $C = -1$ to predominant recoherence (i.e. decoherence back in time). In principle, we also need to consider the case of $C = 0$ (assuming that intensities of decoherence and recoherence exactly match each other) but this case is not realistic. Indeed, if the piston moves very slowly, then the densities of particles must approach the same values on both sides of the membrane $N_A/V_A = N_B/V_B = N_t/V_t$ and, obviously, $N_A(t) = V_A N_t/$

$V_t(t)$. On the one hand, $N_A(t)$ changes but, on the other hand, equation (9) with $C = 0$ enforces that $dN_A/dt = 0$. Therefore, particles must either predominately decohere or predominately recohere. This can be easily determined by moving the piston a bit faster so that the solution of equation (9) deviates from the equilibrium given by $N_A(t) = V_A N_t / V_t(t)$, as illustrated in Figure 3. We can observe either the behaviour indicated by line 3, which corresponds to $C = +1$, or the behaviour indicated by line 4, which corresponds to $C = -1$. The difference between the two cases is in the definition of the direction of time. As we use the conventional definition of the direction of time, where entropy increases toward $t = +\infty$, then $C = +1$ and particles predominantly decohere.

From the perspective of quantum mechanics, the state of equilibrium corresponds to the maximally mixed quantum state, where the density matrix is proportional to the unit matrix and the entropy is maximal. This state of maximal entropy cannot be altered without external interference; neither by unitary evolution (which cannot change entropy), nor by decoherence (which cannot reduce entropy). The effect of decoherence, therefore, is not observable in equilibrium conditions (as it should be — equilibrium states do not evolve). It would be rather unphysical to assume that decoherence, which exists at its full strength under smallest deviations from equilibrium, physically disappears once full equilibrium state is reached. It is the statistical effect of decoherence that disappears, not decoherence itself: it still affects individual particles at microscopic level. This can be illustrated by the Ehrenfest urn model: balls are located in two urns are picked up at random and are placed into another urn (possibly with a fixed probability reflecting the transmission rate between the urns). Each act of redistribution of balls increases uncertainty of ball locations, and therefore, increases the corresponding entropy. Once equilibrium is reached and the two urns have the same number of balls, the process (which still continues physically) does not change the distribution and does not change the entropy.

We observe a very interesting situation: the system stays in complete equilibrium for a very long time and should not be affected by any initial conditions that were imposed on the system or on the whole universe a long time before the experiment. According to the conditions of the experiment, all external influences must be macroscopic. These influences are limited to the movements of the piston, which are conducted in a time-symmetric manner and cannot possibly create any directionality of time. The known laws of classical, quantum and relativistic physics are also time-symmetric. Why do the particles behave in a time-directional manner (decohere and not recohere)? Reichenbach's principle of parallelism of entropy increase clearly requires that $C = +1$ in (9) and, at least under conditions shown in Figure 3, this cannot be directly explained by the low-entropy initial conditions imposed on the universe. Something must be missing.

3.3 The time primer

We, of course, do not suggest that predominance of decoherence ($C = +1$ in (9)) is not related to the low-entropy initial conditions imposed on the universe, but rather observe that there must be a physical mechanism that connects decohering properties of matter to the fundamental state of the universe. There is, however, no obvious or known mechanism that translates a low-entropy Big Bang into the fact that matter predominantly decoheres under conditions when matter is screened from the Big Bang by an equilibrium state (presumably destroying all information about the previous states of the universe). We can call this mechanism the "time primer"[13]. The time primer is related to the most fundamental properties of matter and its primary effect should be predominance of quantum decoherence, resulting in the second law of thermodynamics, causality and in the perceived "flow of time". The time primer must exist and, at least in principle, should be represented by a mechanism that can be detected in experiments but, as discussed in the rest of this paper, this is likely to be a very difficult task. The time primer may, of course, reflect environmental interferences but these interferences should be measurable and enacting the arrow of time without presuming antecedent causality.

The conventional quantum theories [31–33] explain the physical mechanism of decoherence quite well, but only under conditions, in which the direction of time is discriminated by implied causality: setting initial (and not final) conditions is essential for these theories. Therefore, we are trapped in a logical loop: we explain causality by the second law, the second law by decoherence, and decoherence by causality (Figure 4). The time primer points to an unknown physical effect that is needed to break this loop. For the case illustrated in Figure 3 there is no obvious justification for discriminating the directions of time by preferring the initial conditions to the final conditions. Price [18] noted that physical theories often discriminate the directions of time by intuitively implying time-directional causality — these may be valuable theories in many respects but they cannot serve, as physical explanations of the directional properties of time as these properties are presumed and not deduced.

4 Discussion

The current state of arguments about direction of time (illustrated in a simplified form by Figure 4) reflects persisting confusion: philosophers seek the assistance of the physical laws (and especially that of the second law of thermodynamics) in defining antecedent causality, while physicists base their justifications of physical laws and theories on implications of causality (often tacitly or implicitly). This state forms an unsatisfactory explanatory loop, in which antecedent causality is associated with the action of the second law and the second law is explained by the effects of antecedent causality. While the action of the second law can be related at an elementary level to implications of quantum decoherence and collapse, the quantum theory, as was remarked by Einstein half-a-century ago, still cannot provide a uni-
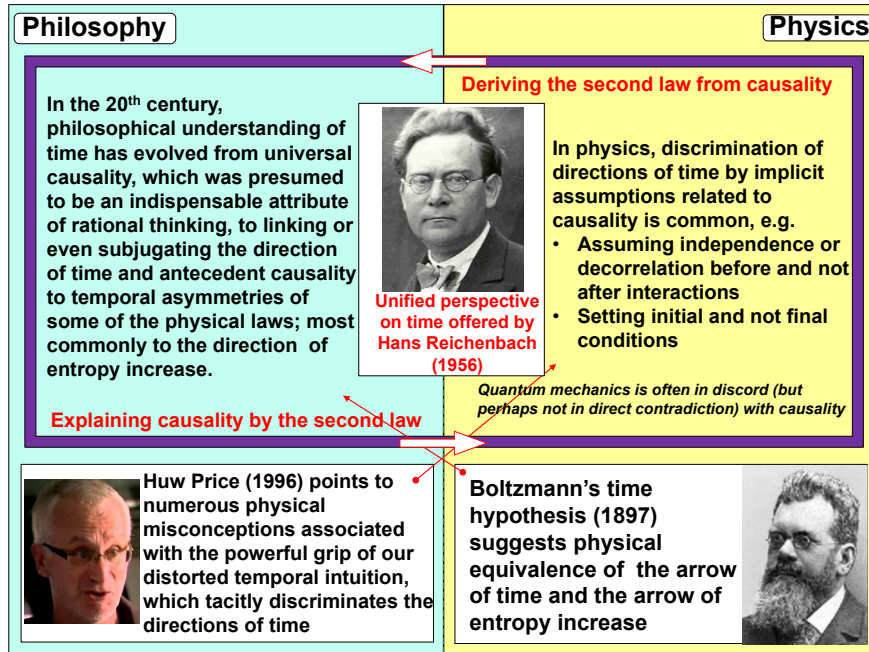
Fig. 4 If considered from a transdisciplinary perspective, arguments commonly used in physics and philosophy in explaining antecedent causality and the second law of thermodynamics form a logical circle

fied picture of physical reality. The time primer is not a physical theory but rather a placeholder for such a theory, recognising that something important is missing in our understanding of thermodynamic time.

Over the last few decades, the direction of time has experienced a gradual drift from the domain of philosophy to the domain of physics. While the influence of physical ideas and theories gradually increases, this transition has not been completed yet since the possibility of experimental validation is a necessary attribute of any physical theory. The possibility of experimental testing of the time priming is discussed in this section.

4.1 Environmental time priming

Decoherence may be induced by relatively weak interactions with the rest of the universe, since our universe is far away from equilibrium and (at least in principle) can induce time-directional effects in the system. Numerous quantum theories point to environmental interferences as the mechanism responsible for decoherence

and thermodynamic behaviour in quantum systems [33–39]. These theories, however, are not specific with respect to the physical mechanism of interactions, which make experimental validation of these interactions rather difficult and uncertain. Environmental interference of CP-violating and CPT-invariant quantum systems is expected to produce apparent CPT violations [13, 40] and detected CPT discrepancies [41] may be related to interference from the environment. The problem with experimental validation of environmental interference is that, even if this interference is detected, there is no guarantee that it is this interference and not something else that represents the principal mechanism controlling the time priming. To prove this point we need to reduce this interference and expect a corresponding reduction in consistency of time priming.

Radiation is likely to be the first suspect for thermodynamic interactions. Since radiation itself must be decoherence-neutral [26], its role should be in connecting the equilibrated system (in Figure 3) to matter that populates the universe and remains far from equilibrium. If the experiment is located in a remote area of the universe, incoming radiation can be interpreted as a random signal. This signal can stimulate decoherence, but it seems that presuming causality is unavoidable under these conditions [32, 33, 35].

If a system is placed far away from all other matter, a reduction in effectiveness of interactions can be expected. At present, however, we do not have any evidence that thermodynamic time slows down when a system is screened from the influence of (or placed far from) other thermodynamic systems. Would radioactive decays become any slower if an radioactive object is placed in a very remote part of the universe? There is no direct evidence that this would be the case. Reichenbach believed that complete insulation of a subsystem would not affect the rate of its entropy increase. This does not rule out environmental mechanisms of time priming, but it does illustrate that obtaining experimental proof of environmental time priming would be very difficult. In principle, there might be a "time field" that is present everywhere, and the direction of time is determined by very weak, yet very important, interactions with this field. This case, however, is practically indistinguishable from intrinsic mechanisms of decoherence.

4.2 Intrinsic mechanisms of time priming

Various theories modifying equations of quantum mechanics to incorporate quantum collapses and decoherences have been suggested [34, 42–45]. These theories, however, assume causality rather than attempt to explain causality (and some are merely empirical). The physical mechanism of entropy-increasing processes at microscopic level remains uncertain. Penrose [42, 46] suggested a physical mechanism that can "prime" the direction of time. This theory (due to Diosi and Penrose) points to gravitational effects as a culprit of irreversibilities observed in the quantum world. Gravity induces quantum violations causing collapses of otherwise reversible uni-

tary evolutions. This provides a very good illustration of how small these violations might be and how difficult it would be to directly detect them in experiments [46].

Considering that radiation is expected to remain decoherence-neutral we might extend this inference to all bosons and expect that the intrinsic source of decoherence must be hidden in the properties of matter, most likely in quark - containing particles (e.g. neutrons and protons) since quarks are known of being capable to violate time symmetry in weak interactions (e.g. known CP violations in mesons, which, in conjunction with CPT invariance, imply T violation). One may prefer to imagine that baryons are capable of accumulating and confining very large quantities of information (i.e. baryons have effectively infinite numbers of internal degrees of freedom that are not externally accessible under normal conditions). In this case, there remain two possibilities: baryons and antibaryons can violate unitarity of external quantum evolutions in a symmetric or antisymmetric manner, which result in either symmetric or antisymmetric extension of thermodynamics from matter into antimatter [27, 47]. Symmetric and antisymmetric versions of thermodynamics respectively correspond to CP- and CPT-invariant time priming and can not be valid simultaneously — only one of them can be (and is) real. The antisymmetric version may or may not correspond to the real world but, conceptually, it is quite attractive due to a number of reasons. One of these reasons is that, if antisymmetric thermodynamics is valid, it kinetically favours conversion of antimatter into matter and, at the same time, explains the present arrow of time by the relative abundance of matter over antimatter [27, 47]. If detected in experiments, antisymmetric thermodynamics can pinpoint at the intrinsic mechanisms of time priming. If it is the symmetric version that is real, then experimental examination of the intrinsic mechanisms of time priming becomes a more difficult task.

4.3 Testing the symmetry of time priming.

From a theoretical perspective, testing whether thermodynamics possesses symmetric or antisymmetric properties may seem straightforward — we just need to create thermodynamically significant quantities of antimatter and see which thermodynamic properties it has. Practically, producing significant quantities of antimatter can be extremely difficult. It might be possible, however, to test the symmetric/antisymmetric properties of thermodynamics at the present level of technology.

It seems that a system with some thermodynamic properties (i.e. quark-gluon plasma [48]) can be created at very small scales as a result of collision of high-energy protons and nuclei. For example, two protons may collide elastically producing two protons with different momenta or inelastically producing jets of multiple particles. While the former collisions are unitary, we are tempted to assume that the latter collisions have some thermodynamic features. If this thermodynamic interpretation of inelastic collisions is correct, collisions of two antiprotons should be the same as collisions of protons according to symmetric thermodynamics, and can be expected to be different from collisions of protons according to antisym-

metric thermodynamics. While the overall energy, momentum and other conserved properties must always be preserved, antisymmetric thermodynamics involves opposite entropy trends for matter and antimatter. Therefore, assuming that thermodynamic effects can play a role within very short times associated with collisions (which is a big assumption, of course), antisymmetric thermodynamics predicts that antiproton-antiproton collisions should tend to have smaller inelastic collision cross-sections than the inelastic cross-sections of the proton-proton collisions under the same conditions. In simple terms, collisions of antiprotons should be biased towards elastic collisions compared to collisions of protons under the same conditions. This attributes the action of the time primer to complex interactions of partons inside baryons, which are clearly revealed only when collision energies are sufficiently high. The extent of the differences between baryons and antibaryons is determined by persistency of the time primer (i.e. it might be difficult to collide two antiprotons inelastically). Symmetric thermodynamics does not predict any differences between inelastic cross-sections of protons and antiprotons.

Note that the implications of antisymmetric thermodynamics may produce an impression of CPT violations: protons and antiprotons can have different overall inelastic collision cross-sections [49]. According to interpretation given above, this conclusion would be incorrect — antisymmetric thermodynamics is based on complete CPT symmetry exhibited both at small and large scales. This effect is similar to apparent CPT violations that can be observed due to the presence of environmental mechanisms of time priming — see Ref [40] for details. It seems that microscopic action of time priming can be detected (due to its interference with unitarity) as apparently present CPT violations in systems that in fact strictly preserve the CPT symmetry.

Another possibility for testing the extension of thermodynamics from matter to antimatter is investigation of photon absorption and radiation by atoms and antiatoms under the same conditions. The antiatoms need to be trapped and cooled down, which is not easy but still possible [50]. The kinetics of light absorption and radiation is the same for atoms and antiatoms in symmetric thermodynamics and different in antisymmetric thermodynamics[26]. In simple terms, if antiatoms are somewhat more reluctant to adsorb photons than the corresponding atoms under the same conditions, then this would indicate validity of antisymmetric thermodynamics. Again, if such effects are detected, they must not be confused with CPT violations — antisymmetric thermodynamics is very much consistent with the CPT invariance.

5 Conclusions

This work briefly reviews and explains the principal ideas about time that were brought by the late Hans Reichenbach in his last book. The Boltzmann time hypothesis and the Reichenbach principle of parallelism of entropy increase seem to be most important among these ideas. While the Boltzmann time hypothesis tends

to be accepted by modern philosophers and physicists (at least by those who have thought about or investigated these issues), the principle of parallelism of entropy increase is still subject to debate. In the present work, we consider a mixing process involving quantum effects and demonstrate that, although the low-entropy initial conditions that characterised early universe are most important, there should be an unknown mechanism that delivers the influence of these initial conditions to thermodynamic subsystems observed in the real world. We call this mechanism the "time primer". The time primer is responsible for prevailing forward-time decoherence in quantum systems, which increases entropy and, according to the Boltzmann time hypothesis, introduces antecedent causality and other components of the perceived flow of time.

The possibility of experimental detection of the time primer is discussed in the last section — in general, this task is quite difficult. If, however, the time primer is CPT-invariant (rather than CP-invariant) and objects with some thermodynamic properties emerge at small scales in inelastic high-energy collisions, the direct effects of the time primer may be detected under the current level of technology.

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Appendix. Quantum tunnelling and decoherence

The quantum outcomes of tunnelling can be expressed by the scattering matrix $\mathbb{S}$, which is a unitary matrix (i.e. $\mathbb{S}\mathbb{S}^{\dagger} = \mathbb{I}$) that connects the amplitudes A^- and B^- of incoming waves with the amplitudes of the outgoing waves A^+ and B^+ (see Figure 5) so that:

$$\underbrace{\begin{bmatrix} \tilde{A}^+ \\ \tilde{B}^+ \end{bmatrix}}_{\psi(t_+)} = \underbrace{\begin{bmatrix} r & q \\ q & r \end{bmatrix}}_{\mathbb{S}} \underbrace{\begin{bmatrix} \tilde{A}^- \\ \tilde{B}^- \end{bmatrix}}_{\psi(t_-)}, \quad \underbrace{\begin{bmatrix} \tilde{A}^- \\ \tilde{B}^- \end{bmatrix}}_{\psi(t_-)} = \underbrace{\begin{bmatrix} r^* & q^* \\ q^* & r^* \end{bmatrix}}_{\mathbb{S}^{\dagger}} \underbrace{\begin{bmatrix} \tilde{A}^+ \\ \tilde{B}^+ \end{bmatrix}}_{\psi(t_+)} \tag{10}$$

where $\tilde{A}^+ = A^+ e^{ik\Delta/2}$, $\tilde{A}^- = A^- e^{-ik\Delta/2}$, $\tilde{B}^+ = B^+ e^{ik\Delta/2}$, $\tilde{B}^- = B^- e^{-ik\Delta/2}$ are the corresponding wave amplitudes evaluated at the boundaries of the barrier at $x = \pm\Delta/2$, the asterisk denotes complex conjugates and the values of q and r are specified below. The quantum barrier is assumed to be symmetric, which corresponds to a symmetric matrix $\mathbb{S}$. The matrix $\mathbb{S}$ should not be confused with the commonly used transfer matrix that links the wave amplitudes on one side of the barrier to the wave amplitudes on the other side. Note that $|q|^2 + |r|^2 = 1$ and $|r^2 - q^2| = 1$ due to the unitary of $\mathbb{S}$. The tunnelling parameters q and r can be determined for specific shape of the potential barrier $U(x)$, which is assumed to have a rectangular shape as shown in Figure 5. The solution of this problem can be found in standard textbooks [51]:

$$r = (k^2 + \kappa^2)\frac{(1 - Q^2)}{W}, \; q = 4ik\kappa\frac{Q}{W}, \; Q = \exp(-\kappa\Delta)$$

$$W = (k + i\kappa)^2 - (k - i\kappa)^2 Q^2, \; k = \frac{\sqrt{2mE}}{\hbar}, \; \kappa = \frac{\sqrt{2m(U_0 - E)}}{\hbar}$$

$$|q|^{-2} = 1 + \frac{1}{4}\frac{(k^2 + \kappa^2)^2}{k^2\kappa^2}\sinh^2(\kappa\Delta) \underset{U_0 \gg E}{\approx} \frac{1}{4}\frac{U_0}{E}\sinh^2\left(\Delta\frac{\sqrt{2mU_0}}{\hbar}\right) \tag{11}$$

where E is the energy of the particle, is the Planck constant and $|q|^2$ is the transmission coefficient. The barrier is assumed to be thin: i.e. its thickness Δ is small but its magnitude U_0 is large. We can assume that $U_0 \gg E$ and therefore $q \ll 1$, $r \sim 1$.
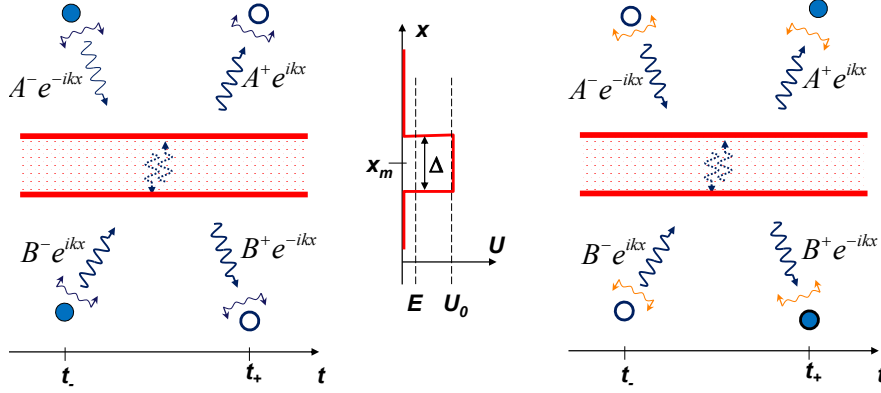

Fig. 5 Tunnelling of a particle through the membrane: left – with decoherence, right – with recoherence, middle – the membrane potential $U = U(x)$

The quantum description of tunnelling specified by (10) is time-symmetric, while its effect on the thermodynamic system considered here (Figure 3) is determined by the decoherence of quantum waves as shown in Figure 5. The decoherence transforms the time-reversible Schrodinger equation into the Pauli master equation, which is the principal equation that combines quantum description with directionality of time [52]. The Pauli master equations are general equations that incorporate decoherence, which determines the direction of the entropy increase, into the quantum world; i.e. different forms of the Pauli master equation are obtained for the same quantum system depending on properties of decoherence and recoherence [26, 27].

Since particles do not interact and classical statistics is implied (i.e. most quantum states are not occupied), one can consider the wave function ψ_j of a single particle. The Pauli master equation for the probabilities $p_j = \psi_j \psi_j^*$ (no summation over j) is given by [27]

$$\frac{dp_j}{dt} = \sum_k C w_j^k p_k - \sum_k C w_k^j p_j \tag{12}$$

where $C = +1$ corresponds to dominant decoherence and $C = -1$ corresponds to dominant recoherence, and $w_j^k = w_k^j$ are transitional probabilities. Note that, unlike in Ref. [26, 27], the predominant direction of the time priming is assumed to be the same for all quantum states. Consider states $a = a_1, a_2, ...$ on side A of the membrane and states $b = b_1, b_2, ...$ on side B of the membrane so that $j = a_1, a_2, ... b_1, b_2, ...$ and the states a_i and b_i, $i = 1, 2, 3, ...$ correspond to interacting waves with the same energy E_i. Evaluation of the two sums over $j = a_1, a_2, ...$ and over $j = b_1, b_2, ...$ in equation (12) while taking into account

$$\sum_a p_a = \frac{N_A}{N_t}, \quad \sum_b p_b = \frac{N_B}{N_t}, \quad N_t = N_A + N_B \tag{13}$$

yields

$$\frac{1}{N_t}\frac{dN_B}{dt} = -\frac{1}{N_t}\frac{dN_A}{dt} = C \sum_b \sum_a \left(w_b^a p_a - w_a^b p_b \right) \tag{14}$$

Substituting the equilibrium distribution g_j° (which are assumed to be classical Gibbs distributions due to $g_j^\circ \ll 1$) and the density of quantum states ρ_j

$$p_a = \frac{N_A}{N_t V_A} \rho_a g_a^\circ, \quad p_b = \frac{N_B}{N_t V_B} \rho_b g_b^\circ, \quad g_{a_i}^\circ = g_{b_i}^\circ = \exp\left(\frac{\mu - E_i}{k_B T}\right) \tag{15}$$

where μ is the chemical potential, we obtain

$$\frac{dN_B}{dt} = -\frac{dN_A}{dt} = C\left(K_1 \frac{N_A}{V_A} - K_2 \frac{N_B}{V_B}\right) \tag{16}$$

and

$$K_1 = \sum_b \sum_a w_b^a \rho_a g_a^\circ = \sum_i w_{b_i}^{a_i} \rho_{a_i} g_{a_i}^\circ, \quad K_2 = \sum_b \sum_a w_a^b \rho_b g_b^\circ = \sum_i w_{a_i}^{b_i} \rho_{b_i} g_{b_i}^\circ \tag{17}$$

since only the corresponding states a_i and b_i interact, i.e. $w_{a_j}^{b_i} = w_{b_j}^{a_i} = 0$ for $j \neq i$. The symmetry of the coefficients $w_a^b = w_b^a$ and equilibrium distributions $g_{a_i}^\circ = g_{b_i}^\circ = g_i^\circ$ and the same conditions on both sides of the membrane $\rho_{a_i} = \rho_{b_i} = \rho_i$ yield equation (9) with $K = K_1 = K_2$. The direction of thermodynamic time in this equation is determined by the temporal direction of decoherence (i.e. by $C = +1$ or $C = -1$). As expected [26], *the transmission rate is proportional to the concentration of decohered particles and does not depend on the concentration of recohered particles, irrespective of the temporal direction of decoherence or recoherence.*

Considering that the scattering matrix is close to unity, one can write $\mathbb{S} = \mathbb{I} + i\mathbb{T}$ where $\mathbb{T}$ is small (since $|q|^2 \ll 1$) and Hermitian $\mathbb{T}^\dagger = \mathbb{T}$ at the leading order. The operator $\mathbb{T}$ can be conventionally be expressed in terms of the interaction Hamiltonian by using perturbation methods, but this is not needed here as we already have the exact solution for the tunnelling problem. Substituting $w_{a_i}^{b_i} = w_{b_i}^{a_i} = \mathfrak{A} u_i |q|_i^2 / 2$, where $\mathfrak{A}$ is the area of the membrane, $|q|_i^2 \approx 4E_i e^{-2\kappa\Delta} / U_0$ is the transmission coefficient, $u_i^2 = 2E_i/m$ and $\kappa = (2mU_0)^{1/2} /$, into (17) results in

$$K = 2^{2\frac{1}{2}} \frac{\mathfrak{A}}{U_0 m^{\frac{1}{2}}} e^{-2\kappa\Delta} \sum_i E_i^{1\frac{1}{2}} \rho_i g_i^\circ \tag{18}$$

A more detailed analysis of tunnelling without decoherence under these conditions can be found in Ref. [30].