

**MATRI** 

# **Topology of Manifolds: Interactions Between High and Low Dimensions**

## **Programme and Abstracts**

**MATRIX, Creswick  
14 – 18 January 2019**

**MATRI** 



**TOPOLOGY OF MANIFOLDS:  
 INTERACTIONS BETWEEN HIGH AND LOW DIMENSIONS  
 CONFERENCE: JANUARY 13TH - 18TH 2019, MATRIX**

**Programme**

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00-9:55	Rubinstein	Lecuona	Piccirillo	Purcell	Kasprowski
9:55-10:05	break	break	break	break	break
10:05-11:00	Levine	Cappell	Thompson	Lambert-Cole	Bleile
11:00-11:35	coffee	coffee	coffee	coffee	coffee
11:35-12:30	Hom	Miller & Land	Khan	Nagy & Wings	Cha
12:30-2:30	lunch	lunch	lunch	lunch	lunch
2:30 - 3:45	Problem Session 2	free	free	free	Problem Session 3

All talks and both problems sessions take place in the lecture theatre, next to reception.

## Monday

HYAM RUBINSETIN  
UNIVERSITY OF MELBOURNE  
MONDAY 9:00 – 9:55

**Title.** *Multi-sections of PL-manifolds*

**Abstract.** Heegaard splittings of 3-manifolds have many useful properties and have been used for invariants such as Heegaard Floer homology and the Casson invariant. Trisections of 4-manifolds are the natural extension to dimension 4. We describe a construction in all dimensions which we call multi sections. A closed PL  $n$ -manifold is divided into  $k+1$  homeomorphic handlebodies ( thickened graphs) meeting along their boundaries in submanifolds with special spines. Here  $n=2k$  or  $2k+1$   $n=3$  gives a Heegaard splitting and  $n=4$  a trisection.

This is joint work with Stephan Tillmann.

ADAM LEVINE  
DUKE UNIVERSITY  
MONDAY 10:05 – 11:00

**Title.** *Simply-connected, spineless 4-manifolds*

**Abstract.** Given an  $m$ -dimensional manifold  $M$  that is homotopy equivalent to an  $n$ -dimensional manifold  $N$  (where  $n \leq m$ ), a spine of  $M$  is a piecewise-linear embedding of  $N$  into  $M$  (not necessarily locally flat) realizing the homotopy equivalence. When  $m-n=2$  and  $m \geq 4$ , Cappell and Shaneson showed that if  $M$  is simply-connected or if  $m$  is odd, then it contains a spine. In contrast, I will show that there exist smooth, compact, simply-connected 4-manifolds which are homotopy equivalent to the 2-sphere but do not contain a spine (joint work with Tye Lidman). I will also discuss some related results about PL concordance of knots in homology spheres (joint with Lidman and Jen Hom).

JEN HOM  
GEORGIA TECH  
MONDAY 11:35 – 12:30

**Title.** *An infinite-rank summand of the homology cobordism group*

**Abstract.** We show that the three-dimensional homology cobordism group admits an infinite-rank summand. It was previously known that the homology cobordism group contains an infinite-rank subgroup and a  $\mathbb{Z}$ -summand. Our proof proceeds by introducing an algebraic variant of the involutive Heegaard Floer package of Hendricks-Manolescu and Hendricks-Manolescu-Zemke. This is joint work with I. Dai, M. Stoffregen, and L. Truong.

OPEN PROBLEM SESSION 2  
TUESDAY 2:30 – 3:30

Participants are asked think of open problems in their research areas. These problems could either be foundational (and hard); e.g. “Does a 4-manifold admit a CW-structure?”, or more specific research problems.

The problem session will proceed by participants using half a whiteboard to write their problem(s) *before* the session begins. During the session participants will have 5 minutes to explain their problem and there may be 5 minutes more for discussion.

Participants will then be asked to write their names next to problems they wish to work on during the week: here “work” means either working to solve the problem or to research its background and relevant literature or both.

An important aim of the conference is to produce a list of well researched problems to stimulate future research in manifold topology. There will be a final problem session on Friday for reports on progress on problems.

## Tuesday

ANA LECUONA  
UNIVERSITY OF GLASGOW  
TUESDAY 9:00 – 9:55

**Title.** *Torus knots and rational homology balls*

**Abstract.** In this talk we will try to motivate and partially answer the question: which integer surgeries on torus knots result in 3 manifolds which bound rational homology balls? Fixing the torus knot to be positive, we have a complete answer in the case of positive surgeries, while the case of negative surgeries is widely open.

Our approach combines Kirby calculus, Heegaard-Floer homology and the combinatorial study of lattice embeddings. This work is a joint project with P. Aceto, M. Golla and K. Larson.

SYLVAIN CAPPELL  
NYU COURANT INSTITUTE  
TUESDAY 10:05 – 11:00

**Title.** *Using Atiyah-Bott classes to produce polynomial invariants of 3-manifolds*

**Abstract.** tba

MAGGIE MILLER  
PRINCETON UNIVERSITY  
TUESDAY 11:30 – 12:00

**Title.** *Extending fibrations from knot complements to ribbon disk complements*

**Abstract.** Casson and Gordon showed that if a fibered knot  $K$  in the 3-sphere bounds a ribbon disk  $D$  in the 4-ball then  $K$  also bounds a fibered disk  $E$  in a homotopy 4-ball  $V$  (That is,  $V \setminus nE$ , where  $nE$  is a tubular neighbourhood of  $E$ , is a fiber bundle over the circle.) One natural question: is  $D=E$ ? (This is a stronger question than "is  $V$  homeomorphic to the 4-ball?".)

I can show that  $D$  is itself fibered when it satisfies a transversality condition. (In particular, it is sufficient for  $D$  to have only two minima, extending a result of Scharlemann about ribbon disks bounded by the unknot.) The proof is constructive. In this talk, I will discuss this motivation and the general strategy for constructing a (singular or nonsingular) fibration of a 4-manifold with boundary over the circle.

MARKUS LAND  
UNIVERSITY OF REGENSBURG  
TUESDAY 12:00 – 12:00

**Title.** *A vanishing theorem for tautological classes of aspherical manifolds*

**Abstract.** I will report on joint work with Hebestreit, Lck, and Randal-Williams about tautological classes for aspherical manifold bundles. Tautological classes define particular classes in the cohomology of the diffeomorphism group of a manifold and are constructed via tangential bundle theoretic information. Prior to our work not much was known about tautological classes of aspherical manifolds.

In the talk I will explain the ideas that go into our main result, which states that many of these tautological classes vanish, provided the fundamental group of the given aspherical manifold satisfies two well-known conjectures.

## Wednesday

LISA PICCIRILLO  
UNIVERSITY OF TEXAS AT AUSTIN  
WEDNESDAY 9:00 – 9:55

**Title.** *The Conway knot is not slice*

**Abstract.** Smooth simply connected 4-manifolds can admit second homology classes not representable by smoothly embedded spheres; knot traces are the prototypical example of 4-manifolds with such classes. I will show that there are knot traces where the minimal genus smooth surface generating second homology is not of the canonical type, resolving question 1.41 on the Kirby problem list. I will also use the same tools to show that Conway knot does not bound a smooth disk in the four ball, which completes the classification of slice knots under 13 crossings and gives the first example of a non-slice knot which is both topologically slice and a positive mutant of a slice knot.

ABIGAIL THOMPSON  
UC DAVIS  
WEDNESDAY 10:05 – 11:00

**Title.** *Trisections and surgery questions on links in 3-manifolds*

**Abstract.** Gay and Kirby have shown that a closed 4-manifold admits a trisection, which is a decomposition of the manifold into three 4-dimensional handlebodies. Understanding the relation between a trisection and a standard handle-decomposition of a 4-manifold leads to natural questions about Dehn surgeries on links in 3-manifolds. I'll describe this relation, and examples of surgery questions that arise, as well as some progress on the simplest of these. This is joint work with Rob Kirby.

QAYUM KHAN  
SAINT LOUIS UNIVERSITY  
WEDNESDAY 11:35 – 12:30

**Title.** Stable existence of incompressible 3-manifolds in 4-manifolds

**Abstract.** Given an injective amalgam at the level of fundamental groups and a specific 3-manifold, is there a corresponding geometric-topological decomposition of a given 4-manifold, in a stable sense? We find an algebraic-topological splitting criterion in terms of the orientation classes and universal covers. Also, we equivariantly generalize the Lickorish-Wallace theorem to regular covers. This is joint work with my former student, Gerrit Smith PhD, and is accepted for publication in *Topology and its Applications*.

## Thursday

JESSICA PURCELL  
MONASH UNIVERSITY  
9.00 – 9:55

**Title.** Combinatorial criteria to determine whether a state surface is a fiber

**Abstract.** Given a link diagram, every choice of Kauffman state gives rise to a spanning surface for the link called a state surface, as well as an associated state graph embedded in the state surface as a spine. State surfaces are easy to describe from a diagram, and arise in both quantum topology and hyperbolic geometry, and so we wish to investigate their properties. For this talk, we are interested in the question of when a state surface is a fiber. We show that whether a state surface is a fiber can be determined from the state graph, in almost linear time. In particular, a state graph can be decomposed along cut-vertices into graphs with induced planar embeddings associated with checkerboard surfaces. The original state surface is a fiber if and only if all the associated checkerboard surfaces are fibers. We give an algebraic condition that characterizes which planar state graphs give fibers. This work is joint with Darlan Girao.

PETER LAMBERT-COLE  
GEORGIA TECH  
10:05 – 11:00

**Title.** Bridge trisections and the Thom conjecture

**Abstract.** The classical degree-genus formula computes the genus of a nonsingular algebraic curve in the complex projective plane. The well-known Thom conjecture posits that this is a lower bound on the genus of smoothly embedded, oriented and connected surface in  $\mathbb{C}P^2$ . The conjecture was first proved twenty-five years ago by Kronheimer and Mrowka, using Seiberg-Witten invariants. In this talk, we will describe a new proof of the conjecture that combines contact geometry with the novel theory of bridge trisections of knotted surfaces. Notably, the proof completely avoids any gauge theory or pseudoholomorphic curve techniques.

CSABA NAGY  
UNIVERSITY OF MELBOURNE  
11:30 – 12:00

**Title.** The Q-form conjecture for some 1-connected manifolds

**Abstract.** I will sketch a proof of the Q-form conjecture for 1-connected  $2q$ -manifolds with normal  $(q-1)$ -type B such that  $H_q(B; \mathbb{Z})$  is torsion-free. As an application, I will give a bordism theoretic computation of the generalised inertia group for certain classes of 8-manifolds.

CHRISTOPH WINGES  
UNIVERSITY OF BONN  
12:00 – 12:30

**Title.** Mapping class groups of high-dimensional, aspherical manifolds

**Abstract.** I will give a quick overview of the Farrell-Jones conjectures. These conjectures predict the values of certain invariants of group rings (like algebraic K-, L- and A-theory) relevant to geometric topology. In particular, I will explain how the conjecture gives a description of the mapping class groups of high-dimensional, aspherical manifolds in terms of generalised Dehn twists, and in which cases this description is known to be correct. This involves joint work with Enkelmann, Kasprowski, Lück, Pieper, Ullmann and Wegner.

## Friday

DANIEL KASPROWSKI  
UNIVERSITY OF BONN  
9:00 – 9:55

**Title.**  $\mathbb{C}P^2$ -stable diffeomorphism of 4-manifolds

**Abstract.** Two 4-manifolds are said to be  $\mathbb{C}P^2$ -stably diffeomorphic if after connected sum with arbitrarily many copies of the complex projective plane, they become diffeomorphic. We show that for many fundamental groups the classification of 4-manifolds up to  $\mathbb{C}P^2$ -stable diffeomorphism is determined by the homotopy 2-type. This is joint work with Mark Powell and Peter Teichner.

BEA BLEILE  
UNIVERSITY OF NEW ENGLAND  
10:05 – 11:00

**Title.** Homotopy Types of Poincaré Duality Complexes

**Abstract.** In dimension 3 homotopy types of Poincaré duality complexes are classified by Turaev's fundamental triple. Using the stable module category Turaev provided a necessary and sufficient condition for such a triple to be realised by a Poincaré duality complex. We discuss generalisations of these results to higher dimensions.

JAE CHOON CHA  
POSTECH  
11:35 – 12:30

**Title.** TBA

**Abstract.** tba

OPEN PROBLEM SESSION 3  
2:30 – 3:30

Participants are asked to briefly report on progress—either towards a solution or to understanding the background—for the problem(s) they nominated to work on in Problem Session 2.

Once again, participants are asked to write a short summary of their problem on half a white-board prior to the problem session.

New problems can also be suggested in this session: previous problems can also be restated.

At the end of this session, there will be a moderated discussion to summarise and evaluate the status and potential of problems discussed.