Pure point spectrum and regular model sets in substitution tilings on $\mathbb{R}^d$

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It has long been known that every regular model set has pure point spectrum, but the converse is not true in general. The relation between regular model sets and pure point spectrum is well studied in [3, 2, 13] in quite a general setting. When we restrict to substitution tilings, it has been shown in [8] that pure point spectrum and inter model set are equivalent. However the inter model set is a projected point set in a cut-and-project scheme (CPS) with an internal space which is constructed with an autocorrelation topology coming from pure point spectrum. It was not easy to extract information from the internal space.

In this joint work with Shigeki Akiyama, we show that the internal space can be a Euclidean space under some additional assumption. This result generalizes the remark [4], which shows the equivalence between regular model set and pure point spectrum in the case of one-dimensional substitution tilings, into d dimensions. From this result, we can think of 'Pisot conjecture' in more general setting of d-dimensional substitution tilings [1, 12]. Rigidity was introduced in [11] and primitive substitution tilings with finite local complexity (FLC) always show this type of rigidity. But the converse is not true as we can observe in an example in [5]. Under the rigidity assumption, pure point spectrum always gives FLC. So we do not asume FLC. Instead we assume rigidity.

We first show how to construct a CPS with Euclidean internal space. This construction was already introduced in [7] for other purposes. We make use of it in a more general setting. Under the assumption of pure point spectrum, we provide conditions under which the representive point set of a primitive repetitive substitution tiling is a model set. Using Keesling's argument [6, 9], the model set is in fact regular. In the process of the proof, we use the equivalence between pure point spectrum and algebraic coincidence which was introduced in [8].

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