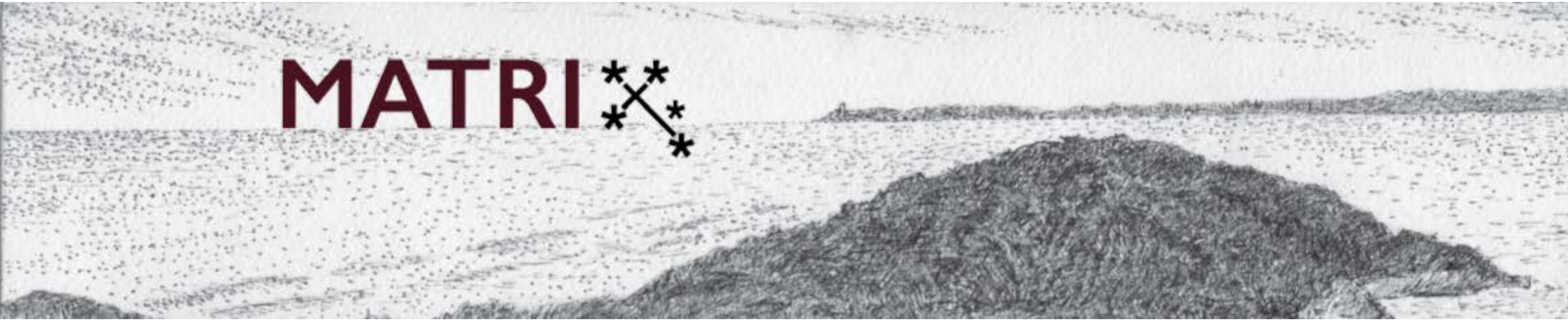

MATRIX Program:

Early Career Researchers Workshop on Geometric Analysis & PDEs

Talk Title and Abstracts

Week 2: 19 – 24 January 2020

MATRIX, Creswick

Abstracts: Matrix ECR Geometric Analysis and PDE

Bourni, Theodora (University of Tennessee)

Translating and ancient solutions to mean curvature flow

We show, in all dimensions $n \geq 2$, that there exists a convex translator lying in a slab of width $\pi \sec \theta$ in R^{n+1} (and in no smaller slab) if and only if $\theta \in [0, \frac{\pi}{2}]$. We will moreover discuss how to construct collapsed ancient solutions in dimension 2 by "gluing" translating ones.

Chodosh, Otis (Stanford University)

On the index of minimal surfaces

I'll survey some recent (and less recent) work on the Morse index of minimal surfaces.

DelaTorre, Azahara (University of Freiburg)

Concentration phenomena for the fractional Q-curvature equation in dimension 3 and fractional Poisson formulas

We study compactness properties of metrics of prescribed fractional Q-curvature of order 3 in R^3. We use an approach inspired from conformal geometry, regarding a metric on a subset of R^3 as the restriction of a metric on R^4_+ with vanishing fourth-order Q-curvature. In particular, in analogy with a 4-dimensional result of Adimurthi, Robert and Struwe, we prove that a sequence of such metrics with uniformly bounded fractional Q-curvature can blow up on a large set (roughly, the zero set of the trace of a nonpositive biharmonic function Φ in R^4_+), and we also construct examples of such behaviour. Towards this result, an intermediate step of independent interest is the construction of general Poisson-type representation formulas (also for higher dimension).

This is a work done in collaboration with María del Mar González, Ali Hyder and Luca Martinazzi.

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Kotschwar, Brett (Arizona State University)

The maximal rate of convergence under the Ricci flow

We will show that a solution to the normalized Ricci flow on a closed manifold which converges modulo di˙eomorphisms to a soliton faster than any fixed exponential rate must itself be self-similar, and estimate from above the rate at which a nontrivial solution may converge.

Kwong, Kwok Kun (University of Wollongong)

Generalized convexity and some sharp comparison theorems

I will show how generalized convexity can be used to prove the classical Toponogov triangle comparison theorem and a sharp isoperimetric type inequality involving the cut distance of a bounded domain. More precisely, I will show that among all domains with cut distance l and with a Ricci curvature lower bound $(n - 1)k$, the ball of radius l in the space form of curvature k has the largest area-to-volume ratio. I will also present a closely related Heintze-Karcher type inequality which relates the volume of the do-main with a boundary integral involving the mean curvature. If time allows, I will give some applications of these results and another isoperimetric-type inequality involving the extrinsic radius of the domain.

Lambert, Ben (University College London)

Lagrangian mean curvature flow with boundary conditions.

The foundational result of Lagrangian Mean Curvature Flow (LMCF) is that in Calabi–Yau manifolds, high codimensional mean curvature flow preserves the Lagrangian condition. A natural question is then to ask if this can this be generalised to manifolds with boundary. Equivalently, what is a well-defined boundary condition for LMCF? In this talk I will provide an answer to this question, and then demonstrate that the resulting flow exhibits good behaviour in two model situations, namely with boundary on the Lawlor neck and Cli˙ord Torus respectively. No prior knowledge of LMCF will be assumed. This work is joint with Chris Evans and Albert Wood.

Langford, Mat (University of Tennessee)

Ancient pancakes

I will discuss recent existence and uniqueness results for ancient solutions to mean curvature flow, including the classification of (noncompact) convex ancient curve shortening flows and the construction and uniqueness of the rotationally symmetric 'ancient pancake'. This is joint work with Theodora Bourni and Giuseppe Tinaglia.

Mazowiecka, Katarzyna (Université catholique de Louvain)

On the size of the singular set of minimizing harmonic maps

Abstract: Minimizing harmonic maps (i.e., minimizers of the Dirichlet inte-gral) with prescribed boundary conditions are known to be smooth outside a singular set of codimension 3. I will consider mappings from an n-dimensional domain with values in the two dimensional sphere. I will present an exten-sion of Almgren and Lieb's linear law on the bound of the singular set. Next, I will investigate how the singular set is a˙ected by small perturbations of the prescribed boundary map and present a stability theorem, which is an extension of Hart and Lin's result. I will also discuss possible extensions to di˙erent target manifolds and the optimality of our assumptions. This is joint work with Michał Mi±kiewicz and Armin Schikorra.

Nurseletov, Medet (The University of Sydney)

Eigenvalue asymptotics for weighted Laplace equations on rough Riemannian manifolds with boundary

We investigate an asymptotic of eigenvalues of the weighted Laplace equa-tion, Δu = λP u, on the Riemannian manifold equiped with the rough metric. Namely, for the di˙erent boundary conditions, we prove the Weyl asymptotic for both negative and positive eigenvalues.

Petrache, Mircea (Pontificia Universidad Catiolica de Chile)

Uniform measures and manifolds all of whose curvatures are con-stant

A uniform measure in Euclidean space R^d is a measure with respect to which balls B(x,r) with center x in the support, are assigned mass dependent of r and independent of the choice x. For example any invariant measure with

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respect to a subgroup of the isometry group of R^d is uniform, and called a homogeneous measure. However we also have a few exotic examples of non-homogeneous uniform measures, such as the volume measure of the "light cone" $\{x^2+y^2+z^2=w^2\}$ in R^4. This class of measures was first s tudied by David Preiss as the crucial ingredient of his 1987 proof of the Besicovitch conjecture. The complete classification o f u niform m easures r emains a diÿ-cult open problem, even restricted to ambient dimension d=2. I will detail the known classification o f 1 -dimensional u niform m easures i n R^d f or gen-eral d, for which, in joint work with Paul Laurain, we show that they are constituted of disjoint unions of helices or of toric knots, or equivalently, of analytic curves all of whose curvatures are constant.

Pluda, Alessandra (University of Pisa)

Willmore energy for 1-dimensional singular structures

In the last decades the study of the Willmore functional for surfaces and curves raised the attention of an increasing number of mathematicians. The reason is twofold: the analysis of the minimization of the Willmore functional presents interesting features and issue from a purely mathematical point of view and the problem naturally arises in mechanical engineering and mate-rials science (for instance studies of the properties of biological membranes, polymer gels, fiber o r p rotein n etworks) I n t his t alk I w ill g ive a n overview of some results related both to the minimization of the Willmore functional in the class of networks (finite union of curves) and to the L^2—gradient flow of this functional. I will also discuss future research directions.

Pulemotov, Artem (The University of Queensland)

The prescribed Ricci curvature problem for naturally reductive metrics on Lie groups

Let G be a compact Lie group. Naturally reductive left-invariant metrics form an important class of Riemannian metrics on G (we denote it M_{nat}) nested between the class of all left-invariant metrics and the class of bi-invariant ones. In the talk, we discuss the prescribed Ricci curvature equa-tion Ric g = cT assuming~$T \in M_{nat}$. The unknowns here are the metric $g \in M_{nat}$ and the scaling factor~$c > 0$. After stating a series of general results, we will consider examples that help understand the nature of so-lutions and reveal several surprising phenomena. Joint work with Romina Arroyo (The University of Queensland) and Wolfgang Ziller (The University of Pennsylvania).

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Sharp, Ben (University of Leeds)

Global estimates for harmonic maps from surfaces

A celebrated theorem of F. Hélein guarantees that a weakly harmonic map from a two-dimensional domain is always smooth. The proof is of a local nature and assumes that the Dirichlet energy is suÿciently small; under this condition it is possible to re-write the harmonic map equation using a suitably chosen frame which uncovers non-linearities with more favourable regularity properties (so-called div-curl or Wente structures). We will prove a global estimate for harmonic maps without assuming a small energy bound, utilising a powerful theory introduced by T. Rivière. This is a joint work with Tobias Lamm.

Wang, Xianfeng (Nankai University)

A new phenomenon involving inverse curvature flows in hyperbolic space

Inverse curvature flows for hypersurfaces in hyperbolic space have been investigated intensively in recent years. In 2015, Hang and Wang constructed an example to show that the limiting shape of the inverse mean curvature flow in hyperbolic space is not necessarily round after scaling. This was extended by Li, Wang and Wei in 2016 to the inverse curvature flow in hy-perbolic space by H^{-p} with power $p \in (0, 1)$. Recently, we discover a new phenomenon involving inverse curvature flows in hyperbolic space. We find that for a large class of symmetric and 1-homogeneous curvature functions F of the shifted Weingarten matrix $W - I$, the inverse curvature flow with initial horospherically convex hypersurface in hyperbolic space and driven by F^{-p} with $p \in (0, 1]$ will expand to infinity in finite time. The flow is asymptotically round smoothly and exponentially as the maximum time is approached, which means that the flow becomes exponentially close to a flow of geodesic spheres. We also construct a counterexample to show that our results cannot be extended to the case with power $p > 1$. This is a joint work with Dr. Yong Wei (ANU) and Dr. Tailong Zhou (Tsinghua University).

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Wei, Yong (Australian National University)

Curvature flows and geometric inequalities in hyperbolic space

I will introduce the geometric structures of horospherically convex (or h-convex) hypersurfaces in hyperbolic space, including horospherical support function, horospherical Gauss map and hyperbolic principal curvature radii, which can be viewed as hyperbolic analogue of the convex geometry in Euclidean space. Then I will discuss the proof of some new geometric inequalities for h-convex hypersurfaces in hyperbolic space, proved using geometric flows driven by functions of hyperbolic principal curvature radii. Joint work with Ben Andrews (ANU) and Xuzhong Chen (Hunan University).

Wu, Yuhan (University of Wollongong)

Short time existence for higher order curvature flows with and without boundary conditions

We prove short time existence for higher order curvature flows of plane curves with and without generalised Neumann boundary condition and describe how these results fit into a broader framework for analysis of the behaviour of these flows.

Xiong, Changwei (Australian National University)

Proof of Escobar's conjecture in the case of nonnegative sectional curvature

J. Escobar in 1999 conjectured that for an n-dimensional ($n \geq 3$) compact Riemannian manifold with nonnegative Ricci curvature and with bound-ary principal curvatures bounded below by $c > 0$, its first nonzero Steklov eigenvalue is no less than c, with equality holding only for Euclidean balls with radius $1/c$. We confirm Escobar's conjecture in the case of nonnegative sectional curvature. In this talk we will outline the proof and also discuss related problems. This is a joint work with Chao Xia (Xiamen University, China).