A note on the scattering for 3D quantum Zakharov system with non-radial data in L^2

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Abstract In this note, we give a remark on the scattering for quantum Zakharov system with non-radial small initial data in L^2 with one order additional angular regularity using the generalized Strichartz estimate with wider range and the normal form transformation.

1 Introduction

We study the scattering of solutions to the 3D quantum Zakharov system

$$\begin{cases} iu_t + \Delta u - \varepsilon^2 \Delta^2 u = nu, \\ n_{tt} - \Delta n + \varepsilon^2 \Delta^2 n = \Delta(|u|^2), \\ u(0,x) = u_0,\ n(0,x) = n_0,\ \partial_t n(0,x) = n_1, \end{cases} \tag{1}$$

where $u(t,x) : \mathbb{R}^1 \times \mathbb{R}^3 \to \mathbb{C}$ is the envelope electric field and $n(t,x) : \mathbb{R}^1 \times \mathbb{R}^3 \to \mathbb{R}$ describes the plasma density fluctuation. The quantum parameter $0 < \varepsilon \leq 1$ is the ratio between the ion plasmon energy and the electron thermal energy. For detailed background of this system, see [6].
The solutions (u,n) of (1) preserve the mass $\|u(t)\|_{L^2}$ and the energy

$$E(u,n,\partial_t n) = \int_{\mathbb{R}^d} |\nabla u(t)|^2 + \varepsilon^2 |\Delta u(t)|^2 + n|u|^2 + \frac{1}{2}(|D^{-1} n_t|^2 + n^2 + \varepsilon^2 |\nabla n(t)|^2) dx.$$

When $\varepsilon = 0$, (1) reduces to the classical Zakharov system.
For simplicity, we change (1) to a lower order system by letting

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$$N = n - \frac{in_t}{\sqrt{-\Delta + \varepsilon^2 \Delta^2}}. \tag{2}$$

Then (1) is transformed to

$$\begin{cases} iu_t - (-\Delta + \varepsilon^2 \Delta^2) u = (\bar{N}u + Nu)/2, \\ iN_t + \sqrt{-\Delta + \varepsilon^2 \Delta^2} N = \frac{\Delta}{\sqrt{-\Delta + \varepsilon^2 \Delta^2}} (|u|^2), \end{cases} \tag{3}$$

with

$$u(0) = u_0, \quad N(0) = n_0 - i(-\Delta + \varepsilon^2 \Delta^2)^{-\frac{1}{2}} n_1.$$

The treatment for $\bar{N}u$ is similar to Nu, we may assume that the nonlinear term in the first equation of (3) is $\bar{N}u$. The global well-posedness of (1) in energy space was obtained in [5] when $d = 1,2,3$. As pointed out in [1] that L^2 is the most important function space in mathematics and it is also important for Zakharov type system since it measures the total electric energy in physics, to this motivation the authors studied the local well-posedness with large data($1 \le d \le 8$), global well-posedness($1 \le d \le 5$) and scattering for small initial data($4 \le d \le 8$) of (3) in $L^2(\mathbb{R}^d) \times L^2(\mathbb{R}^d)$, but for $1 \le d \le 3$, scattering is not obtained in [1]. One of the main difficulties of proving scattering for quantum Zakharov systemin in low dimensions is the quadratic nonlinearities. Recently, the scattering for 3D quantum Zakharov system in $L^2(\mathbb{R}^3) \times L^2(\mathbb{R}^3)$ with small radial initial data was proved in [7] using normal form transformation and radial improved Strichartz estimates. In this note, we explain that the radial condition can be removed if we assume additional angular regularity of degree one. The Sobolev space with one order angular regularity $H^{0,1}_{2,\sigma}$ is defined in (5), the angular derivative D_σ is defined in Subsection 1.1, the Strichartz norm S and W are defined in (1). The main result is the following

Theorem 1.1 $d = 3$. *Suppose that* $\|(u_0, N_0)\|_{H^{0,1}_{2,\sigma}(\mathbb{R}^3) \times H^{0,1}_{2,\sigma}(\mathbb{R}^3)} = \varepsilon_0 > 0$ *which is small enough, then there exists a unique global solution* (u,N) *of* (3) *satisfying* $\|(u,N)\|_{S \times W} \le C\varepsilon_0$ *and scatters in this space. Namely, there exists a solution* $(u^\pm, N^\pm) \in H^{0,1}_{2,\sigma}(\mathbb{R}^3) \times H^{0,1}_{2,\sigma}(\mathbb{R}^3)$ *to the linear system*

$$\begin{cases} iu_t - (-\Delta + \varepsilon^2 \Delta^2) u = 0, \\ iN_t + \sqrt{-\Delta + \varepsilon^2 \Delta^2} N = 0, \end{cases} \tag{4}$$

satisfying

$$\|u(t) - u^\pm(t)\|_{L^2} + \|N(t) - N^\pm(t)\|_{L^2} + \|D_\sigma(u(t) - u^\pm(t))\|_{L^2} + \|D_\sigma(N(t) - N^\pm(t))\|_{L^2} \to 0, \quad .$$

as $t \to \pm\infty$.

Next we introduce some notations used in this note.

1.1 Notation

For $x \in \mathbb{R}^n$, write $\langle x \rangle := (1+|x|^2)^{1/2}$. We use $\hat{f}$ or $\mathscr{F} f$ to denote the Fourier transform of f. Write $D := \sqrt{-\Delta} = \mathscr{F}^{-1}|\xi|\mathscr{F}$ and $\langle D \rangle^s := \mathscr{F}^{-1}(1+|\xi|^2)^{s/2}\mathscr{F}$. Let $\eta : \mathbb{R}^3 \to \mathbb{R}$ be a smooth bump function supported in $B_2(0)$ and equal to 1 in $B_1(0)$. For $k \in \mathbb{Z}$, let $\chi_k(\xi) = \eta(\xi/2^k) - \eta(\xi/2^{k-1})$ and $\chi_{\leqslant k}(\xi) = \eta(\xi/2^k)$. The Littlewood-Paley operators are defined by

$$\widehat{P_k(\xi)} = \chi_k(|\xi|)\widehat{u}(\xi), \quad \widehat{P_{\leq k}(\xi)} = \chi_{\leqslant k}(|\xi|)\widehat{u}(\xi).$$

Let Δ_σ be the Laplace-Beltrami operator on the unit sphere $\mathbb{S}^{d-1}$ endowed with standard metric g and measure $d\sigma$. Denote $D_\sigma = \sqrt{-\Delta_\sigma}$ and $\Lambda_\sigma = \sqrt{1-\Delta_\sigma}$. For $1 \leq i,j \leq d$, $X_{i,j} = x_i\partial_j - x_j\partial_i$ are rotational vector fields and for $f \in C^2(\mathbb{R}^d)$, $\Delta_\sigma(f)(x) = \sum_{1\leq i,j\leq d} X_{i,j}^2(f)(x)$.

$L^p(\mathbb{R}^d)$ denotes the usual Lebesgue space and $\mathscr{L}^p(\mathbb{R}^+) = \mathscr{L}^p(\mathbb{R}^+ : \rho^{d-1}d\rho)$. We follow the notations in [2] and write $L^p_\sigma = L^p_\sigma(\mathbb{S}^{d-1})$, $\mathscr{H}^s_p = \mathscr{H}^s_p(\mathbb{S}^{d-1}) = \Lambda_\sigma^{-s}L^p_\sigma$. $\mathscr{L}^p_\rho L^q_\sigma$ and $\mathscr{L}^p_\rho \mathscr{H}^s_q$ are Banach spaces defined by the norms $\|f\|_{\mathscr{L}^p_\rho L^q_\sigma} = \|\|f(\rho\sigma)\|_{L^q_\sigma}\|_{\mathscr{L}^p_\rho}$ and $\|f\|_{\mathscr{L}^p_\rho \mathscr{H}^s_q} = \|\|f(\rho\sigma)\|_{\mathscr{H}^s_q}\|_{\mathscr{L}^p_\rho}$.

For $s \in \mathbb{R}$, $1 \leq p \leq \infty$, H^s_p denotes Banach space of elements $u \in \mathscr{S}'(\mathbb{R}^d)$ such that $\mathscr{F}^{-1}(1+|\xi|^2)^{s/2}\hat{u} \in L^p(\mathbb{R}^d)$ and $H^s(\mathbb{R}^n) = H^s_2(\mathbb{R}^n)$. The homogeneous Sobolev space $\dot{H}^s$ is defined by $\dot{H}^s(\mathbb{R}^d) = \{u \in \mathscr{S}'(\mathbb{R}^d) : \|u\|_{\dot{H}^s} = \||\xi|^s \hat{f}(\xi)\|_{L^2_\xi} < \infty\}$.

For $s \in \mathbb{R}$ and $1 \leq p,q,r \leq \infty$, $\dot{B}^s_{p,q}(\mathbb{R}^d)$ is the standard homogeneous Besov space on $\mathbb{R}^d$ with norm $\|u\|_{\dot{B}^s_{p,q}(\mathbb{R}^d)} := \left(\sum_{k\in\mathbb{Z}} 2^{qsk}\|P_k u(x)\|_p^q\right)^{1/q}$. $\dot{B}^s_{(p,q),r}$ denotes the Besov type space with norm

$$\|u\|_{\dot{B}^s_{(p,q),r}} := \left(\sum_{k\in\mathbb{Z}} 2^{rsk}\|P_k u\|^r_{\mathscr{L}^p_\rho L^q_\sigma}\right)^{1/r}.$$

For $0 \leq \alpha \leq 1$, $H^{s,\alpha}_{p,\sigma}$ is the space with norm

$$\|f\|_{H^{s,\alpha}_{p,\sigma}} = \|\Lambda^\alpha_\sigma f\|_{H^s_p}. \tag{5}$$

$\dot{H}^{s,\alpha}_{p,\sigma}$, $\dot{B}^{s,\alpha}_{p,q,\sigma}$ and $\dot{B}^{s,\alpha}_{(p,q),r,\sigma}$ are defined similarly. For simplicity, we write $\dot{B}^{s,\alpha}_{p,\sigma} = \dot{B}^{s,\alpha}_{p,2,\sigma}$ and $B^{s,\alpha}_{p,\sigma} = B^{s,\alpha}_{p,2,\sigma}$.

Let X be any Banach space of functions on $\mathbb{R}^n$, we define $L^q_t X$ to be the space on $\mathbb{R}\times\mathbb{R}^n$ with space-time norm $\|u\|_{L^q_t X} := \left(\int_{\mathbb{R}} \|u\|^q_X dt\right)^{1/q}$.

p' denotes the conjugate of $p \in [1,\infty]$ given by $\frac{1}{p}+\frac{1}{p'} = 1$.

1.2 Normal form transform

In this subsection, we use the normal form transform technique(which was first used by Shatah[8] in quadratic Klein-Gordon equations) for the quantum Zakharov system. Normal form transform method is one of the most powerful tools to exploit nonlinear structures. Write

$$\omega_1(D) = D^2 + \varepsilon^2 D^4, \quad \omega_2(D) = D\sqrt{1+\varepsilon^2 D^2},$$

and

$$\omega_1(|\xi|) = |\xi|^2 + \varepsilon^2|\xi|^4, \quad \omega_2(|\xi|) = |\xi|\sqrt{1+\varepsilon^2|\xi|^2}.$$

Define $S(t) = e^{it\omega_1(D)} := \mathscr{F}^{-1}e^{-it\omega_1(\xi)}\mathscr{F}$ to be the fourth order Schrödinger semigroup and $W(t) = e^{it\omega_2(D)} := \mathscr{F}^{-1}e^{it\omega_2(\xi)}\mathscr{F}$ to be the wave semigroup.
For any u and v, define the low-high, high-low and high-high interactions by

$$(uv)_{LH} := \sum_{k\in\mathbb{Z}}(P_{\le k-5}u)(P_k v), \quad (uv)_{HL} := \sum_{k\in\mathbb{Z}}(P_k u)(P_{\le k-5}v),$$
$$(uv)_{HH} := \sum_{\substack{|k_1-k_2|\le 4\\ k_1,k_2\in\mathbb{Z}}}(P_{k_1}u)(P_{k_2}v),$$

then $uv = (uv)_{LH} + (uv)_{HL} + (uv)_{HH}$. To make a distinction with resonant and non-resonant terms, we write

$$(uv)_{1L} := \sum_{|k|\le 1}(P_k u)(P_{\le k-5}v), \quad (uv)_{L1} := (vu)_{1L},$$
$$(uv)_{XL} := \sum_{|k|>1}(P_k u)(P_{\le k-5}v), \quad (uv)_{LX} := (vu)_{XL},$$

then

$$(uv)_{HL} = (uv)_{1L} + (uv)_{XL}, \quad (uv)_{LH} = (uv)_{L1} + (uv)_{LX}.$$

We use $\varphi_{XL}, \varphi_{LX}$, etc. to denote the bilinear symbol of operators u_{XL}, u_{LX}, etc.,

$$\mathscr{F}(uv)_{XL} = \int \varphi_{XL}\hat{u}(\xi-\eta)\hat{v}(\eta)d\eta, \quad \mathscr{F}(uv)_{LX} = \int \varphi_{LX}\hat{u}(\xi-\eta)\hat{v}(\eta)d\eta. \quad (6)$$

Symbols φ_{XL}, φ_{LX}, etc., can be expressed in terms of $\chi_k(\xi)$, i.e., $\varphi_{XL} = \sum_{|k|>1}\chi_k(\xi-\eta)\chi_{\le k-5}(\eta)$. Similarly as in [7], (3) are transformed to the following equivalent integral equations

$$u = S(t)u_0 - \Omega_1(\bar{N},u)(t) + S(t)\Omega_1(\bar{N},u)(0) - i\int_0^t S(t-s)\Omega_2(D|u|^2,u)(s)ds$$
$$-i\int_0^t S(t-s)\Omega_1(\bar{N},\bar{N}u)(s)ds - i\int_0^t S(t-s)(\bar{N}u)_{HH+LH+1L}(s)ds. \quad (7)$$

$$\bar{N} = W(t)\bar{N}_0 - D\Omega_3(u,u)(t) + W(t)D\Omega_3(u,u)(0) - i\int_0^t W(t-s)(D\Omega_3(\bar{N}u,u)$$
$$- D\Omega_3(u,\bar{N}u))ds - i\int_0^t W(t-s)\frac{D}{\sqrt{1+\varepsilon^2 D^2}}(u\bar{u})_{HH+L1+1L}(s)ds, \tag{8}$$

where $\Omega_j (j=1,2,3)$ are bilinear multipliers

$$\Omega_1(f,g) = \mathscr{F}^{-1}\int \varphi_{XL}\Phi_\varepsilon^{-1}\hat{f}(\xi-\eta)\hat{g}(\eta)d\eta,$$
$$\Omega_2(f,g) = \mathscr{F}^{-1}\int \frac{\varphi_{XL}}{\Phi_\varepsilon\sqrt{1+\varepsilon^2|\xi-\eta|^2}}\hat{f}(\xi-\eta)\hat{g}(\eta)d\eta,$$
$$\Omega_3(f,g) = \mathscr{F}^{-1}\int \varphi_{XL+LX}\frac{\hat{f}(\xi-\eta)\hat{\bar{g}}(\eta)}{\tilde{\Phi}_\varepsilon\sqrt{1+\varepsilon^2|\xi|^2}}d\eta,$$

in which $\Phi_\varepsilon := \omega_1(|\xi|) - \omega_1(|\eta|) - \omega_2(|\xi-\eta|)$ and $\tilde{\Phi}_\varepsilon = \omega_2(|\xi|) + \omega_1(|\eta|) - \omega_1(|\xi-\eta|)$ are resonance functions for the Schrödinger and wave component in (3). After normal form transform, the transformed new system is:

$$(i\partial_t + \omega_1(D))(u + \Omega_1(\bar{N},u)) = (\bar{N}u)_{HH+LH+1L} - i\Omega_2(D|u|^2,u) - i\Omega_1(\bar{N},\bar{N}u),$$
$$(i\partial_t + \omega_2(D))(\bar{N} + D\Omega_3(u,u)) = \frac{D}{\sqrt{1+\varepsilon^2 D^2}}(u\bar{u})_{HH+L1+1L} - iD\Omega_3(\bar{N}u,u) + iD\Omega_3(u,\bar{N}u). \tag{9}$$

Remark 1.2 *In proving scattering, the most difficult terms are the high-low interaction terms $(Nu)_{XL}$, $(u\bar{u})_{XL}$ and $(u\bar{u})_{XL}$. The Schrödinger component and wave component have different propagation speed in these cases. These terms are highly non-resonant which could be observed from the resonant functions Φ_ε and $\tilde{\Phi}_\varepsilon$. After normal form transform, these quadratic terms are transformed into trilinear terms and then have more freedom of space and time integrability which is crucial to close the argument.*

2 Angular Strichartz estimates and nonlinear estimates

In this section, we first recall the generalized spherically averaged Strichartz estimate proved in [2].

Lemma 2.1 ([2]) $d=3$, $k\in\mathbb{Z}$.
(1) Let $\frac{10}{3} < r \le +\infty$, for any initial data $\phi \in L_x^2(\mathbb{R}^3)$, we have

$$\|S(t)P_k\phi\|_{L_t^2\mathscr{L}_\rho^r L_\sigma^2} \lesssim 2^{k(\frac{1}{2}-\frac{3}{r})}\|\phi\|_{L_x^2}. \tag{1}$$

(2) Let $4 < r \le +\infty$, for any initial data $\phi \in L_x^2(\mathbb{R}^3)$, there holds

$$\|W(t)P_k\phi\|_{L_t^2\mathscr{L}_\rho^r L_\sigma^2} \lesssim 2^{k(1-\frac{3}{r})}\|\phi\|_{L_x^2}. \tag{2}$$

To state the angular Strichartz estimate, we first give a definition on angular admissible pair:

Definition 2.2 *Assume that* $2 \leq q,p \leq \infty$.
(1)A pair (q,p) *is called angular Schrödinger-admissible if*

$$\frac{2}{q}+\frac{5}{p}<\frac{5}{2} \text{ or } (q,p)=(\infty,2). \tag{3}$$

(2)A pair (q,p) *is called angular wave-admissible if*

$$\frac{1}{q}+\frac{2}{p}<1 \text{ or } (q,p)=(\infty,2). \tag{4}$$

Using Lemma 2.1 and interpolating with the classical Strichartz estimate, we obtain the following:

Lemma 2.3 (Angular Strichartz estimates for the fourth order Schrödinger operator) *Assume that* $2 \leq q,\tilde{q},p,\tilde{p} \leq \infty$, $(q,p),(\tilde{q},\tilde{p})$ *are both angular Schrödinger-admissible pairs and* $\tilde{q}>2$*, then we have the homogeneous Strichartz estimate:*

$$\|S(t)u_0\|_{L_t^q \dot{B}_{(p,2),2}^{\frac{2}{q}+\frac{3}{p}-\frac{3}{2}}} \lesssim \|u_0\|_{L_x^2}, \tag{5}$$

and the inhomogeneous Strichartz estimate

$$\left\|\int_0^t S(t-s)F(s)ds\right\|_{L_t^q \dot{B}_{(p,2),2}^{\frac{2}{q}+\frac{3}{p}-\frac{3}{2}}} \lesssim \|F\|_{L_t^{\tilde{q}'} \dot{B}_{(\tilde{p}',2),2}^{\frac{3}{2}-\frac{3}{\tilde{p}}-\frac{2}{\tilde{q}}}}, \tag{6}$$

where the implicit constants are independent of ε, $\frac{1}{\tilde{q}}+\frac{1}{\tilde{q}'}=1$ *and* $\frac{1}{\tilde{p}}+\frac{1}{\tilde{p}'}=1$.

Lemma 2.4 (Angular Strichartz estimates for the wave operator) *Suppose that* $2 \leq q,\tilde{q},p,\tilde{p} \leq \infty$, $(q,p),(\tilde{q},\tilde{p})$ *are angular wave-admissible pairs and* $\tilde{q}>2$*, then there holds the homogeneous Strichartz estimate*

$$\|W(t)u_0\|_{L_t^q \dot{B}_{(p,2),2}^{\frac{1}{q}+\frac{3}{p}-\frac{3}{2}}} \lesssim \|u_0\|_{L_x^2}, \tag{7}$$

and the inhomogeneous Strichartz estimate

$$\left\|\int_0^t W(t-s)F(s)ds\right\|_{L_t^q \dot{B}_{(p,2),2}^{\frac{1}{q}+\frac{3}{p}-\frac{3}{2}}} \lesssim \|F\|_{L_t^{\tilde{q}'} \dot{B}_{(\tilde{p}',2),2}^{\frac{3}{2}-\frac{3}{\tilde{p}}-\frac{1}{\tilde{q}}}}, \tag{8}$$

where the implicit constants are independent of ε, $\frac{1}{\tilde{q}}+\frac{1}{\tilde{q}'}=1$ *and* $\frac{1}{\tilde{p}}+\frac{1}{\tilde{p}'}=1$.

For $(q,p) \neq (\infty,2)$, we have slightly stronger Strichartz estimates:

Corollary 2.5 *For $2 \le q, \tilde{q}, p, \tilde{p} \le \infty$, $(q,p) \neq (\infty, 2)$ and $q > \tilde{q}'$.*
(a)Suppose that $(q,p), (\tilde{q}, \tilde{p})$ are Schrödinger admissible pairs, then

$$\|S(t)u_0\|_{L_t^q \dot{B}^{\frac{2}{q}+\frac{3}{p}-\frac{3}{2}}_{(p,2+),2}} \lesssim \|u_0\|_{L_x^2}, \tag{9}$$

$$\left\| \int_0^t S(t-s)F(s)ds \right\|_{L_t^q \dot{B}^{\frac{2}{q}+\frac{3}{p}-\frac{3}{2}}_{(p,2+),2}} \lesssim \|F\|_{L_t^{\tilde{q}'} \dot{B}^{\frac{3}{2}-\frac{3}{\tilde{p}}-\frac{2}{\tilde{q}}}_{(\tilde{p}',2),2}}. \tag{10}$$

(b)Suppose that $(q,p), (\tilde{q}, \tilde{p})$ are wave admissible pairs, then

$$\|W(t)u_0\|_{L_t^q \dot{B}^{\frac{1}{q}+\frac{3}{p}-\frac{3}{2}}_{(p,2+),2}} \lesssim \|u_0\|_{L_x^2}, \tag{11}$$

$$\left\| \int_0^t W(t-s)F(s)ds \right\|_{L_t^q \dot{B}^{\frac{1}{q}+\frac{3}{p}-\frac{3}{2}}_{(p,2+),2}} \lesssim \|F\|_{L_t^{\tilde{q}'} \dot{B}^{\frac{3}{2}-\frac{3}{\tilde{p}}-\frac{1}{\tilde{q}}}_{(\tilde{p}',2),2}}. \tag{12}$$

3 Nonlinear Estimates

For the variables u and N in the transformed system, we use the following angular Strichartz norms with wider range as working spaces

$$\begin{aligned} &u \in S = L_t^\infty H^{0,1}_{2,\sigma} \cap L_t^2 \dot{B}^{1/4+\delta,1}_{(q(\delta),2+),\sigma} \cap L_t^2 B^{0,1}_{6,\sigma}, \\ &N \in W = L_t^\infty H^{0,1}_{2,\sigma} \cap L_t^2 \dot{B}^{-1/4-\delta,1}_{(q(-\delta),2+),\sigma}, \end{aligned} \tag{1}$$

where $0 < \delta \ll 1$ is a fixed small enough number and q is defined by $\frac{1}{q(\delta)} = \frac{1}{4} + \frac{\delta}{3}$.

For $0 < \delta \ll 1$ small enough, there holds

$$\frac{10}{3} < q(\delta) < 4 < q(-\delta) < \infty,$$

then the norms defined in (1) are angular Strichartz admissible.

For the resonant terms containing $(\bar{N}u)_{HH+LH+1L}$ and $(u\bar{u})_{HH+L1+1L}$ in (7) and (8), we apply the inhomogeneous generalized Strichartz estimates to estimate them. We have

$$\begin{aligned} &\| \int_0^t S(t-s)(\bar{N}u)_{HH+LH+1L}(s)ds\|_S \\ &\lesssim \|(\bar{N}u)_{LH}\|_{L_t^1 H^{0,1}_{2,\sigma}} + \|(\bar{N}u)_{HH}\|_{L_t^1 H^{0,1}_{2,\sigma}} + \|(\bar{N}u)_{1L}\|_{L_t^{\tilde{q}'} \dot{B}^{\frac{3}{2}-\frac{2}{\tilde{q}}-\frac{3}{\tilde{r}},1}_{(\tilde{r}',2),\sigma}}, \end{aligned} \tag{2}$$

and

$$\| \int_0^t W(t-s) \frac{D}{\sqrt{1+\varepsilon^2 D^2}} (u\bar{u})_{HH+L1+1L}(s) ds \|_W$$
$$\lesssim \| \frac{D}{\sqrt{1+\varepsilon^2 D^2}} (u\bar{u})_{HH} \|_{L_t^1 H_{2,\sigma}^{0,1}} + \| \frac{D}{\sqrt{1+\varepsilon^2 D^2}} (u\bar{u})_{1L+L1} \|_{L_t^{\tilde{q}_1'} \dot{B}_{(\tilde{r}_1',2),\sigma}^{\frac{3}{2}-\frac{1}{\tilde{q}}-\frac{3}{\tilde{r}},1}}, \quad (3)$$

where $(\tilde{q}', \tilde{r}')$ is the dual angular Schrödinger admissible pair and $(\tilde{q}_1', \tilde{r}_1')$ is the dual angular wave admissible pair.
To deal with the other nonlinear terms, we follow [3] to use representation theory of $SO(3)$. Let μ be Haar measure of $SO(3)$ and write $L_A^q = L^q(SO(3), \mu)$. Then

$$\|f\|_{\mathscr{L}_\rho^p L_\sigma^q} \sim \|f(Ax)\|_{L_x^p L_A^q}, \quad \forall 1 \le p, q \le \infty.$$

Lemma 3.1 ([10]) *For any* $1 < q < \infty$*,*

$$\|f\|_{\mathscr{L}_\rho^p \mathscr{H}_q^1} \sim \|f\|_{\mathscr{L}_\rho^p L_\sigma^q} + \sum_{i,j} \|X_{i,j} f\|_{\mathscr{L}_\rho^p L_\sigma^q},$$

where $X_{i,j} = x_i \partial_j - x_j \partial_i$.

Let T_m be a bilinear operator on $\mathbb{R}^n$ defined as

$$T_m(f,g)(x) = \int_{\mathbb{R}^{2n}} m(\xi,\eta) \hat{f}(\xi) \hat{g}(\eta) e^{ix(\xi+\eta)} d\xi d\eta.$$

We recall a bilinear multiplier estimate proved in [3].

Lemma 3.2 ([3]) *Let* $1 \le p, p_1, p_2 \le \infty$ *and* $1/p = 1/p_1 + 1/p_2$*. Assume* $m(\xi,\eta) = h(|\xi|,|\eta|)$ *for some function h, m is bounded and satisfies for all* α, β

$$|\partial_\xi^\alpha \partial_\eta^\beta m(\xi,\eta)| \le C_{\alpha\beta} |\xi|^{-|\alpha|} |\eta|^{-|\beta|}, \quad \xi, \eta \ne 0.$$

Then for $q > 2$*,*

$$\|T_m(P_{k_1} f, P_{k_2} g)\|_{\mathscr{L}_\rho^p \mathscr{H}_q^1} \leqslant C \|f\|_{\mathscr{L}_\rho^{p_1} \mathscr{H}_q^1} \|g\|_{\mathscr{L}_\rho^{p_2} \mathscr{H}_q^1},$$

for any $k_1, k_2 \in \mathbb{Z}$ *with an uniform C.*

With Lemma 3.1 and applying Lemma 3.2 for every bilinear dyadic piece, we have the following two lemmas following the proof of [4] and [7] with slightly modifications:

Lemma 3.3 (Bilinear Estimates) *Let* δ *be a small number.*
(1)For any N and u, there holds

$$\|(\bar{N}u)_{LH}\|_{L_t^1 H_{2,\sigma}^{0,1}} \lesssim \|N\|_{L_t^2 \dot{B}^{-1/4-\delta,1}_{(q(-\delta),2+),\sigma}} \|u\|_{L_t^2 \dot{B}^{1/4+\delta,1}_{(q(\delta),2+),\sigma}},$$
$$\|(\bar{N}u)_{HH}\|_{L_t^1 H_{2,\sigma}^{0,1}} \lesssim \|N\|_{L_t^2 \dot{B}^{-1/4-\delta,1}_{(q(-\delta),2+),\sigma}} \|u\|_{L_t^2 \dot{B}^{1/4+\delta,1}_{(q(\delta),2+),\sigma}},$$
$$\|(\bar{N}u)_{1L}\|_{L_t^{\tilde{q}'} \dot{B}^{\frac{3}{2}-\frac{2}{\tilde{q}}-\frac{3}{\tilde{r}},1}_{(\tilde{r}',2),\sigma}} \lesssim \|N\|_{L_t^2 \dot{B}^{-1/4-\delta,1}_{(q(-\delta),2+),\sigma}} \|u\|_{L_t^\infty H_{2,\sigma}^{0,1} \cap L_t^2 \dot{B}^{1/4+\delta,1}_{(q(\delta),2),\sigma}},$$

where in the third estimate $0 \le \theta \le 1$, $\frac{1}{\tilde{q}} = \frac{1}{2} - \frac{\theta}{2}$, $\frac{1}{\tilde{r}} = \frac{1}{4} + \frac{\theta}{3} + \frac{\delta}{3}$.
(2)For any u, there holds

$$\|\frac{D}{\sqrt{1+\varepsilon^2 D^2}}(u\bar{u})_{HH}\|_{L_t^1 H_{2,\sigma}^{0,1}} \lesssim \|u\|_{L_t^2 \dot{B}^{1/4-\delta,1}_{(q(-\delta),2+),\sigma}} \|u\|_{L_t^2 \dot{B}^{1/4+\delta,1}_{(q(\delta),2+),\sigma}},$$
$$\|\frac{D}{\sqrt{1+\varepsilon^2 D^2}}(u\bar{u})_{1L+L1}\|_{L_t^{\tilde{q}'_1} \dot{B}^{\frac{3}{2}-\frac{1}{\tilde{q}}-\frac{3}{\tilde{r}},1}_{(\tilde{r}'_1,2),\sigma}} \lesssim \|u\|^2_{L_t^\infty H_{2,\sigma}^{0,1} \cap L_t^2 \dot{B}^{1/4+\delta,1}_{(q(\delta),2+),\sigma}},$$

where in the last inequality $0 \le \theta \le 1$, $\frac{1}{\tilde{q}} = \frac{1}{2} - \frac{\theta}{2}$, $\frac{1}{\tilde{r}} = \frac{1}{4} + \frac{\theta}{3} - \frac{\delta}{3}$.

Sketch of the Proof. We briefly sketch the proof of the first estimate. By dyadic decomposition,

$$\begin{aligned}\|(\bar{N}u)_{LH}\|^2_{H_{2,\sigma}^{0,1}} &\lesssim \sum_{k_2} \|\sum_{k_1<k_2-5} \Lambda_\sigma^1 (P_{k_1}\bar{N}P_{k_2}u)\|^2_{\mathscr{L}_\rho^2 L_\sigma^{2+}} \\ &\lesssim \sum_{k_2} (\sum_{k_1<k_2-5} \|P_{k_1}\bar{N}P_{k_2}u\|_{\mathscr{L}_\rho^2 \mathscr{H}_{2+}^1})^2.\end{aligned}$$

When $q > 2$, $\mathscr{H}_q^1$ is an algebra. Using the bilinear estimate, i.e., Lemma 3.2, we have

$$\begin{aligned}\|(\bar{N}u)_{LH}\|^2_{H_{2,\sigma}^{0,1}} &\lesssim \sum_{k_2} (\sum_{k_1<k_2-5} \|P_{k_1}\bar{N}\|_{\mathscr{L}_\rho^{q(-\delta)} \mathscr{H}_{2+}^1} \|P_{k_2}u\|_{\mathscr{L}_\rho^{q(\delta)} \mathscr{H}_{2+}^1})^2 \\ &= \sum_{k_2} (\sum_{k_1<k_2-5} 2^{k_1(-\frac{1}{4}-\delta)} \|P_{k_1}\bar{N}\|_{\mathscr{L}_\rho^{q(-\delta)} \mathscr{H}_{2+}^1} 2^{k_1(\frac{1}{4}+\delta)} \|P_{k_2}u\|_{\mathscr{L}_\rho^{q(\delta)} \mathscr{H}_{2+}^1})^2 \\ &\lesssim \|N\|^2_{\dot{B}^{-1/4-\delta,1}_{(q(-\delta),2+),\sigma}} \|u\|^2_{\dot{B}^{1/4+\delta,1}_{(q(\delta),2+),\sigma}},\end{aligned}$$

which implies the first estimate by using Hölder's inequality in time. The other terms can be estimated similarly, we skip the details.
For the boundary terms, applying homogeneous Strichartz estimates, we obtain

Lemma 3.4 (Boundary terms I) *For any* N_0 *and* u_0,

$$\|S(t)\Omega_1(\bar{N},u)(0)\|_S \lesssim \|\Omega_1(\bar{N}_0,u_0)\|_{H_{2,\sigma}^{0,1}} \lesssim \|N_0\|_{H_{2,\sigma}^{0,1}} \|u_0\|_{H_{2,\sigma}^{0,1}},$$
$$\|W(t)D\Omega_3(u,u)(0)\|_W \lesssim \|D\Omega_3(u_0,u_0)\|_{H_{2,\sigma}^{0,1}} \lesssim \|u_0\|^2_{H_{2,\sigma}^{0,1}}.$$

Then for any N and u, there holds

$$\|\Omega_1(\bar{N},u)\|_{L_t^\infty H^{0,1}_{2,\sigma}} \lesssim \|N\|_{L_t^\infty H^{0,1}_{2,\sigma}} \|u\|_{L_t^\infty H^{0,1}_{2,\sigma}}, \quad \|D\Omega_3(u,u)\|_{L_t^\infty H^{0,1}_{2,\sigma}} \lesssim \|u\|^2_{L_t^\infty H^{0,1}_{2,\sigma}}.$$

We need the Coifman-Meyer bilinear multiplier estimates to deal with the other nonlinear terms

Lemma 3.5 *Assume m is bounded and satisfying the following estimates:*

$$|\partial_\xi^\alpha \partial_\eta^\beta m(\xi,\eta)| \le C_{\alpha\beta} |\xi|^{-|\alpha|} |\eta|^{-|\beta|}, \quad \forall \alpha,\beta.$$

Let $1 \leqslant p,q,r \leqslant \infty$, $1/r = 1/p + 1/q$, *then for any* $k_1, k_2 \in \mathbb{Z}$, *we have*

$$\|T_m(P_{k_1} f, P_{k_2} g)\|_{L^r} \leqslant C \|f\|_{L^p} \|g\|_{L^q}.$$

Since $X_{i,j}$ commutes with the radial Fourier multiplier operator and $X_{ij}(fg) = gX_{ij}f + fX_{ij}g$, applying X_{ij} to the multiplier on dyadic piece and then estimating with Lemma 3.5 similarly as in [7], we have the following bilinear and trilinear estimates:

Lemma 3.6 (Boundary terms II) *For any N and u, there holds*

$$\|\Omega_1(\bar{N},u)\|_{L_t^2 \dot{B}^{1/4+\delta,1}_{(q(\delta),2),\sigma}} \lesssim \|N\|_{L_t^\infty H^{0,1}_{2,\sigma}} \|u\|_{L_t^2 B^{0,1}_{6,\sigma}},$$
$$\|D\Omega_3(u,u)\|_{L_t^2 \dot{B}^{-1/4-\delta}_{(q(-\delta),2),\sigma}} \lesssim \|u\|_{L_t^2 B^{0,1}_{6,\sigma}} \|u\|_{L_t^\infty H^{0,1}_{2,\sigma}},$$

where the implicit constant is independent of ε.

Sketch of the Proof. For the first estimate, using dyadic decomposition, Sobolev embedding and Lemma 3.1

$$\begin{aligned}
\|\Omega_1(\bar{N},u)\|^2_{\dot{B}^{1/4+\delta,1}_{(q(\delta),2),\sigma}} &\lesssim \|D\Omega_1(\bar{N},u)\|^2_{H^{0,1}_{2,\sigma}} \\
&\lesssim \sum_{k_2} \|\Lambda^1_\sigma P_{k_2} \langle D\rangle^{-1} \sum_{k_1 \le k_2-5} D\langle D\rangle \Omega_1(P_{k_2}\bar{N}, P_{k_1}u)\|^2_{L^2} \\
&\lesssim \sum_{k_2} \left(\sum_{k_1 \le k_2-5} \langle 2^{k_2}\rangle^{-1} \|P_{k_2} D\langle D\rangle \Omega_1(P_{k_2}\bar{N}, P_{k_1}u)\|_{L^2} \right)^2 \\
&\quad + \sum_{k_2} \left(\sum_{k_1 \le k_2-5} \sum_{i,j} \langle 2^{k_2}\rangle^{-1} \|P_{k_2} D\langle D\rangle \Omega_1(X_{i,j}P_{k_2}\bar{N}, P_{k_1}u)\|_{L^2} \right)^2 \\
&\quad + \sum_{k_2} \left(\sum_{k_1 \le k_2-5} \sum_{i,j} \langle 2^{k_2}\rangle^{-1} \|P_{k_2} D\langle D\rangle \Omega_1(P_{k_2}\bar{N}, X_{i,j}P_{k_1}u)\|_{L^2} \right)^2.
\end{aligned}$$

In which $D\langle D\rangle \Omega_1(P_{k_2}\bar{N}, P_{k_1}u)$ is a bilinear multiplier with symbol

$$m_1(\xi,\eta) = \frac{|\xi+\eta| \langle \xi+\eta\rangle \chi_{k_2}(\xi) \chi_{k_1}(\eta)}{\omega_1(|\xi+\eta|) - \omega_1(|\eta|) - \omega_2(|\xi|)}.$$

One can check that $m_1(\xi,\eta)$ satisfies the conditions in the Coifman-Meyer multiplier estimate, i.e., Lemma 3.5. Then

$$\begin{aligned}\|\Omega_1(\bar{N},u)\|^2_{\dot{B}^{1/4+\delta,1}_{(q(\delta),2),\sigma}} &\lesssim \sum_{k_2}\Big(\sum_{k_1\le k_2-5}\langle 2^{k_2}\rangle^{-1}\|\Lambda^1_\sigma P_{k_2}\bar{N}\|_{L^2}\|\Lambda^1_\sigma P_{k_1}u\|_{L^\infty}\Big)^2\\ &\lesssim \|N\|^2_{H^{0,1}_{2,\sigma}}\|u\|^2_{B^{0,1}_{6,\sigma}},\end{aligned}$$

which implies the first estimate. For the boundary term containing Ω_3, we skip the details and refer to [7] for the detailed proof.
For the cubic terms, we have

Lemma 3.7 (Trilinear estimates) *For any N and u, we get*

$$\Big\|\int_0^t S(t-s)\Omega_2(D|u|^2,u)(s)ds\Big\|_S \lesssim \|\Omega_2(D|u|^2,u)\|_{L^1_tH^{0,1}_{2,\sigma}} \lesssim \|u\|^2_{L^2_tB^{0,1}_{6,\sigma}}\|u\|_{L^\infty_tH^{0,1}_{2,\sigma}}.$$

$$\Big\|\int_0^t S(t-s)\Omega_1(\bar{N},\bar{N}u)(s)ds\Big\|_S \lesssim \|\Omega_1(\bar{N},\bar{N}u)\|_{L^2_t\dot{B}^{0,1}_{(\frac{6}{5},2),\sigma}} \lesssim \|u\|_{L^2_tB^{0,1}_{6,\sigma}}\|N\|^2_{L^\infty_tH^{0,1}_{2,\sigma}},$$

$$\Big\|\int_0^t W(t-s)(D\Omega_3(\bar{N}u,u)-D\Omega_3(u,\bar{N}u))ds\Big\|_W \lesssim \|D\Omega_3(\bar{N}u,u)\|_{L^1_tH^{0,1}_{2,\sigma}} \lesssim \|u\|^2_{L^2_tB^{0,1}_{6,\sigma}}\|N\|_{L^\infty_tH^{0,1}_{2,\sigma}}.$$

Sketch of the Proof. We only sketch the main idea of the proof for the first trilinear estimate. Using dyadic decomposition and Bernstein's inequality(see for instance [9]),

$$\begin{aligned}\|\Omega_2(D|u|^2,u)\|^2_{H^{0,1}_{2,\sigma}} &\lesssim \sum_{k_2}\Big\|\sum_{k_1\le k_2-5}\Lambda^1_\sigma P_{k_2}\Omega_2(P_{k_2}D|u|^2,P_{k_1}u)\Big\|^2_{L^2}\\ &\lesssim \sum_{k_2}2^{2k_2}\Big\|\sum_{k_1\le k_2-5}\Lambda^1_\sigma P_{k_2}\Omega_2(P_{k_2}D|u|^2,P_{k_1}u)\Big\|^2_{L^{\frac{6}{5}}}.\end{aligned}$$

Recall that the resonant function for the Schrödinger component is

$$\Phi_\varepsilon := \omega_1(|\xi|)-\omega_1(|\eta|)-\omega_2(|\xi-\eta|).$$

In the support of the symbol of Ω_2, for the low frequency part($|\xi|\lesssim 1, |\eta|\ll|\xi|\sim|\xi-\eta|$),

$$|\Phi_\varepsilon|\sim|\xi|.$$

While for the high frequency part ($|\xi|\gg 1, |\eta|\ll|\xi|\sim|\xi-\eta|$),

$$|\Phi_\varepsilon|\sim|\xi|^4.$$

Therefore it can absorb four derivatives for the high frequency in terms of Coifman-Meyer multiplier estimate(Lemma 3.5) which helps to close the argument. In detail,

$$\|\Omega_2(D|u|^2,u)\|^2_{H^{0,1}_{2,\sigma}} \lesssim \sum_{k_2}2^{2k_2}\langle 2^{k_2}\rangle^{-6}\Big(\sum_{k_1\le k_2-5}\|\Lambda^1_\sigma P_{k_2}\langle D\rangle^3\Omega_2(P_{k_2}D|u|^2,P_{k_1}u)\|_{L^{\frac{6}{5}}}\Big)^2$$

Noticing that $\langle D\rangle^3\Omega_2(P_{k_2}D|u|^2, P_{k_1}u)$ is a bilinear multiplier with symbol

$$m_2(\xi,\eta)=\frac{\langle\xi+\eta\rangle^3|\xi|\chi_{k_2}(\xi)\chi_{k_1}(\eta)}{(\omega_1(|\xi+\eta|)-\omega_1(|\eta|)-\omega_2(|\xi|))\sqrt{1+\varepsilon^2|\xi|^2}},$$

which satisfies the conditions in Lemma 3.5. Therefore

$$\begin{aligned}\|\Omega_2(D|u|^2,u)\|^2_{H^{0,1}_{2,\sigma}} \lesssim& \sum_{k_2}\langle 2^{k_2}\rangle^{-6}2^{2k_2}\left(\sum_{k_1\le k_2-5}\|P_{k_2}|u|^2\|_{L^{\frac{3}{2}}}\|P_{k_1}u\|_{L^6}\right)^2\\ &+\sum_{k_2}\langle 2^{k_2}\rangle^{-6}2^{2k_2}\left(\sum_{k_1\le k_2-5}\sum_{i,j}\|X_{i,j}P_{k_2}|u|^2\|_{L^{\frac{3}{2}}}\|P_{k_1}u\|_{L^6}\right)^2\\ &+\sum_{k_2}\langle 2^{k_2}\rangle^{-6}2^{2k_2}\left(\sum_{k_1\le k_2-5}\sum_{i,j}\|P_{k_2}|u|^2\|_{L^{\frac{3}{2}}}\|X_{i,j}P_{k_1}u\|_{L^6}\right)^2\\ \lesssim& \|u\|^4_{B^{0,1}_{6,\sigma}}\|u\|^2_{H^{0,1}_{2,\sigma}},\end{aligned}$$

which yields the first estimate as desired. The other terms can be estimated similarly. For instance, for the third estimate containing Ω_3, one only need to notice that the resonant function for the wave component is

$$\tilde{\Phi}_\varepsilon=\omega_2(|\xi|)+\omega_1(|\eta|)-\omega_1(|\xi-\eta|),$$

and $|\tilde{\Phi}_\varepsilon|$ behaves like $\langle\xi\rangle^3|\xi|$(when $|\eta|\ll|\xi|$) which can again absorb four derivatives in high frequency. We skip the details of proof.

Remark 3.8 *When $\varepsilon=0$, namely for the original Zakharov system, the resonant function for the Schrödinger component behaves like*

$$|\Phi_\varepsilon|\sim\langle\xi\rangle|\xi|,$$

which can only absorb two derivatives for the high frequency. This is one of the main reason that quantum Zakharov system has much better properties than the original Zakharov system.

4 Scattering in L^2

For any small initial data $(u_0,N_0)\in H^{0,1}_{2,\sigma}(\mathbb{R}^3)\times H^{0,1}_{2,\sigma}(\mathbb{R}^3)$, we define the operators

$$\begin{aligned}\Psi^1_{u_0}(u,N)=&S(t)u_0-\Omega_1(\bar{N},u)(t)+S(t)\Omega_1(\bar{N},u)(0)-i\int_0^t S(t-s)\Omega_2(D|u|^2,u)(s)ds\\ &-i\int_0^t S(t-s)\Omega_1(\bar{N},\bar{N}u)(s)ds-i\int_0^t S(t-s)(\bar{N}u)_{HH+LH+1L}(s)ds,\end{aligned}$$

and

$$\Psi^2_{\bar{N}_0}(u,N) = W(t)\bar{N}_0 - D\Omega_3(u,u)(t) - i\int_0^t W(t-s)(D\Omega_3(\bar{N}u,u) - D\Omega_3(u,\bar{N}u))ds$$
$$+ W(t)D\Omega_3(u,u)(0) - i\int_0^t W(t-s)\frac{D}{\sqrt{1+\varepsilon^2D^2}}(u\bar{u})_{HH+L1+1L}(s)ds.$$

Write $\Psi : (u,N) \to (\Psi^1_{u_0}, \Psi^2_{\bar{N}_0})$ and choose the resolution space as

$$\mathscr{D} = \{(u,N) : \|(u,N)\|_X \le \alpha\},$$

with the norm $\|(u,N)\|_X = \|u\|_S + \|N\|_W$, α is a small number to be determined. Applying the Strichartz and nonlinear estimates, for any $(u,N) \in \mathscr{D}$, we have

$$\|\Psi^1_{u_0}(u,N)\|_S \lesssim \|u_0\|_{H^{0,1}_{2,\sigma}} + \|N\|_{L^\infty_t H^{0,1}_{2,\sigma}}\|u\|_{L^\infty_t H^{0,1}_{2,\sigma}} + \|N_0\|_{H^{0,1}_{2,\sigma}}\|u_0\|_{H^{0,1}_{2,\sigma}}$$
$$+ \|N\|_{L^\infty_t H^{0,1}_{2,\sigma}}\|u\|_{L^2_t B^{0,1}_{6,\sigma}} + \|u\|^2_{L^2_t B^{0,1}_{6,\sigma}}\|u\|_{L^\infty_t H^{0,1}_{2,\sigma}} + \|u\|_{L^2_t B^{0,1}_{6,\sigma}}\|N\|^2_{L^\infty_t H^{0,1}_{2,\sigma}}$$
$$+ \|N\|_{L^2_t \dot{B}^{-1/4-\delta}_{(q(-\delta),2+),\sigma}}\|u\|_{L^2_t \dot{B}^{1/4+\delta}_{(q(\delta),2+),\sigma}},$$

and

$$\|\Psi^2_{\bar{N}_0}(u,N)\|_W \lesssim \|N_0\|_{H^{0,1}_{2,\sigma}} + \|u\|^2_{L^\infty_t H^{0,1}_{2,\sigma}} + \|u_0\|^2_{H^{0,1}_{2,\sigma}}$$
$$+ \|u\|_{L^2_t B^{0,1}_{6,\sigma}}\|u\|_{L^\infty_t H^{0,1}_{2,\sigma}} + \|u\|^2_{L^2_t B^{0,1}_{6,\sigma}}\|N\|_{L^\infty_t H^{0,1}_{2,\sigma}} + \|u\|_{L^2_t \dot{B}^{1/4-\delta}_{(q(-\delta),2+),\sigma}}\|u\|_{L^2_t \dot{B}^{1/4+\delta}_{(q(\delta),2+),\sigma}}.$$

Then

$$\|\Psi(u,N)\|_X = \|\Psi^1_{u_0}(u,N)\|_S + \|\Psi^2_{\bar{N}_0}(u,N)\|_W$$
$$\lesssim \|u_0\|_{H^{0,1}_{2,\sigma}} + \|N_0\|_{H^{0,1}_{2,\sigma}} + (\|u_0\|_{H^{0,1}_{2,\sigma}} + \|N_0\|_{H^{0,1}_{2,\sigma}})^2 + \|(u,N)\|^2_X + \|(u,N)\|^3_X.$$

If the initial data is sufficiently small, namely, $\beta_0 = \|u_0\|_{H^{0,1}_{2,\sigma}} + \|N_0\|_{H^{0,1}_{2,\sigma}} \ll 1$, we choose $\alpha = C\beta_0$, then $\Psi : \mathscr{D} \to \mathscr{D}$. Similarly Ψ is a contraction mapping on $\mathscr{D}$. Therefore there exists a unique solution on $\mathscr{D}$ with global space-time bound. By the standard techniques, we obtain that the solution $(u(t),N(t))$ to (3) scatters in $H^{0,1}_{2,\sigma} \times H^{0,1}_{2,\sigma}$.

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