Structured Random Matrices in Down Under (July 26th — August 6th)

Workshop Program

All times are given in UTC. Thus, to obtain your local time:

- subtract 7h for San Diego, San Fransisco
- subtract 5h for Chicago, Mexico City etc.
- subtract 4h for New York City, Dearborn (Michigan)
- add 1h for London, Lisbon etc.
- add 2h for Berlin, Brussels, Rom, Paris and Warsaw
- add 3h Athens, Jerusalem, Moscow
- add 5.5h for New Delhi, Sri Lanka
- add 8h Beijing, Hong Kong, Singapore, Shanghai
- add 9h for Seoul, Tokyo
- add 9.5h for Adelaide
- add 10h for Melbourne

Please, be aware that for the informal discussion it may happen that there is a shift of the day. That will be highlighted.

Length of the open workshop talks and schedule:

45min talk + 15min discussions and changing

Zoom Link:
https://unimelb.zoom.us/j/88303799510?pwd=bSthM2pHak9xODZEaFpMSTRoSmp1QT09

No date changes occur	11am-12pm UTC	12pm-1pm UTC
July 26th (Monday)	Matthew McKay: Guiding rational vaccine design with random matrix theory	Wenjian Liu: Bayesian Phylogenetic Inference of Stochastic Block Models on Infinite Trees
July 27th (Tuesday)	Thomas Guhr: Non--Stationarity: Identifying Market States and their Dynamics	Jiyuan Zhang: A new class of random unitary matrices, their products, and eigenvalue statistics
July 28th (Wednesday)	László Erdös: Eigenstate thermalisation hypothesis and Gaussian fluctuations for Wigner matrices	Rishabh Dudeja: Phase Retrieval with Structured Sensing Matrices
July 29th (Thursday)	Stephane Dartois: PPT criterion for random bosonic states: a graph polynomial approach	Fan Yang: Delocalization and quantum diffusion of random band matrices in high dimensions
July 30th (Friday)	Zdzisław Burda: Cleaning large-dimensional covariance matrices for correlated samples	Arno Kuijlaars: The spherical ensemble with external sources
August 2nd (Monday)	Sergey Berezin: Gap probability for the product of Ginibre matrices in the critical regime	Jac Verbaarschot: The (non-)Hermitian Sachdev-Ye-Kitaev Model
August 3rd (Tuesday)	Karol Życzkowski: Structured Random Matrices and Lindblad dynamics	Qiang Wu: Multi-species Sherrington-Kirkpatrick spin glass in the replica symmetry regime
August 4th (Wednesday)	Benoit Collins: On the operator norm of tensors	Thomas Seligman: Dynamics of market states in the space of correlation matrices
August 5th (Thursday)	Shurong Zheng: Linear Spectral Statistics of Large Dimensional Sample Correlation Matrices	Shinsuke Nishigaki: Tracy-Widom method for Janossy densities
August 6th (Friday)	Peter Forrester: Some deformation of $\beta = 2$ ensembles in RMT	Manan Vyas: Conditional q-normal form of strength functions for fermionic embedded ensembles

Abstracts

1. **Matthew McKay**: *Guiding rational vaccine design with random matrix theory*
Abstract: This talk will describe how random matrix theory (RMT) can aid the rational design of vaccines. Considering the hepatitis C virus (HCV) and the human immunodeficiency virus (HIV) as examples, high-dimensional RMT approaches will be introduced and applied to genetic sequence data measured from infected individuals. These approaches will be used to inform how the function/structure of viral proteins is mediated by correlated sets of genetic mutations, and to identify potential weaknesses that may be targeted by a vaccine. These results, used together with population-level immune system data, will be leveraged to specify novel vaccine designs. Vaccine candidates based on these designs may be implemented using delivery techniques that are being widely employed during the COVID-19 pandemic (e.g., viral vectors as used by the AstraZeneca vaccine, or mRNA methods as being employed by Pfizer-BioNTech). The computational algorithms rely on spectral results of spiked correlation-coefficient matrices, which will also be discussed.

   * Vaccine design analysis is joint work with Ahmed Abdul Quadeer and Syed Faraz Ahmed (HKUST), and David Morales-Jimenez (University of Malaga) ** Spike model analysis is joint work with Iain Johnstone and Jeha Yang (Stanford), and David Morales-Jimenez (University of Malaga)

2. **Wenjian Liu**: *Bayesian Phylogenetic Inference of Stochastic Block Models on Infinite Trees*
Abstract: This talk involves a classification problem on a deep network, by considering a broadcasting process on an infinite communication tree, where information is transmitted from the root to all the vertices according to transition matrices of mutation probabilities. The information reconstruction problem on an infinite tree, is to collect and analyze massive data samples at the nth level of the tree to identify whether there is non-vanishing information of the root, as n goes to infinity. Its connection to the clustering problem in the setting of the stochastic block model, which has wide applications in machine learning and data mining, has been well established. For the stochastic block mode, an "information theoretically solvable but computationally hard" region, or say "hybrid-hard phase", appears whenever the reconstruction bound is not tight of the corresponding reconstruction on the tree problem. Inspired by the recently proposed $q_1 + q_2$ block matrices, we extend the classical works on the Ising model and the Potts model, by studying a general model which incorporates the characteristics of both Ising and Potts through different in-community and out-community transition probabilities, and rigorously establishing the exact conditions for reconstruction.

3. **Thomas Guhr**: *Non–Stationarity: Identifying Market States and their Dynamics*
Abstract: Random Matrix Theory is capable of modeling generic or universal statistical features of correlation matrices. But is the spectral statistics of real-life correlation matrix consistent with the random matrix assumption? - It turns out that non-stationarity, which is often improperly handled or even ignored when assessing real systems, almost always sets severe limits to universality. Nevertheless, a separation of universal and non-universal regimes is possible in a dynamical sense. A detailed analysis of financial data is presented to identify and quantify non-stationarity.

4. **Jiyuan Zhang**: *A new class of random unitary matrices, their products, and eigenvalue statistics*
Abstract: The framework of spherical transforms and Pólya ensembles is of utility in deriving structured analytic results for sums and products of random matrices in a unified way. In this talk, we will carry over this framework to study products of unitary matrices. Those are not distributed via the Haar measure, but still are drawn from distributions where the eigenvalue and eigenvector statistics factorise. They include the circular Jacobi ensemble, known in relation to the Fisher-Hartwig singularity in the theory of Toeplitz determinants, as well as the heat kernel for Brownian motion on the unitary group. We define cyclic Pólya frequency functions and show their relation to the cyclic Pólya ensembles, give a uniqueness statement for the corresponding weights, and derive the determinantal point processes of the eigenvalue statistics at fixed matrix dimension. This work is available on arXiv [2012.11993].

5. **László Erdös:**: *Eigenstate thermalisation hypothesis and Gaussian fluctuations for Wigner matrices*
Abstract: We prove that any deterministic matrix is approximately the identity in the eigenbasis of a

large random Wigner matrix W with an optimal error inversely proportional to the square root of the dimension. This verifies a strong form of Quantum Unique Ergodicity with an optimal convergence rate and we also prove Gaussian fluctuations around this convergence. The key technical tool is a new multi-resolvent local law for Wigner ensemble and the Dyson Brownian motion for eigenvector overlaps.

6. **Rishabh Dudeja**: *Phase Retrieval with Structured Sensing Matrices*
Abstract: Phase Retrieval is a statistical inference problem where one seeks to recover an unknown complex-valued n-dimensional signal vector from the magnitudes of m linear measurements. The linear measurements are specified using a mxn sensing matrix. This problem is a mathematical model for imaging systems arising in x-ray crystallography and other applications where it is infeasible to acquire the phase of the measurements. In this talk, I will describe some results regarding the analysis of this problem in the high-dimensional asymptotic regime where the number of measurements and the signal dimension diverge proportionally so that their ratio remains fixed. A limitation of existing high-dimensional analysis of this problem is that they model the sensing matrix as a random matrix with independent and identically distributed (i.i.d.) Gaussian entries. In practice, this matrix is highly structured with limited randomness. I will describe a correction to the i.i.d. Gaussian sensing model, known as the sub-sampled Haar sensing model which faithfully captures a crucial orthogonality property of realistic sensing matrices. For the sub-sampled Haar sensing model, I will present a precise asymptotic characterization of the performance of commonly used spectral estimators for solving the phase retrieval problem. This result can be leveraged to tune certain parameters involved in the spectral estimator optimally. I will demonstrate an empirical universality phenomenon: the performance curves derived for the sub-sampled Haar model accurately describe the empirical performance curves for realistic sensing matrices. I will also present a result regarding information-theoretic lower bounds for the sub-sampled Haar sensing model, which shows that appropriately tuned spectral methods are information-theoretically optimal. Finally, I will describe recent progress towards obtaining a theoretical understanding of the empirical universality phenomena, which causes realistic sensing matrices to behave like sub-sampled Haar sensing matrices. This talk is based on the papers arXiv:1903.02676, arXiv:1910.11849, and arXiv:2008.10503.

7. **Stephane Dartois**: *PPT criterion for random bosonic states: a graph polynomial approach*
Abstract: In this presentation, I propose to describe the properties of the partial transpose of a random symmetric density matrix in the sense that it describes random quantum states of bosons. I will study in details the limiting distribution of eigenvalues which shows features that are specific to the symmetric case. In order to study the limiting distribution we use the moment method together with graph polynomials techniques that allowing to describe the relevant different scales.

8. **Fan Yang**: *Delocalization and quantum diffusion of random band matrices in high dimensions*
Abstract: We consider Hermitian random band matrices $H = (h_{xy})$ on the d-dimensional lattice $(\mathbb{Z}/L\mathbb{Z})^d$. The entries h_{xy} are independent centred complex Gaussian random variables with variances $s_{xy} = \mathbb{E}|h_{xy}|^2$. The variance matrix $S = (s_{xy})$ has a banded structure so that s_{xy} is negligible if $|x-y|$ exceeds the band width W. In dimensions $d \geq 8$, we prove that, as long as $W \geq L^\epsilon$ for a small constant $\epsilon > 0$, with high probability most bulk eigenvectors of H are delocalized in the sense that their localization lengths are comparable to L. Denote by $G(z) = (H-z)^{-1}$ the Green's function of H. For $\mathrm{Im}\, z \gg W^2/L^2$, we also prove a widely used criterion in physics for quantum diffusion of this model, namely, the leading term in the Fourier transform of $\mathbb{E}|G_{xy}(z)|^2$ with respect to $x-y$ is of the form $(\mathrm{Im}\, z + a(p))^{-1}$ for some $a(p)$ quadratic in p, where p is the Fourier variable. Joint work with Horng-Tzer Yau and Jun Yin.

9. **Zdzisław Burda**: *Cleaning large-dimensional covariance matrices for correlated samples*
Abstract: Using the free probability calculus we derive an analytic expression for a rotationally invariant estimator of large-dimensional covariance matrices in the big-data regime for correlated samples. The result generalises the Ledoit-Péché non-linear shrinkage estimator from uncorrelated to correlated

samples. The result is implemented as an efficient algorithm which allows de-noising of experimental covariance matrices. As an example we discuss exponentially decaying correlations.

10. **Arno Kuijlaars**: *The spherical ensemble with external sources*
 Abstract: We study a model of a large number of points on the unit sphere under the influence of a finite number of fixed repelling charges. In the large n limit the points fill a region that is known as the droplet. For small external charges the droplet is the complement of the union of a number of spherical caps, one around each of the external charges. When the external charges grow, the spherical caps will start to overlap and the droplet ondergoes a non-trivial deformation.

 We explicitly describe the transition for the case of equal external charges that are symmetrically located around the north pole. In our approach we first identify a motherbody that, due to the symmetry in the problem, will be located on a number of meridians connecting the north and south poles. After projecting onto the complex plane, and undoing the symmetry, we characterize the motherbody by means of the solution of a vector equilibrium problem from logarithmic potential theory.

 The talk is based on joint work with Juan Criado del Rey.

11. **Sergey Berezin**: *Gap probability for the product of Ginibre matrices in the critical regime*
 Abstract: Consider a product of M i.i.d. matrices of size $N \times N$, drawn from the complex Ginibre ensemble. Of recent interest is the scaling limit for which both N and M go to infinity in such a way that M/N converges to a positive constant. In this scenario, the singular values of the product form a determinantal point process. We will be interested in the gap probability. This probability will be expressed in terms of the unique solution of a certain Riemann-Hilbert problem; the asymptotics of the latter will be discussed. Based on joint work with E. Strahov.

12. **Jac Verbaarschot**: *The (non-)Hermitian Sachdev-Ye-Kitaev Model*
 Abstract: We discuss replica symmetry breaking in a non-Hermitian SYK model. Due to the imaginary parts of the eigenvalues the disconnected part of the two-replica partition function becomes subleading with respect to the connected part which is determined by the two-point correlation function of the Ginibre ensemble. Applications to replica wormholes and the statistical theory of S-matrices are discussed.

13. **Karol Życzkowski**: *Structured Random Matrices and Lindblad dynamics*
 Abstract: A model of structured unitary matrices determined by a graph is investigated. With any graph consisting of k vertices and m bonds of a given topology one associates a physical system consisting of $2m$ subsystems of local dimension n each and defines a corresponding ensemble of structured random unitary matrices of dimension $N = n^{2m}$. Statistical properties of the corresponding ensemble vary from Poissonian for a disconnected graph to circular unitary ensemble for a complete graph.

 The other model of non-Hermitian structured random matrices is related to quantum dynamics in a continuous time. A Lindblad operator generates Markov evolution in the space of density matrices of a fixed size n. A typical operator of this kind can be modelled by a sum of a real Ginibre matrix G_R and a Hermitian part in the form of the direct sum, $C \otimes \mathbb{I}_n + \mathbb{I}_n \otimes C$, where C denotes of a random matrix of order n pertaining to Gaussian orthogonal ensemble. We show universal spectral features of such operators, including the lemon-like shape of the spectrum in the complex plane and study changes of the spectrum under the quantum-to-classical transition – see Tarnowski et al. preprint arXiv:2105.02369.

14. **Qiang Wu**: *Multi-species Sherrington-Kirkpatrick spin glass in the replica symmetry regime*
 Abstract: Multi-species spin glass model is an inhomogeneous extension of the classical Sherrington-Kirkpatrick model. The disorder coupling matrix is in particular a structured random matrix with given variance profile. Specifically, if the spins take value from a N-dimensional sphere, this is also know as spherical spin glass model, this problem becomes a finite temperature parametrized random matrix problem. The ground state of this model corresponds to the largest eigenvalue of the associated

structured random matrix. In this talk, we will discuss some fluctuation results of the free energy about multi-species SK model, and some connections with structured random matrices.

15. **Benoit Collins**: *On the operator norm of tensors*
Abstract: If one considers tensors of random iid unitaries, it is known that they are asymptotically free. The absence of outliers for NC polynomials in these models, aka strong asymptotic freeness, was established for one-leg-tensors (classical iid Haar unitaries) by Collins and Male. We extend this result to arbitrary tensors, provided we look on the orthogonal of the fixed point space. This can be seen as a 0-1 law on tensors and irreducible representation of the unitary group, and the techniques involve refinements of Weingarten calculus, as well as operator-valued non-backtracking theory. This is based on joint work with Charles Bordenave.

16. **Thomas Seligman**: *Dynamics of market states in the space of correlation matrices*
Abstract: We consider the dynamics of a system such as e.g. financial markets the space of correlation matrices. We characterize "states" or "regions" through which the system under consideration evolves over time to do so w try to understand the system based on the occurrences of correlation structures as well as the trajectories in the space of states, which reduces the complexity and provides a better understanding of the system.

17. **Shurong Zheng**: *Linear Spectral Statistics of Large Dimensional Sample Correlation Matrices*
Abstract: Under the high-dimensional setting that the dimension tends to infinity proportionally with the sample size, we establish the central limit theorems (CLT) for linear spectral statistics (LSS) of sample correlation matrices under two settings: (1). The population follows an independent component structure; (2). The population follows an elliptical structure. It shows that the CLTs of LSS of sample correlation matrices are very different under the two settings. Especially, even if the population correlation matrix is an identity matrix, the CLTs are different under the two settings. An application of our established two CLTs is given.

18. **Shinsuke Nishigaki**: *Tracy-Widom method for Janossy densities*
Abstract: Janossy density (JD) for a determinantal point process is the probability distribution that an interval contains exactly p points except for k points at designated loci. I observe that the JD associated with an integral kernel is expressed as a Fredholm determinant of a modified kernel, which is TW-type if the original kernel is TW-type. This observation enables application of the TW method, leading to JDs expressed in terms of the TW systems of PDEs. My approach does not explicitly refer to the isomonodromic system employed in the preceding works by Forrester and Witte. As concrete examples I compute JDs with $k = 1, p = 0$ for Airy and Bessel kernels, related to joint distributions of the first two largest and smallest eigenvalues of Laguerre UE.

19. **Peter Forrester**: *Some deformation of $\beta = 2$ ensembles in RMT*
Abstract: Historically a deformation of the $\beta = 2$ symmetry class was proposed by Gaudin in 1965. It interpolates between Poisson and RMT statistics. I'll discuss this model in the broader context of L-ensembles. Another deformation occurred in the 1984 work of Fisher on vicious random walkers. Further interpretations will be given, along with a discussion in the context of q-orthogonal polynomial ensembles.

20. **Manan Vyas**: *Conditional q-normal form of strength functions for fermionic embedded ensembles*
Abstract: For finite quantum many-particle systems modeled with say m fermions in N single particle states and interacting with k-body interactions ($k \leq m$), the wavefunction structure is studied using random matrix theory. It is shown that the first four moments of strength functions $F_k(E)$ are essentially same as that of the conditional q-normal distribution given in: P.J. Szabowski, Electronic Journal of Probability 15, 1296 (2010). This naturally gives asymmetry in $F_k(E)$ with respect to E as E_k increases and also the peak value changes with E_k. Thus, the wavefunction structure in quantum many-fermion systems with k-body interactions follows in general the conditional q-normal distribution.

Length of the informal talks including discussions and schedule:

Only for participants invited for the specific sessions

Up to 60min talk + open end research discussion

Note that the times vary from day to day as the respective speakers and participants come from different time zones.

Due to the possibility of a switch of the date, we give the date and time in UTC, EDT (New York), and AET (Melbourne)

Date/Time	
UTC: **EDT:** **AET:**	TBA
UTC: 6am July 27th (Tuesday) **EDT: 2am July 27th (Tuesday)** **AET: 4pm July 27th (Tuesday)**	RMT (mathematical) Anthony Mays: Some non-classical skew-orthogonal polynomials for the GOE and GSE
UTC: **EDT:** **AET:**	
UTC: 6am July 29th (Thursday) **EDT: 2am July 29th (Thursday)** **AET: 4pm July 29th (Thursday)**	Quantum Information Nick Witte: Moments of Entropy & Mutual Information arising in Classical and Quantum Information Theory
UTC: **EDT:** **AET:**	TBA
UTC: 11am July 31st (Saturday) **EDT: 7am July 31st (Saturday)** **AET: 9pm July 31st (Saturday)**	Time Series Analysis Mario Kieburg: Broadening of Outliers in Short Time Series
UTC: 11am August 1st (Sunday) **EDT: 7am August 1st (Sunday)** **AET: 9pm August 1st (Sunday)**	Time Series Analysis Hirdesh Pharasi: Universal scaling and phase separation in financial markets
UTC: **EDT:** **AET:**	TBA
UTC: 6am August 3rd (Tuesday) **EDT: 2am August 3rd (Tuesday)** **AET: 8am August 3rd (Tuesday)**	Quantum Information Benoit Collins: Concrete estimates for output sets in quantum information theory
UTC: 6am August 4th (Wednesday) **EDT: 2am August 4th (Wednesday)** **AET: 4pm August 4th (Wednesday)**	RMT (mathematical) Jesper Ipsen: Multi-time orthogonal Ensembles and Markov Processes
UTC: **EDT:** **AET:**	TBA
UTC: 6am August 6th (Friday) **EDT: 2am August 6th (Friday)** **AET: 4pm August 6th (Friday)**	Quantum Information Lucas Hackl: The Page Curve for Fermionic Gaussian States