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THE UNIVERSITY OF SYDNEY

Mathematical Research Institute

MATRIX-SMRI Research Symposium: Singularities in Geometric Flows: An Ancient Perspective

5 – 14 January 2022

Talk Titles and Abstracts

Wednesday, 5 January 2022

Lu Wang

Title: Relative expander entropy and its applications

Abstract:

In this talk, I will discuss basic properties for a notion of relative entropy motivated by self-expanding solutions to the mean curvature flow. I will also discuss a version of the forward monotonicity formula for mean curvature flow. This is joint with Jacob Bernstein.

Thursday, 6 January 2022

Kyeongsu Choi

Title: Translating flows by sub-affine-critical powers of Gauss curvature

Abstract:

The Gauss curvature flow with sub-affine-critical powers generically develops Type II singularities, while the flow with super-affine-critical powers converges to the round point. Therefore, to analyze the singularities with small powers, one needs to the translators as the model case of the limit ancient flows. In this talk, we show that blow-downs of translators are the graphs of homogeneous functions such that the level sets of the graphs are the shrinkers of the alpha-curve shortening flows. Then, we discuss how to classify the translators by using the Morse index and Jacobi fields of the shrinkers.

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Friday, 7 January 2022

Stephen Lynch

Title: Collapsing and noncollapsing convex ancient solutions

Abstract:

An important problem in mean curvature flow is to find conditions on initial data that rule out 'collapsing' singularity models, such as the Grim Reaper. A prevalent class of singularity models are the convex ancient solutions. We will discuss a result proven with T. Bourni and M. Langford which asserts that a convex ancient solution is collapsing if and only if it admits a sequence of rescalings converging to a Grim Reaper. This makes it possible to rule out collapsing via curvature pinching. The methods generalise to flows by other curvature functions and open up new approaches to proving gradient estimates needed eg. for surgery.

Ramiro Lafuente

Title: Non-compact Einstein manifolds with symmetry.

Abstract:

In this talk we will discuss recent joint work in collaboration with Christoph Böhm in which we obtain structure results for non-compact Einstein manifolds admitting a cocompact isometric action of a connected Lie group. As an application, we prove the Alekseevskii conjecture (1975): any connected homogeneous Einstein space of negative scalar curvature is diffeomorphic to a Euclidean space.

Tim Buttsworth

Title: Cohomogeneity one Ricci solitons and ancient flows on four manifolds

Abstract:

We will discuss a number of recent constructions of steady and shrinking gradient Ricci solitons and ancient Ricci flows that are complete on some four-dimensional manifolds that admit cohomogeneity one symmetries.

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Saturday, 8 January 2022

Brett Kotschwar

Title: Propagation of geometric structure under the Ricci flow and applications to shrinking solitons.

Abstract:

We will present some unique-continuation results which ensure the propagation of symmetry and other geometric features (e.g., warped product structures) backward in time along a solution to the Ricci flow and discuss their applications to the classification problem for shrinking Ricci solitons.

Otis Chodosh

Title: Ancient flows and generic regularity

Abstract:

I will give a broad overview of an approach to the study of generic regularity of MCF (developed with Choi, Mantoulidis, Schulze) based on a connection to "one-sided" ancient flows. I will explain the link with the Colding--Minicozzi theory of generic singularities as well as some consequences of our approach.

Monday, 10 January 2022

Ben Andrews

Title: Limiting shapes of fully nonlinear flows of convex hypersurfaces

Abstract:

I will discuss some questions about the long-time behaviour of hypersurfaces evolving by functions of curvature which are homogeneous of degree greater than 1.

Felix Schulze

Title: A relative entropy and a unique continuation result for Ricci expanders

Abstract:

We prove an optimal relative integral convergence rate for two expanding gradient Ricci solitons coming out of the same cone. As a consequence, we obtain a unique continuation result at infinity and we prove that a relative entropy for two such self-similar solutions to the Ricci flow is well-defined. This is joint work with Alix Deruelle.

Huy Nguyen

Title: Brakke Regularity for the Allen--Cahn Flow

Abstract:

In this paper we prove an analogue of the Brakke's $\e$-regularity theorem for the parabolic Allen--Cahn equation. In particular, we show uniform $C^{2,\alpha}$ regularity for the transition layers converging to smooth mean curvature flows as $\e\rightarrow 0$. A corresponding gap theorem for entire eternal solutions of the parabolic Allen--Cahn is also obtained. As an application of the regularity theorem, we give an affirmative answer to a question of Ilmanen that there is no cancellation in BV convergence in the mean convex setting.

Tuesday, 11 January 2022

Theodora Bourni

Title: Ancient solutions of mean curvature flow.

Abstract: Ancient solutions, which are solutions that have existed for all times in the past, are of interest in the study of geometric flows as they model singularities of the flows. In this talk we will present some recent developments concerning convex ancient solutions. The main focus will be discussing a new way of constructing and classifying collapsed solutions, which, for instance, can be used to provide new examples of eternal, non translating, convex solutions. Moreover, we will present a characterization of entire and non-entire convex ancient solutions and furthermore provide a new classification result for ancient solutions of curve shortening flow with free boundary on a circle. This is joint work with Mat Langford, Stephen Lynch and Giuseppe Tinaglia.

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Wednesday, 12 January 2022

Robert Haslhofer

Title: Classification of noncollapsed translators in R^4

Abstract: I will describe our recent classification of noncollapsed singularity models for the mean curvature flow of 3-dimensional hypersurfaces. Specifically, we show that every noncollapsed translating hypersurface in R^4 is either R×2d-bowl, or a 3d round bowl, or belongs to the one-parameter family of 3d oval bowls constructed by Hoffman-Ilmanen-Martin-White. This is joint work with Choi and Hershkovits.

Thursday, 13 January 2022

Christos Mantoulidis

Title: Variational aspects of ancient flows

Abstract:

This talk is based on joint work with O. Chodosh, K. Choi, and F. Schulze. We'll discuss a variational characterization of ancient gradient flows, working toward a characterization of ancient mean curvature flows lying on one side of a compact or an asymptotically conical shrinker. This has applications to the mean curvature flow of generic closed hypersurfaces, in many situations of interest.

Friday, 14 January 2022

Nick Edelen

Title: Degeneration of 7-dimensional minimal hypersurfaces with bounded index

Abstract: A 7D minimal and locally-stable hypersurface will in general have a discrete singular set, provided it has no singularities modeled on a union of half-planes. We show in this talk that the geometry/topology/singular set of these surfaces has uniform control, in the following sense: if M_i is a sequence of 7D minimal hypersurfaces with uniformly bounded index and area, and discrete singular set, then up to a subsequence all the M_i are bi-Lipschitz equivalent, and uniform Lipschitz bounds on the maps. As a consequence, we prove the space of C^2 embedded minimal hypersurfaces in a fixed 8-manifold, having index $\leq I$, area $\leq \Lambda$, and discrete singular set, divides into finitely-many diffeomorphism types.