Super Non-Abelian T-Duality

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Outline

1. Introduction
2. Super Non-Abelian T-Duality of Principal Chiral Models
3. Principal Chiral Model on $OSp(1|2)$
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5. Conclusions

Introduction: T-Duality in a Nutshell

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Another sigma model is obtained, in which the Lagrange multipliers play the role of dual coordinates. This is referred to as *T-Dual model*.

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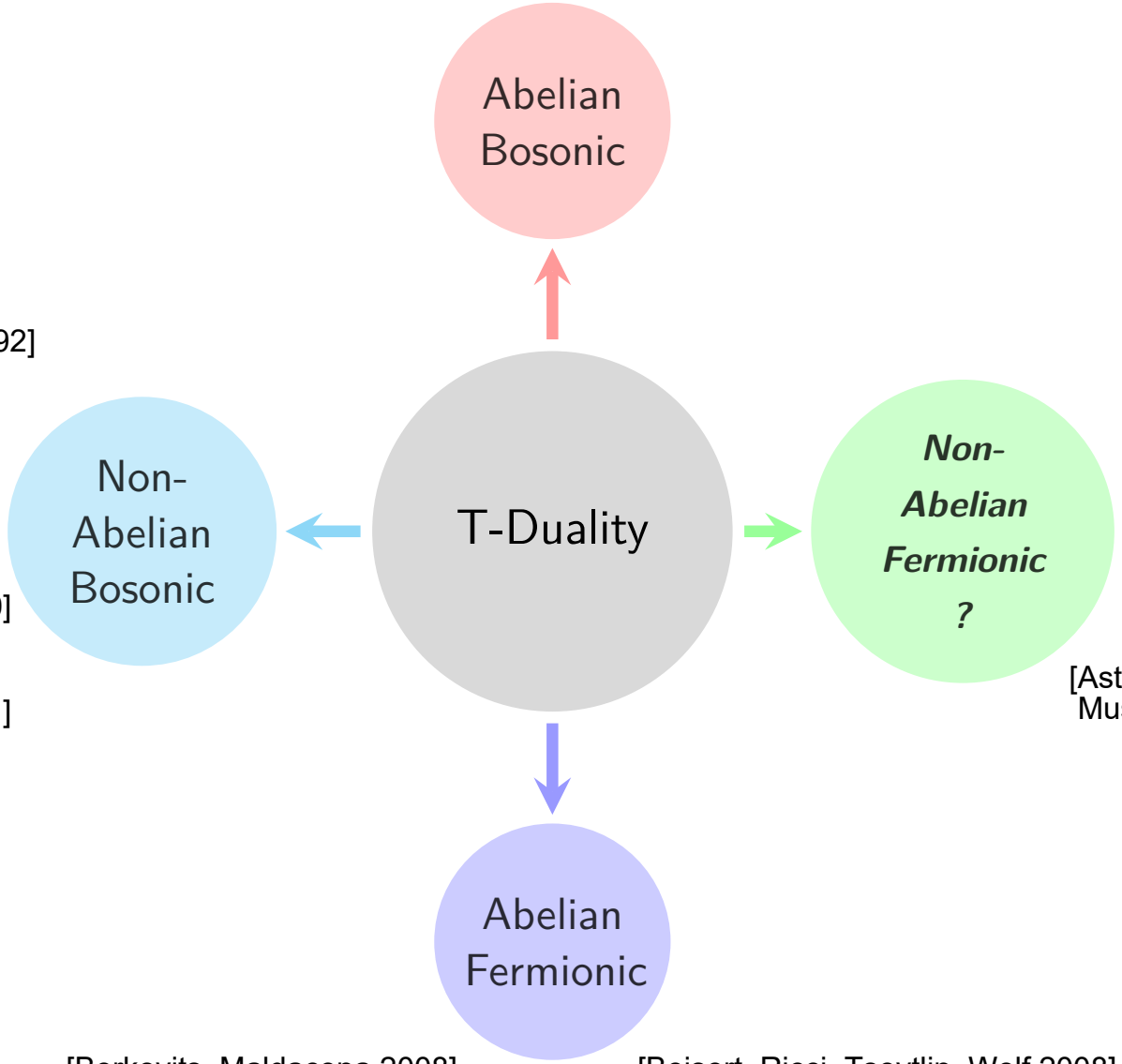
- Backgrounds with a non-abelian set of isometries generically break G to some smaller group, hence the procedure cannot be repeated in the reverse direction. T-Duality is not a symmetry of string theory **[Giveon, Rocek 1993]**.

Introduction: The Landscape of T-Dualities

[Busher 1987-1988] [Rocek, Verlinde 1991] [Giveon, Rocek 1991]



[de la Ossa, Quevedo 1992]

[Alvarez, Alvarez-Gaume, Barbon, Lozano 1993]

[Giveon, Rocek 1993]

[Sfetsos, Thompson 2010]

[Lozano, Colgain, Sfetsos, Thompson 2011]

[Grassi, Mezzalira 2011]

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- Self-duality of certain backgrounds under combined dualisation of bosonic and fermionic isometries **[Abbott, Murugan, Penati, Pittelli, Sorokin, Sundin, Tarrant, Wolf, Wulff 2015]**.
- Self-duality of $AdS_5 \times S^5$ **[Berkovits, Maldacena 2008; Beisert, Ricci, Tseytlin, Wolf 2008]**, which explained the connection between scattering amplitudes and Wilson loops and the origin of the so-called dual superconformal symmetry.

Introduction: Relevance of Bosonic Non-Abelian Case

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- Relation between Yang-Baxter sigma models **[Klimcik 2002]** and integrable deformations of non-abelian T-Dual models **[Osten, Van Tongeren 2016; Hoare, Tseytlin 2016; Borsato, Wulff 2016-2017-2018]**

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The super non-abelian setting remains the least studied and understood one. Despite the works **[Mezzalira, Grassi 2011; Borsato, Wulff 2017-2018; Astrakhantsev, Bakhmatov, Musaev 2021]** explicit examples in which a (sub)supergroup of isometries is dualised seem to be still missing.

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We study 3 classes of models, working out the exchange of Maurer-Cartan equations with EOM and constructing the T-Dual Lax connections and analysing in detail the dualisation of the PCM on $OSp(1|2)$.

SNATD of PCMs: Initial Model

Given a Lie (super)group G with Lie (super)algebra $\mathfrak{g}$, a principal chiral model (PCM) is a two-dimensional sigma model on G defined as

$$S_{PCM} := \frac{1}{2}\int_{\Sigma}\langle j, \star j\rangle \qquad \textit{with} \qquad \begin{cases} j := g^{-1}dg \\ g : \Sigma \to \mathrm{G} \end{cases}$$

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- The equations of motion (EOM) read $d(\star j) = 0$
- EOM and MC encoded in the flatness $F_J = 0$ of a Lax connection

$$J(z) := -\frac{1}{4}(z - z^{-1})^2 j + \frac{1}{4}(z^2 - z^{-2}) \star j$$

SNATD of PCMs: Topological Deformation

Originally considered in **[Borsato, Wulff 2016-2017-2018]** the deformation reads

$$S_\Omega := S_{PCM} + \frac{\zeta}{2} \int_\Sigma \Omega(j,j)$$

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We shall mostly rely on the Lie algebra endomorphism $D : \mathfrak{g} \to \mathfrak{g}$, defined by the Riesz representation theorem as the unique endomorphism relating the cocycle to the inner product

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Graded antisymmetry of Ω and cocycle condition respectively imply that

$$\langle D(U), V \rangle = -\langle U, D(V) \rangle \qquad D([U,V]) = [D(U), V] + [U, D(V)]$$

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$$S_\omega := S_{PCM} + \tfrac{1}{2}\int_\Sigma \langle D(j_\omega), j_\omega\rangle + \int_\Sigma \langle \Lambda + D(g)g^{-1}, F_\omega\rangle$$

the gauged action is invariant under K_L gauge transformations provided that $\Lambda \rightarrow k^{-1}\Lambda k + k^{-1}D(k)$

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- Integrating out ω the T-Dual action is obtained, which depends on both the initial coordinates and multipliers.
- K-gauge invariance of the dual action can be used to remove the extra degrees of freedom, i.e. the coordinates of the initial model.

SNATD of PCMs: Dual Action

Varying the master action with respect to ω one finds

$$\star j_\omega + d\tilde{\Lambda} - D_{\tilde{\Lambda}}(j_\omega) = 0 \qquad \textit{with} \qquad D_{\tilde{\Lambda}} := D + ad_{\tilde{\Lambda}}$$

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which can be solved for j_ω as

$$j_\omega = -\frac{1}{1 - D_{\tilde{\Lambda}}} P^+(d\tilde{\Lambda}) + \frac{1}{1 + D_{\tilde{\Lambda}}} P^-(d\tilde{\Lambda}) \qquad \textit{with} \qquad P^\pm := \tfrac{1}{2}(1 \pm \star)$$

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Substituting back the solution and rearranging, the dual action reads

$$\tilde{S}_{PCM} = \int_\Sigma \langle d\tilde{\Lambda}, \frac{1}{1 - D_{\tilde{\Lambda}}} P^+(d\tilde{\Lambda}) \rangle = \tfrac{1}{2} \int_\Sigma \langle \tilde{e}, \star \tilde{e} \rangle + \tfrac{1}{2} \int_\Sigma \langle \tilde{e}, D_{\tilde{\Lambda}}(\tilde{e}) \rangle$$

where in the last step we introduced dual vielbeine $\tilde{e} := -\frac{1}{1 - D_{\tilde{\Lambda}}}(d\tilde{\Lambda})$

SNATD of PCMs: Dual Lax Connection

Upon varying the dual action with respect to the dual coordinates $\tilde{\Lambda}$ one finds MC-like equations of motion

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Exchange of MC eqs and EOM allows to construct the dual Lax connection

$$\tilde{J}(\tilde{z}) = -\tfrac{1}{4}(\tilde{z} - \tilde{z}^{-1})^2\tilde{j} + \tfrac{1}{4}(\tilde{z}^2 - \tilde{z}^{-2}) \star \tilde{j}$$

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The dual Noether current associated to the residual isometries reads

$$\tilde{J}_N = \star D_{\tilde{\Lambda}}(\tilde{j})$$

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- The PCM on $SL(2, \mathbb{R})$ has been dualised in **[Alvarez, Alvarez-Gaume, Barbon, Lozano 1993]**. The dual model containts non-trivial B_2 field and a metric corresponding to the spacetime of a black hole. It is interesting to understand whether the theory could be embedded in a 3d supergravity.

PCM on $OSp(1|2)$: Initial Setup

Consider the $\mathfrak{osp}(1|2)$ algebra, generated by 3 bosonic generators $L_{\alpha\beta} = L_{\beta\alpha}$ and 2 fermionic generators Q_α

$$[L_{\alpha\beta}, L_{\gamma\delta}] = -i\left(\epsilon_{\gamma(\alpha} L_{\beta)\delta} + \epsilon_{\delta(\alpha} L_{\beta)\gamma}\right)$$

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Since $H^2(\mathfrak{osp}(1|2))$ vanishes **[Sheunert, Zhang 1997]** we shall work with the undeformed model.

PCM on $OSp(1|2)$: Initial Model and Torsion Constraint

Imposing the supergravity constraints on torsion and connection **[Gates, Grisaru, Rocek, Siegel 1983; Buchbinder, Kuzenko 1998]**, the Cartan structure equation $de^A - e^B \wedge \omega_B{}^A = -T^A$ become

$$\begin{cases} de^{\alpha\beta} - e^{\gamma(\alpha} \wedge \omega_\gamma{}^{\beta)} = -\frac{1}{2} e^\alpha \wedge e^\beta \\ de^\alpha - e^\beta \wedge \omega_\beta{}^\alpha = -\frac{1}{2} e^{\delta\epsilon} \wedge e^{\beta\gamma} T_{\beta\gamma\ \delta\epsilon}{}^\alpha - e^\delta \wedge e^{\beta\gamma} T_{\beta\gamma\ \delta}{}^\alpha \end{cases}$$

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The initial current $j = j^{\alpha\beta} L_{\alpha\beta} + j^\alpha Q_\alpha$ satisfies MC eqs

$$dj + \frac{1}{2}[j,j] = 0 \qquad \Rightarrow \qquad \begin{cases} dj^{\alpha\beta} + ij^{\gamma(\alpha} \wedge j_\gamma{}^{\beta)} = \frac{1}{2} j^\alpha \wedge j^\beta \\ dj^\alpha + ij^\beta \wedge j_\beta{}^\alpha = 0 \end{cases}$$

PCM on $OSp(1|2)$: Initial Model and Torsion Constraint

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which agree with the torsion constraints after identifying

$$e^{\alpha\beta} = -j^{\alpha\beta} \qquad e^\alpha = -j^\alpha \qquad \omega_\alpha{}^\beta = -ij_\alpha{}^\beta$$

PCM on $OSp(1|2)$: Initial Model and H_3-form

In 3d supergravity theories the components of the H_3-form should be functions of a scalar superfield L and its derivatives **[Buchbinder, Hutomo, Kuzenko, Tartaglino-Mazzucchelli 2017]**

$$H_3 = e_\alpha \wedge e^{\alpha\beta} \wedge e_\beta L + e_\beta \wedge e_\gamma{}^\beta \wedge e^{\gamma\alpha} \mathcal{D}_\alpha L - \frac{i}{6} e_\alpha{}^\beta \wedge e_\beta{}^\gamma \wedge e_\gamma{}^\alpha (i\mathcal{D}^2 + 8\mathcal{S}) L$$

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One could include in the PCM the H_3-form

$$H_3 = e_\alpha \wedge e^{\alpha\beta} \wedge e_\beta + \frac{4i}{3l} e_\alpha{}^\beta \wedge e_\beta{}^\gamma \wedge e_\gamma{}^\alpha$$

which would dynamically generate the cosmological constant in the EOM, thus preserving the super-AdS_3 interpretation.

PCM on $OSp(1|2)$: T-Dualisation of $OSp(1|2)_L$

We construct the T-Dual model by gauging and dualising $OSp(1|2)_L$.

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- Absence of Ω-deformations implies the replacement $D_{\tilde{\Lambda}} \to ad_{\Lambda}$

$$\tilde{S}_{PCM} = \frac{1}{2}\int_{\Sigma} \langle \tilde{e}, \star \tilde{e} \rangle + \frac{1}{2}\int_{\Sigma} \langle \tilde{e}, ad_{\Lambda}(\tilde{e}) \rangle \quad \textit{with} \quad \tilde{e} := -\frac{1}{1 - ad_{\Lambda}}(d\Lambda)$$

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- Using the definition of $\tilde{e}$ and $\tilde{B}_2 := \frac{1}{2}\langle \tilde{e}, ad_\Lambda(\tilde{e})\rangle$ one finds

$$d\tilde{e} + \frac{1}{2}[\tilde{e}, \tilde{e}] = -\frac{1}{2}\frac{1}{1-ad_\Lambda}[\tilde{e}, \tilde{e}] \qquad \tilde{H}_3 := d\tilde{B}_2 = \frac{1}{2}\langle \tilde{e}, \frac{1}{1-ad_\Lambda}[\tilde{e}, \tilde{e}]\rangle$$

PCM on $OSp(1|2)$: T-Dual Torsion Constraint and $\tilde{H}_3$

- Writing $\tilde{e} = \tilde{e}^{\alpha\beta} L_{\alpha\beta} + \tilde{e}^{\alpha} Q_{\alpha}$ one can compute

$$[\tilde{e}, \tilde{e}] = (2i\tilde{e}^{\gamma(\alpha} \wedge \tilde{e}_{\gamma}{}^{\beta)} - \tilde{e}^{\alpha} \wedge \tilde{e}^{\beta}) L_{\alpha\beta} + (2i\tilde{e}^{\alpha\beta} \wedge \tilde{e}_{\beta}) Q_{\alpha}$$

and analyse the projection of the torsion constraint on $L_{\alpha\beta}$

$$d\tilde{e}^{\alpha\beta} - \tilde{e}^{\gamma(\alpha} \wedge \tilde{\omega}_{\gamma}{}^{\beta)} = -\tfrac{1}{2}\tilde{e}^{\alpha} \wedge \tilde{e}^{\beta}$$

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Computing $d\tilde{e}^{\alpha\beta}$ and isolating terms proportional to $\tilde{e}^{\alpha} \wedge \tilde{e}^{\beta}$ one finds a complicated structure which is not compatible with the constant coefficient on the right-hand side of the constraint.

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- Computing $\tilde{H}_3$ expliticly one finds it contains extra pieces of the form

$$\tilde{H}_3 = \tilde{e}^{\alpha} \wedge \tilde{e}^{\beta} \wedge \tilde{e}^{\gamma} \tilde{H}_{\gamma\beta\alpha} + \ldots$$

PCM on $OSp(1|2)$: A Potential Solution

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$$\nabla_A \tilde{e}_g + \frac{1}{2}[\tilde{e}_g, \tilde{e}_g] = -\frac{1}{2}\frac{1}{1 - ad_{\Lambda_g}}[\tilde{e}_g, \tilde{e}_g] \quad \textit{with} \quad \begin{cases} A := g^{-1}dg \\ \nabla_A := d + [A, -] \end{cases}$$

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Computing the projection of $d\tilde{e}_g$ on $L_{\alpha\beta}$ one finds that while some pieces could be cancelled from the torsion constraint, there are still terms of the form $\tilde{e}_g{}^{\gamma} \wedge \tilde{e}_g{}^{\delta}\tilde{x}_{(\gamma}{}^{(\alpha}\tilde{x}_{\delta)}{}^{\beta)}$ which cannot be removed.

PCM on $OSp(1|2)$: A possible explanation of the result

Incompatibility of the T-Dual model with supergravity also arises under dualisation of the maximal bosonic subgroup $SL(2,\mathbb{R})_L \subset OSp(1|2)_L$.

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This might be explained by the fact that the initial model was not a 3d Green-Schwarz superstring model on AdS_3, as in **[Borsato, Wulff 2018]** it has been shown that super non-abelian T-Duality of 10d Green-Schwarz models preserves compatibility with supergravity.

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It would thus be instructive to study the AdS_3 superstring sigma model

$$S_{GS} = -\frac{1}{2}\int_{\Sigma} j^{\alpha\beta} \wedge \star j_{\alpha\beta} + k\int_{M_3} H_3$$

to which the gauging procedure adopted in this work would however be inapplicable **[Hull, Spence 1989; Sfetsos 1994; Alvarez, Alvarez-Gaume, Lozano 1994]**.

SNATD of Cosets: Symmetric and Semi-Symmetric Spaces

Symmetric and Semi-Symmetric Spaces represent two interesting classes of cosets G/H in which the subgroup H respectively arises as the fixed point set of an automorphism $\sigma : \mathfrak{g} \to \mathfrak{g}$ of order 2 ($\sigma^2 = 1$) and 4 ($\sigma^4 = 1$) **[Serganova 1983; Young 2005; Zarembo 2010]**.

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- Symmetric (S): $\mathfrak{g} \simeq \mathfrak{h} \oplus \mathfrak{m}$ with

$$\mathfrak{h} = \tfrac{1}{2}(1+\sigma)(\mathfrak{g}) := P_{\mathfrak{h}}(\mathfrak{g}) \qquad \mathfrak{m} = \tfrac{1}{2}(1-\sigma)(\mathfrak{g}) := P_{\mathfrak{m}}(\mathfrak{g})$$

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- Semi-Symmetric (SS): $\mathfrak{g} \simeq \mathfrak{h} \oplus \mathfrak{p} \oplus \mathfrak{m} \oplus \mathfrak{q}$ with

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The above decompositions reflect on the current j as

$$\begin{cases} S: \quad j = A + m \\ SS: \; j = A + p + m + q \end{cases} \qquad A := P_{\mathfrak{h}}(j) \;\; p := P_{\mathfrak{p}}(j) \;\; m := P_{\mathfrak{m}}(j) \;\; q := P_{\mathfrak{q}}(j)$$

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and in turn on the transformation laws under the local H_R action

$$g \to gh \quad \Rightarrow \quad j \to h^{-1} j h + h^{-1} dh \quad \Rightarrow \quad \begin{cases} A \to h^{-1} A h + h^{-1} dh \\ \{p, m, q\} \to h^{-1} \{p, m, q\} h \end{cases}$$

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Locally H_R-invariant and globally G_L-invariant actions thus read

- Symmetric: $\quad S_S = \frac{1}{2} \int_\Sigma \langle m, \star m \rangle$
- Semi-Symmetric: $\quad S_{SS} = \frac{1}{2} \int_\Sigma \langle m, \star m \rangle + \frac{1}{2} \int_\Sigma \langle p, q \rangle$

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The EOM for these two classes of models read

$$S: \quad \nabla_A \star m = 0 \qquad\qquad SS: \quad \begin{cases} \nabla_A \star m - \frac{1}{2}[p,p] + \frac{1}{2}[q,q] = 0 \\ [m, p + \star p] = 0 \\ [m, q - \star q] = 0 \end{cases}$$

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EOM and MC eqs can be encoded in the flatness of a Lax connection

$$\begin{cases} S: \quad J(z) := A + \frac{1}{2}(z^2 + z^{-2})m - \frac{1}{2}(z^2 - z^{-2}) \star m \\ SS: J(z) := A + zp + \frac{1}{2}(z^2 + z^{-2})m + z^{-1}q - \frac{1}{2}(z^2 - z^{-2}) \star m \end{cases}$$

SNATD of Cosets: General Features

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- To deform the models while retaining the local H_R-invariance, the two-cocycle Ω should vanish on $\mathfrak{h}$ and this in turn implies that $D : \mathfrak{m} \leftrightarrow \mathfrak{h}$. In semi-symmetric models this further leads to $D : \mathfrak{p} \leftrightarrow \mathfrak{q}$

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- Integrating out ω is potentially obstructed by the lack, in the initial actions, of terms lying in $\mathfrak{h}$. This forces a case by case analysis.

We proceed by isolating the potential obstruction. The resulting T-Dual actions allow to work out explicitly the exchange of MC eqs and EOM and thus to write down the T-Dual Lax connections.

SNATD of Cosets: Symmetric Spaces - EOM for ω

The master action for symmetric spaces takes the form

$$S_\omega = \frac{1}{2}\int_\Sigma \langle m_\omega, \star m_\omega\rangle + \frac{1}{2}\int_\Sigma \langle D(j_\omega), j_\omega\rangle + \int_\Sigma \langle \tilde{\Lambda}, F_{j_\omega}\rangle$$

with $j_\omega := A_\omega + m_\omega$.

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with $j_\omega := A_\omega + m_\omega$. The EOM for ω read

$$\star m_\omega + \nabla_{j_\omega}\tilde{\Lambda} - D(j_\omega) = 0$$

decomposing $\tilde{\Lambda} = \tilde{\Lambda}_{\mathfrak{h}} + \tilde{\Lambda}_{\mathfrak{m}}$ and renaming $\tilde{A} := A_\omega$ the latter become

$$\mathfrak{h}: \quad \nabla_{\tilde{A}}\tilde{\Lambda}_{\mathfrak{h}} = D_{\tilde{\Lambda}_{\mathfrak{m}}}(m_\omega) \qquad \mathfrak{m}: \quad m_\omega - [\tilde{\Lambda}_{\mathfrak{h}}, \star m_\omega] = -\star\left(d\tilde{\Lambda}_{\mathfrak{m}} - D_{\tilde{\Lambda}_{\mathfrak{m}}}(\tilde{A})\right)$$

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While the equation in $\mathfrak{m}$ can be solved for m_ω in general form, the equation in $\mathfrak{h}$ cannot be solved for $\tilde{A}$.

$$m_\omega = -\frac{1}{1 - ad_{\tilde{\Lambda}_\mathfrak{h}}} P^+(d\tilde{\Lambda}_\mathfrak{m} - D_{\tilde{\Lambda}_\mathfrak{m}}(\tilde{A})) + \frac{1}{1 + ad_{\tilde{\Lambda}_\mathfrak{h}}} P^-(d\tilde{\Lambda}_\mathfrak{m} - D_{\tilde{\Lambda}_\mathfrak{m}}(\tilde{A}))$$

SNATD of Cosets: Symmetric Spaces - Dual Action

Substituting m_ω back in the action and rearranging one finds

$$\tilde{S}_S = \int_\Sigma \langle d\tilde{\Lambda}_{\mathfrak{m}} - D_{\tilde{\Lambda}_{\mathfrak{m}}}(\tilde{A}), \frac{1}{1 - ad_{\tilde{\Lambda}_{\mathfrak{h}}}} P^+(d\tilde{\Lambda}_{\mathfrak{m}} - D_{\tilde{\Lambda}_{\mathfrak{m}}}(\tilde{A}))\rangle + \int_\Sigma \langle \tilde{\Lambda}_{\mathfrak{h}}, F_{\tilde{A}} \rangle$$

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EOM for $\tilde{\Lambda}_{\mathfrak{h}}$ and $\tilde{\Lambda}_{\mathfrak{m}}$ take the form of MC eqs for the initial model

$$\mathfrak{h}: \quad F_{\tilde{A}} + \tfrac{1}{2}[\tilde{m}, \tilde{m}] = 0 \qquad \mathfrak{m}: \quad \nabla_{\tilde{A}} \tilde{m} = 0 \qquad \textit{with} \qquad \tilde{m} := m_\omega$$

while the EOM for $\tilde{A}$ is the unsolved equation on the previous slide.

SNATD of Cosets: Symmetric Spaces - Dual Action

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$$\tilde{S}_S = \int_\Sigma \langle d\tilde{\Lambda}_{\mathfrak{m}} - D_{\tilde{\Lambda}_{\mathfrak{m}}}(\tilde{A}), \frac{1}{1 - ad_{\tilde{\Lambda}_{\mathfrak{h}}}} P^+(d\tilde{\Lambda}_{\mathfrak{m}} - D_{\tilde{\Lambda}_{\mathfrak{m}}}(\tilde{A}))\rangle + \int_\Sigma \langle \tilde{\Lambda}_{\mathfrak{h}}, F_{\tilde{A}} \rangle$$

EOM for $\tilde{\Lambda}_{\mathfrak{h}}$ and $\tilde{\Lambda}_{\mathfrak{m}}$ take the form of MC eqs for the initial model

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while the EOM for $\tilde{A}$ is the unsolved equation on the previous slide.

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SNATD of Cosets: SS-Spaces - Dual Action

The master action for semi-symmetric spaces is of the form

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with $j_\omega := A_\omega + p_\omega + m_\omega + q_\omega$.

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Decomposing the latter on the various subspaces, 4 equations are obtained. Those for $\mathfrak{p}, \mathfrak{m}, \mathfrak{q}$ can be solved for $p_\omega, m_\omega, q_\omega$, while the one in $\mathfrak{h}$ reads

$$-[\tilde{\Lambda}_\mathfrak{q}, p_\omega] - D_{\tilde{\Lambda}_\mathfrak{m}}(m_\omega) - [\tilde{\Lambda}_\mathfrak{p}, q_\omega] = -\nabla_{\tilde{A}}\tilde{\Lambda}_\mathfrak{h}$$

and cannot be solved for $\tilde{A}$ in general form.

SNATD of Cosets: SS-Spaces - Dual Action

Substituting the solutions for $p_\omega, m_\omega, q_\omega$ in the action one finds

$$\tilde{S}_{SS} = \int_\Sigma \langle T, \frac{1}{1-S} P^+ T \rangle + \int_\Sigma \langle \tilde{\Lambda}_{\mathfrak{h}}, F_{\tilde{A}} \rangle +$$
$$+ \frac{1}{2} \int_\Sigma \langle \nabla_{\tilde{A}} \tilde{\Lambda}_{\mathfrak{p}}, R_{21}(\nabla_{\tilde{A}} \tilde{\Lambda}_{\mathfrak{p}}) + R_{22}(\nabla_{\tilde{A}} \tilde{\Lambda}_{\mathfrak{q}}) \rangle +$$
$$+ \frac{1}{2} \int_\Sigma \langle \nabla_{\tilde{A}} \tilde{\Lambda}_{\mathfrak{q}}, R_{11}(\nabla_{\tilde{A}} \tilde{\Lambda}_{\mathfrak{p}}) + R_{12}(\nabla_{\tilde{A}} \tilde{\Lambda}_{\mathfrak{q}}) \rangle$$

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where we have introduced the following quantities

- the operators R_{ij} are entries of a 2×2 matrix appearing when solving the equations in $\mathfrak{p}$ and $\mathfrak{q}$. They solely depend on $ad_{\tilde{\Lambda}_{\mathfrak{h}}}$ and $D_{\tilde{\Lambda}_{\mathfrak{m}}}$.

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- S is an operator starting with $ad_{\tilde{\Lambda}_\mathfrak{h}}$ as for symmetric spaces, but also receives contributions from compositions of R_{ij} and $ad_{\tilde{\Lambda}_\mathfrak{p}}$, $ad_{\tilde{\Lambda}_\mathfrak{q}}$.

SNATD of Cosets: SS-Spaces - Dual Lax Connection

The dual EOM for $\tilde{\Lambda}_{\mathfrak{h}}, \tilde{\Lambda}_{\mathfrak{p}}, \tilde{\Lambda}_{\mathfrak{m}}, \tilde{\Lambda}_{\mathfrak{q}}$ take the form of MC for the intial model

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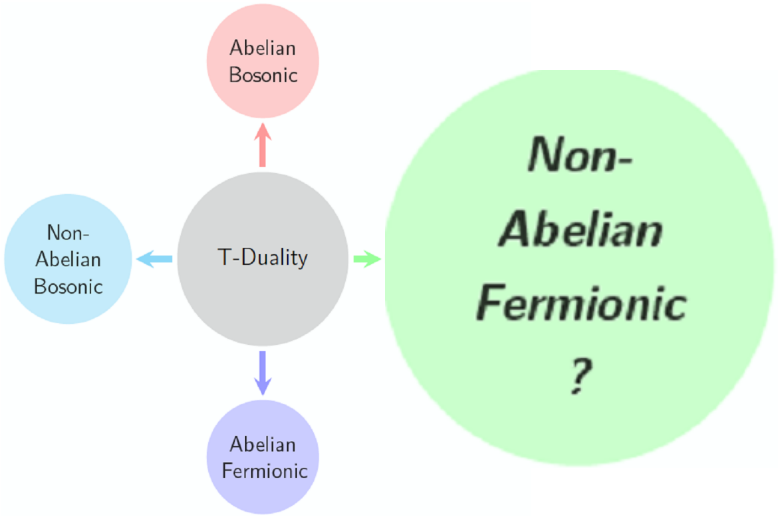
Conclusions: Short Summary



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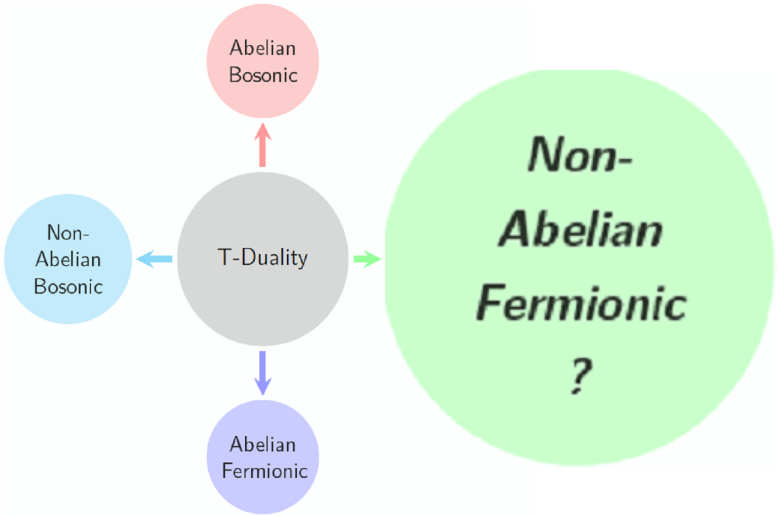

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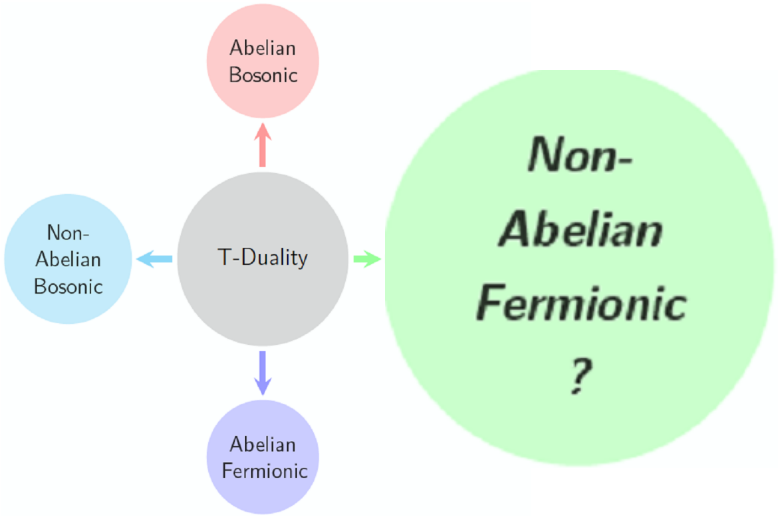

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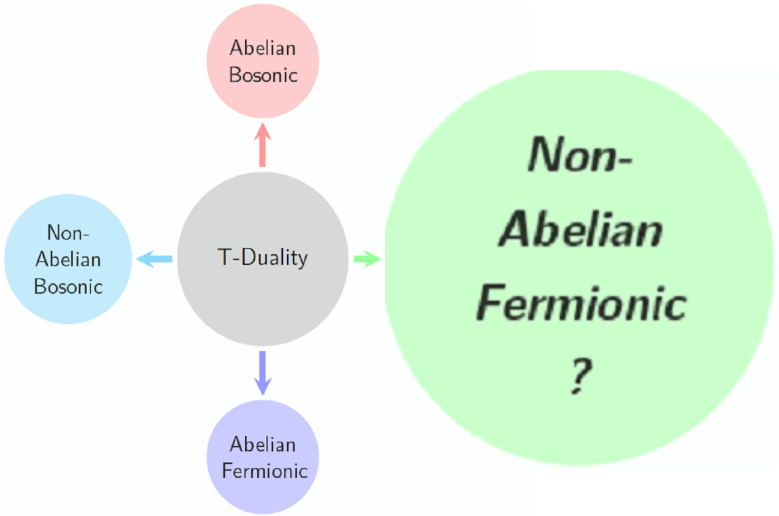

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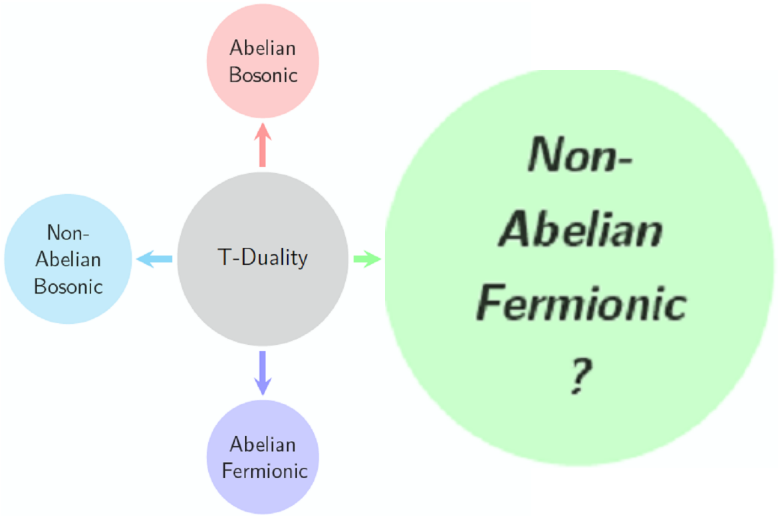

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- From the general perspective
  - Start considering explicit examples of symmetric and semi-symmetric spaces to uncover further properties of the super non-Abelian setting
  - Attempt dualisation of notable backgrounds which have so far eluded the proof of self duality, such as $AdS_4 \times \mathbb{CP}^3$

Thank You for the Attention!