Holography and Irrelevant Operators

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Roadmap.

Goal: expand upon the relationship between certain brane solutions in type IIB supergravity and irrelevant deformations of CFTs.

The talk is based on 2202.????? with Chih-Kai Chang and Savdeep Sethi.

The plan is as follows:

1. Introduction and background. Why are irrelevant deformations and these solutions related to holography beyond AdS? What is known?
2. Generalized solutions. How are our undecoupled spacetimes obtained?
3. Charges. Review of covariant phase space formalism and derivation of square-root formula.
4. Puzzles. Charge integrability; tachyon interpretation of large-λ instability; generalization to (p, q) branes.

Part 1: Introduction and background.

Why irrelevant deformations?

First, some intuition for why holography beyond AdS – e.g. in spacetimes similar to flat space – might be related to irrelevant deformations.

The entropy of big black holes has different scaling with energy in d-dimensional AdS versus flat space:

$$S_{\text{BBH}} \sim E^p \,, \quad p = \begin{cases} \frac{d-2}{d-1} & \mathrm{AdS} \\ \frac{d-2}{d-3} & \text{flat space} \end{cases} .$$

The AdS scaling matches the high-energy density of states in a $(d-1)$-dimensional CFT. But the flat space scaling disagrees with such a CFT at high energies.

This suggests that the flat space dual theory has modified UV behavior, as one would expect (for instance) from an irrelevant deformation of a CFT.

$T\overline{T}$-type deformations.

In the case of $\mathrm{AdS}_3/\mathrm{CFT}_2$, a natural well-behaved irrelevant deformation on the field theory side is $T\overline{T}$:

$$\frac{\partial \mathcal{L}_\lambda}{\partial \lambda} = \det \left(T^{(\lambda)}_{\mu\nu} \right) = \frac{1}{2} \left(\left(T^{(\lambda)\mu}{}_{\mu} \right)^2 - T^{(\lambda)\mu\nu} T^{(\lambda)}_{\mu\nu} \right) .$$

$T\overline{T}$ has several nice properties which follow from translation invariance:

1. This combination of stress tensors defines a local operator by point-splitting, up to total derivatives.
2. $\langle T\overline{T} \rangle$ factorizes into products of stress tensor one-point functions, which implies a flow equation for the cylinder energy levels:

$$E_n(\lambda) = \frac{R}{2\lambda} \left(\sqrt{1 + \frac{4\lambda E_n}{R} + \frac{4\lambda^2 P_n^2}{R^2}} - 1 \right) .$$

3. Observables in $T\overline{T}$-deformed theories (e.g. torus partition function or S-matrix) can be determined in terms of undeformed quantities.
4. High-energy density of states in a $T\overline{T}$-deformed theory is Hagedorn.

Holographic interpretation.

In AdS_3 holography, one can also define a single-trace version of the $T\overline{T}$ operator. If the dual field theory is the symmetric product orbifold* $(CFT_2)^N/S_N$, then the double-trace $T\overline{T}$ and single-trace $D(x)$ are

$$T\overline{T} = \left(\sum_{i=1}^{N} T_i \right) \left(\sum_{j=1}^{N} \bar{T}_j \right) \qquad \text{v.s.} \qquad D(x) = \sum_{i=1}^{N} T_i \overline{T}_i .$$

We expect that double-trace deformations modify boundary conditions while single-trace deformations change the bulk geometry.

Indeed, (an operator like) single-trace $T\overline{T}$ is believed to deform an AdS_3 geometry to one which interpolates from AdS_3 in the IR to linear dilaton in the UV. This geometry describes a vacuum of little string theory.

*The dual to strings on AdS_3 is not expected to be the symmetric product orbifold but a deformation of the orbifold by a marginal operator.

Some worldsheet input.

Much of the progress in understanding deformations like $D(x)$ has come from worldsheet considerations. In [Giveon, Itzhaki, Kutasov '17], it was proposed that this deformation corresponds to a worldsheet vertex operator

$$D(x) = \int d^2 z \, \left((\partial_x J)\partial_x + 2(\partial_x^2 J) \right) \left((\partial_{\bar{x}} \bar{J})\partial_{\bar{x}} + 2(\partial_{\bar{x}}^2 \bar{J}) \right) \Phi_1 \, ,$$

where $J(x;z) = 2xJ_3(z) - J^+(z) - x^2 J^-(z)$ and $\Phi_h(x;z)$ correspond to the $SL(2,\mathbb{R})_L$ currents in AdS_3 and to eigenfunctions of the Laplacian.

One can exchange the worldsheet integral $d^2 z$ for a boundary integral $d^2 x$:

$$\int d^2 x \, D(x) \sim \int d^2 z \, J^-(z) \bar{J}^-(\bar{z}) \, .$$

Thus there is a simple, exactly marginal deformation in the worldsheet theory that corresponds to the operator $D(x)$ in the boundary theory.

Spacetime picture.

So far we have described a deformation via the CFT operator $D(x)$ and the marginal worldsheet $J^- \bar{J}^-$ operator.

From the spacetime perspective, this deformation is believed to "re-couple" a linear dilaton region.

Recall that the usual decoupling procedure in the F1-NS5 frame is:

1. Begin with a gravity solution with F1-strings and NS5-branes.
2. First decouple by sending the asymptotic $g_s \to 0$. This goes to the near-horizon region of the five-branes (not necessarily the strings).
3. Further decouple by sending $\alpha' \to 0$. This goes to the AdS_3 in the near-horizon region of both the strings and fivebranes.

In this language, the irrelevant deformation begins from the totally decoupled AdS in (3) and goes back up to (2).

Goals of this work.

Although worldsheet techniques are powerful, the subtlety in defining $D(x)$ away from the orbifold point (and its relationship to the usual $T\overline{T}$) suggests that an orthogonal approach may yield fresh insights.

We will instead perform a **pure gravity analysis** without using data about the worldsheet or dual CFT.

Some questions we will aim to answer:

1. Does the single-trace $T\overline{T}$ lead to the same square-root formula as the double-trace version? Is it visible from gravity?
2. What is the totally undecoupled version of the partly decoupled solutions obtained by turning on single-trace $T\overline{T}$?
3. What is the interpretation of the parameter λ in the gravity theory?

Part 2: Generalized solutions.

The F1-NS5 solution.

We saw that the "single-trace" deformation has an interpretation in terms of little string theory and linear dilaton spacetimes.

These spacetimes are related to a type IIB supergravity solution with Q_1 fundamental strings and Q_5 NS5-branes:

$$ds^2 = -\frac{1}{f_1}\, dt^2 + \frac{1}{f_1}\, dx_5^2 + f_5\, dr^2 + f_5\, r^2\, d\Omega_3^2 + \left(dx_6^2 + \cdots + dx_9^2 \right) ,$$

$$e^{-2\Phi} = \frac{1}{g_s^2} \frac{f_1}{f_5} , \qquad H_3 = \frac{1}{r^3 f_5^{3/2}} \left(c_1 e^{2\Phi} \epsilon_3^{\mathcal{M}_3} + c_5 \epsilon_3^{S^3} \right) ,$$

$$f_1 = 1 + \frac{r_1^2}{r^2} , \qquad f_5 = 1 + \frac{r_5^2}{r^2} .$$

Here r_1 and r_5 depend on Q_1 and Q_5, and $\mathcal{M}_3 \sim (t, r, x_5)$.

In the decoupling limit $g_s \to 0$, $\alpha' \to 0$,* this solution looks like $\mathrm{AdS}_3 \times S^3 \times T^4$ and we have $\mathrm{AdS}/\mathrm{CFT}$.

*Alternatively one can think of this limit as taking $r \ll r_1, r_5$.

A mild generalization.

Let's treat this as a math problem: can we find a more general class of solutions with these asymptotics ($\mathrm{AdS}/\mathrm{BTZ}$ at small r, linear dilaton at intermediate r, flat at large r)?

Modify the previous solution as follows:

$$ds^2 = -\frac{f_e}{f_1}\, dt^2 + \frac{1}{f_1}\, (dx_5 + f_j\, dt\,)^2 + f_r\, dr^2 + f_5\, r^2\, d\Omega_3^2 + \left(dx_6^2 + \cdots + dx_9^2\right),$$

$$e^{-2\Phi} = \frac{1}{\tilde{g}_s^2}\frac{f_1}{f_5}, \qquad H_3 = \frac{1}{r^3 f_5^{3/2}}\left(c_1 e^{2\Phi}\epsilon_3^{\mathcal{M}_3} + c_5 \epsilon_3^{S^3}\right),$$

$$f_1 = k_1 + \frac{r_1^2}{r^2}, \qquad f_5 = k_5 + \frac{r_5^2}{r^2}.$$

Here k_1, k_5 are constants and f_e, f_j, f_r are functions of r to be determined.

Note that $g_s^2 \equiv \lim_{r\to\infty} e^{2\phi} = \tilde{g}_s^2 \frac{k_5}{k_1}$ so we have called this constant $\tilde{g}_s$ rather than g_s.

Flux quantization.

Although not visible in supergravity, in string theory flux quantization through the S^3 requires that

$$\frac{1}{4\pi^2\alpha'}\int_{S^3} H_3 = m_5 \in \mathbb{Z}\,,$$

where m_5 is an integer corresponding to the NS5-brane charge. There is another quantization condition on the dual field strength; if we define

$$H_7 = e^{-2\Phi} * H_3\,,$$

then for any 7-cycle Σ_7 this form satisfies

$$\frac{1}{(2\pi)^6(\alpha')^3}\int_{\Sigma_7} H_7 = m_1 \in \mathbb{Z}\,.$$

The form of H_3 on the previous slide is chosen to satisfy both of these quantization conditions, which automatically implies that H_3 satisfies its equation of motion $\nabla^\mu\left(e^{-2\Phi}H_{\mu\nu\rho}\right) = 0$.

Asymptotics and equations of motion.

We still require that our ansatz reduce to an $\mathrm{AdS}/\mathrm{BTZ}$-type solution at small r, which implies

$$f_e = k_e + \frac{r_e^2}{r^2} + \frac{r_j^4}{r^4}\,, \qquad f_j = \frac{r_j^2}{r^2}\,.$$

Finally we impose the Einstein equations and dilaton equation of motion, which fix f_r and give two algebraic conditions:

$$f_r = \frac{k_e f_5}{f_e}\,, \qquad c_5^2 k_e - 4 k_e r_5^2 + 4 k_5 r_5^2 r_e^2 = 0\,,$$
$$4 r_1^2 \left(k_e r_1^2 r_e^2 - k_1 r_e^4 + 2 k_1 k_e r_j^4 \right) - c_1 \tilde{g}_s^4 k_e r_e^2 = 0\,.$$

We now have a sensible family of solutions to the equations of motion of type IIB supergravity, *if* the constants are chosen so that the quadratic constraints have real solutions.

Parameters and λ.

The r_i and k_i in our solutions can be mapped onto physical parameters of the bulk BTZ, along with the F1-string and NS5-brane charges:

$$c_1 = \frac{32 m_1 \pi^4 \alpha'^3}{V_4} \,, \quad c_5 = 2\alpha' m_5 \,, \quad \frac{r_e^2}{r_1^2} = -8MG \,, \quad \frac{r_j^2}{r_1^2} = -4GJ \,, \quad \cdots \,.$$

The important parameter is $k_1 \equiv \frac{\lambda}{\alpha'}$. When k_1 is too large, the quadratic constraint on the previous slide requires $\tilde{g}_s^2$ to be imaginary:

$$\tilde{g}_s^2 = \frac{2}{c_1} \cdot \sqrt{\frac{k_e r_1^4 - k_1 r_1^2 r_e^2 + k_1^2 r_j^4}{k_e}} \,.$$

Therefore, changing $k_1 \sim \lambda$ in string frame has the effect of both changing the asymptotic g_s and re-scaling certain asymptotic metric components.

Why is this not trivial?

Usually such a re-scaling of coordinates just re-scales the mass. For BTZ,

$$ds^2 = -\left(\frac{r^2}{\ell^2} - M\right) dt^2 + \left(\frac{r^2}{\ell^2} - M\right)^{-1} dr^2 + r^2 \, d\phi^2,$$

where $\phi \sim \phi + 2\pi$. If we change to $\phi \sim \phi + 2\pi a$ for some dimensionless constant a, then define $\phi' = a\phi$, $r' = \frac{r}{a}$, $t' = at$, the metric returns to

$$ds^2 = -\left(\frac{r'^2}{\ell^2} - \frac{M}{a^2}\right) dt'^2 + \left(\frac{r'^2}{\ell^2} - \frac{M}{a^2}\right)^{-1} dr'^2 + r'^2 \, d\phi'^2$$

with a re-scaled mass parameter $M' = \frac{M}{a^2}$.

The effect of changing λ also leads to a different mass parameter, but it is not a trivial re-scaling; we simultaneously change t, x_5, and g_s in a correlated way which is different at large r and small r.

Towards a mass interpretation.

The formula for $\tilde{g}_s^2$ looks very similar to the square-root formula for double-trace $T\overline{T}$-deformed energies. The dilaton is

$$e^{2\Phi} = \frac{2}{c_1} \cdot \frac{k_5 + \frac{r_5^2}{r^2}}{k_1 + \frac{r_1^2}{r^2}} \cdot \sqrt{\frac{k_e r_1^4 - k_1 r_1^2 r_e^2 + k_1^2 r_j^4}{k_e}} \, .$$

When we convert from string frame to Einstein frame, the x_5 circle will be re-scaled in a λ-dependent and M-dependent way, differently at large r and small r. That is, the CFT cylinder radius (small r) is different from the UV theory cylinder radius (large r).

This is similar to the implicit solution $E_n(R, \lambda) = E_n\left(R + \lambda E_n(R, \lambda), 0\right)$ for $T\overline{T}$-deformed energies when $P_n = 0$.

To make this precise, we need a robust way of computing the mass of a type IIB solution with non-trivial dilaton and flux. We do this next.

Part 3: Charges.

Preliminaries on the formalism.

The covariant phase space formalism allows us to define conserved surface charges in generally covariant theories. First we define some basic notions.

For a theory with Lagrangian $\boldsymbol{L}(\Phi^i)$, where Φ^i represents an arbitrary collection of fields, we vary and integrate by parts to write

$$\delta \boldsymbol{L} = \frac{\partial \boldsymbol{L}}{\partial \Phi^i} \delta \Phi^i + \frac{\partial \boldsymbol{L}}{\partial(\partial_\mu \Phi^i)} \partial_\mu \delta \Phi^i + \cdots = \underbrace{\frac{\delta \boldsymbol{L}}{\delta \Phi^i}}_{\text{eom}} \delta \Phi^i - d\boldsymbol{\Theta}[\delta \Phi^i; \Phi^i] \, .$$

The object $\boldsymbol{\Theta}[\delta \Phi^i; \Phi^i]$ is called the presymplectic potential and depends both on the fields Φ^i and their variations $\delta \Phi^i$ (we will suppress the i).

To define conserved charges, it is convenient to define a related object $\boldsymbol{\omega}$ called the presymplectic form via

$$\boldsymbol{\omega}[\delta_2 \Phi, \delta_1 \Phi; \Phi] = \delta_2 \boldsymbol{\Theta}[\delta_1 \Phi; \Phi] \, .$$

Variations from diffeomorphisms.

We are interested in field variations $\delta_\xi \Phi$ associated with an infinitesimal diffeomorphism $x^\mu \to x^\mu + \xi^\mu$. For instance, under such a transformation any tensor field $T_{\mu_1 \cdots \mu_p}$ transforms via the Lie derivative $\mathcal{L}_\xi$ as

$$\delta_\xi T_{\mu_1 \cdots \mu_p} = \mathcal{L}_\xi T_{\mu_1 \cdots \mu_p} \, .$$

We will consider the case where the field configuration Φ satisfies the equations of motion for the theory, the first variation $\delta_1 \Phi = \delta \Phi$ solves the linearized equations of motion about the solution Φ, and the second variation $\delta_2 \Phi = \delta_\xi \Phi$ is generated by a diffeomorphism of this form.

In this case, one can show that $\boldsymbol{\omega}[\delta_\xi \Phi, \delta \Phi; \Phi]$ is exact, so that

$$\boldsymbol{\omega}[\delta_\xi \Phi, \delta \Phi; \Phi] = d \boldsymbol{k}_\xi [\delta \Phi; \Phi] \, ,$$

where $\boldsymbol{k}_\xi$ can also be expressed (up to total derivative terms) as

$$\boldsymbol{k}_\xi [\delta \Phi; \Phi] = - \delta \boldsymbol{Q}_\xi [\delta \Phi; \Phi] + i_\xi \boldsymbol{\Theta} [\delta \Phi; \Phi] \, .$$

Defining the charge.

We are primarily interested in the integral

$$\delta Q_\xi[\delta\Phi;\Phi] = \oint_S \boldsymbol{k}_\xi[\delta\Phi;\Phi]\,,$$

where S is a closed codimension-2 surface, e.g. a large sphere at fixed time t (so the total derivative ambiguity of $\boldsymbol{k}_\xi$ does not matter).

We specialize to the case where ξ is a Killing vector. In this case, δQ_ξ represents the change in the conserved quantity Q_ξ associated with the Killing vector ξ between the solutions Φ and $\Phi + \delta\Phi$.

In particular, if we have a parameterized family of solutions $\Phi(\alpha)$ for some α, then $\delta\Phi = \frac{\partial\Phi}{\partial\alpha}\,\delta\alpha$ is trivially a linearized solution. In this case $\delta Q_\xi[\delta\Phi;\Phi]$ is interpreted as $\frac{\partial Q_\xi}{\partial\alpha}\,\delta\alpha$.

Covariant phase space summary.

In short, one can think of this formalism as a black box whose inputs are

1. a Lagrangian $\mathcal{L}$ for some fields Φ^i;
2. a family of solutions to the equations of motion, labeled by some parameters α_i; and
3. a Killing vector ξ of the solutions.

The output of the black box is some expression for

$$\frac{\partial Q_\xi}{\partial \alpha_i} \, ,$$

which measures how the would-be charge Q_ξ changes as we vary one of the parameters α_i. If mixed partials agree, we can integrate to find $Q_\xi(\alpha_i)$.

Expressions for $\boldsymbol{k}_\xi$ in type IIB.

We will focus on actions of the form

$$S = \frac{1}{16\pi G} \int d^D x \sqrt{-g} \left(R - \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2} e^{-\beta \Phi} |H|^2 \right) .$$

There are three contributions to $\boldsymbol{k}_\xi$ from the three dynamical fields:

$$\begin{aligned} k_\xi^{g,\mu\nu} &= \xi^{[\nu} \nabla^{\mu]} \delta g - \xi^{[\nu} \nabla_\alpha \delta g^{\mu]\alpha} + \xi_\alpha \nabla^{[\nu} \delta g^{\mu]\alpha} + \frac{1}{2} \delta g \nabla^{[\nu} \xi^{\mu]} \\ &\quad - \frac{1}{2} \delta g^{\alpha[\nu} \nabla_\alpha \xi^{\mu]} + \frac{1}{2} \delta g^{\alpha[\nu} \nabla^{\mu]} \xi_\alpha \, , \\ k_\xi^{\mu\nu,\Phi} &= (\delta\Phi) \cdot \xi^{[\nu} \partial^{\mu]} \Phi , \\ k_\xi^{B,\mu\nu} &= \frac{1}{p} e^{-\beta\Phi} \xi^{[\mu} H^{\nu]\alpha_1 \cdots \alpha_p} \delta B_{\alpha_1 \cdots \alpha_p} - \frac{1}{2} e^{-\beta\Phi} \mathcal{L}_\xi H^{[\mu}{}_{\alpha_1 \cdots \alpha_p} \delta B^{\nu]\alpha_1 \cdots \alpha_p} + \cdots . \end{aligned}$$

The total change in a charge Q_ξ as a parameter α of the solution is varied, therefore, is given by $\frac{\partial Q_\xi}{\partial \alpha} = \oint_S \boldsymbol{k}_\xi$, where $\boldsymbol{k}_\xi = \boldsymbol{k}_\xi^\Phi + \boldsymbol{k}_\xi^B + \boldsymbol{k}_\xi^g$. The integral picks out $k_\xi^{t x_5}$ and we integrate at fixed t and large r.

Mass formula.

When $J = 0$ and $\xi = \partial_t$, we use the preceding formulas to find

$$\frac{\partial Q_\xi}{\partial r_e} = \oint_S \left(\boldsymbol{k}_\xi^g + \boldsymbol{k}_\xi^B + \boldsymbol{k}_\xi^\Phi \right) = - \frac{c_1}{8\pi G} \frac{r_e \, \delta r_e}{\sqrt{k_e r_1^4 - k_1 r_1^2 r_e^2}} \, .$$

This can be integrated to find

$$Q_\xi = \frac{c_1 \sqrt{k_e}}{8\pi G k_1} \left(\sqrt{1 - \frac{k_1}{k_e} \frac{r_e^2}{r_1^2}} - 1 \right) \, .$$

and since $r_e^2 \sim -M$, $k_1 \sim \lambda$, up to normalization of constants this is

$$Q_\xi = \frac{R}{2\lambda} \left(\sqrt{1 + \frac{4\lambda M}{R}} - 1 \right) \, .$$

The square-root formula of double-trace $T\overline{T}$ emerges directly from gravity.

Adding spin?

If $J \neq 0$, this prescription does not give the correct square-root formula for the mass. However, if we use a rotated Killing vector

$$\xi = \partial_t + \left(\frac{k_1 r_j^2}{r_1^2} \right) \partial_{x_5} \, ,$$

then we again recover a square-root dependence on M.

Interpretation: the t and x_5 coordinates in the asymptotic region, which should parameterize the space where the $T\overline{T}$-deformed theory lives, are different from (t, x_5) in the deep bulk near the BTZ black hole, which are the coordinates on which our undeformed CFT lives.

The deep bulk region sees the rotation of the black hole whereas the asymptotic region is approximately static; a spin-dependent change of variables is needed to compare quantities in the two regions.*

*This is similar to Kerr/CFT, where one defines the vacuum near the horizon not with respect to the usual Killing vector ∂_t but rather using a Killing vector $\partial_t - \Omega \partial_\phi$.

Part 4: Puzzles.

Charge integrability.

There are a few puzzles regarding these solutions whose resolution might provide additional insight.

In a (super)gravity solution with multiple parameters α_i, we can unambiguously define surface charges Q_ξ only if

$$\partial_{\alpha_i} \left(\partial_{\alpha_j} Q_\xi \right) = \partial_{\alpha_j} \left(\partial_{\alpha_i} Q_\xi \right) \qquad \text{for all } i \neq j \, .$$

However, it is known [Dias, Hartnett, Santos '19] that a general type IIB supergravity solution with multiple parameters does *not* satisfy this integrability condition.

For an AdS solution dual to a CFT, one can use additional input from holography (scale invariance) to identify preferred notions of charge. But our dual theory is not scale invariant in the deformed context.

Failure of integrability.

Charge in our solutions fail to be integrable in two ways:

1. For $J = 0$, charges are non-integrable in the space of λ, M. This is okay since M parameterizes different solutions in one theory, but λ changes between theories. [Apolo, Detournay, Song '19]
2. For $J \neq 0$, charges are non-integrable in the space of M, J. This is surprising; should be a prescription that picks out a notion of energy satisfying a square root equation and a momentum that doesn't flow.

One obstruction to charge integrability is a non-vanishing presymplectic current on the boundary:

$$\omega[\Phi, \delta\Phi]\Big|_{\text{bdry}} \neq 0 \iff Q_\xi \text{ not integrable.}$$

Can be related either to radiation through the boundary, or in our case, to boundary degrees of freedom.

This suggests that some data about the DoF in the dual theory (deformed by an irrelevant operator) are captured by the non-vanishing boundary ω.

Winding tachyon.

The algebraic constraint

$$4r_1^2\left(k_e r_1^2 r_e^2 - k_1 r_e^4 + 2k_1 k_e r_j^4\right) - c_1 \tilde{g}_s^4 k_e r_e^2 = 0$$

seems to require $\tilde{g}_s^2$ to be complex when $\lambda \sim k_1$ is too large.

Since large λ corresponds to a small x_5 circle at infinity, it is tempting to interpret this as the development of a closed string winding tachyon.

This suggests that, at large λ, we are expanding about the wrong vacuum. What is the correct vacuum? Is there a way to "cure" the large-λ theory and lift it to something consistent?

More general brane constructions.

Much of the progress in understanding the single-trace $T\overline{T}$ deformation relies on a worldsheet description, available since the fluxes are purely NS.

But what about a general background of (m, n) strings and (p, q) five-branes? In this case, both NS and Ramond fluxes are turned on, so there is no tractable worldsheet description.

However, the gravity analysis via covariant phase space techniques is still available (if more complicated). For instance, fluxes in $\mathcal{M}_3$ now look like

$$H_3 = (m_1 + n_1 C_0) e^{2\Phi} \frac{\kappa_3^2}{\pi\alpha'} \epsilon_3, \quad F_3 = \Big(n_1 + C_0 e^{2\Phi} (m_1 + C_0 n_1) \Big) \frac{\kappa_3^2}{\pi\alpha'} \epsilon_3.$$

to satisfy flux quantization, and one can generate 3-brane charge due to the form of $\widetilde{F}_5 = dC_4 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3$.

Can one perform a similar analysis in this context and find a relationship with irrelevant deformations?