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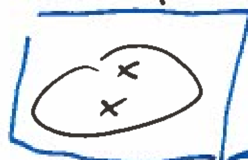
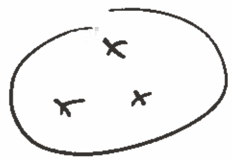
A gentle introduction to the AdS_3/CFT_2 mirror TBA

We want to compute AdS/CFT observables

CFT: $\langle \mathcal{O}^\dagger(x)\, \mathcal{O}(y) \rangle \simeq$  $+$  $+ \dots$

$\langle \mathcal{O}_1\, \mathcal{O}_2\, \mathcal{O}_3 \rangle \simeq$  $+$  $+ \dots$

$\mathcal{O}(x)$ is generic local op. (not protected)

planar spectrum

$$\frac{1}{|x-y|^{2\Delta}}$$

We want the string energy spectrum

Examples: Flat space strings

- very degenerate
- very simple

$$E(N,\tilde{N}) \simeq \sqrt{R^2 + \lambda(N+\tilde{N})}$$

$N = n_1 + n_2 + \dots + n_M$

$\tilde{N} = \dots$

Marboe - Volin 2014

(example of $AdS_5 \times S^5$ spectrum)

4. Summary of results and discussion

4.1. Results

As an example of the structure of the obtained results, the 10-loop conformal dimension of the Konishi operator is:

$$
\begin{aligned}
\Delta \;=\;& 4 + 12g^2 - 48g^4 + 336g^6 + g^8\big(-2496 + 576\,\zeta_3 - 1440\,\zeta_5\big) \\
&+g^{10}\big(15168 + 6912\,\zeta_3 - 5184\,\zeta_3^2 - 8640\,\zeta_5 + 30240\,\zeta_7\big) \\
&+g^{12}\big(-7680 - 262656\,\zeta_3 - 20736\,\zeta_3^2 + 112320\,\zeta_5 + 155520\,\zeta_3\,\zeta_5 + 75600\,\zeta_7 - 489888\,\zeta_9\big) \\
&+g^{14}\big(-2135040 + 5230080\,\zeta_3 - 421632\,\zeta_3^2 + 124416\,\zeta_3^3 - 229248\,\zeta_5 + 411264\,\zeta_3\,\zeta_5 \\
&\quad - 993600\,\zeta_5^2 - 1254960\,\zeta_7 - 1935360\,\zeta_3\,\zeta_7 - 835488\,\zeta_9 + 7318080\,\zeta_{11}\big) \\
&+g^{16}\Big(54408192 - 83496960\,\zeta_3 + 7934976\,\zeta_3^2 + 1990656\,\zeta_3^3 - 19678464\,\zeta_5 - 4354560\,\zeta_3\,\zeta_5 \\
&\quad - 3255552\,\zeta_3^2\,\zeta_5 + 2384640\,\zeta_5^2 + 21868704\,\zeta_7 - 6229440\,\zeta_3\,\zeta_7 + 22256640\,\zeta_5\,\zeta_7 \\
&\quad + 9327744\,\zeta_9 + 23224320\,\zeta_3\,\zeta_9 + \frac{65929248}{5}\,\zeta_{11} - 106007616\,\zeta_{13} - \frac{684288}{5}\,Z_{11}^{(2)}\Big) \\
&+g^{18}\Big(-1014549504 + 1140922368\,\zeta_3 - 51259392\,\zeta_3^2 - 20155392\,\zeta_3^3 + 575354880\,\zeta_5 \\
&\quad - 14294016\,\zeta_3\,\zeta_5 - 26044416\,\zeta_3^2\,\zeta_5 + 55296000\,\zeta_5^2 + 15759360\,\zeta_3\,\zeta_5^2 - 223122816\,\zeta_7 \\
&\quad + 34020864\,\zeta_3\,\zeta_7 + 22063104\,\zeta_3^2\,\zeta_7 - 92539584\,\zeta_5\,\zeta_7 - 113690304\,\zeta_7^2 - 247093632\,\zeta_9 \\
&\quad + 119470464\,\zeta_3\,\zeta_9 - 245099520\,\zeta_5\,\zeta_9 - \frac{186204096}{5}\,\zeta_{11} - 278505216\,\zeta_3\,\zeta_{11} - 253865664\,\zeta_{13} \\
&\quad + 1517836320\,\zeta_{15} + \frac{15676416}{5}\,Z_{11}^{(2)} - 1306368\,Z_{13}^{(2)} + 1306368\,Z_{13}^{(3)}\Big) \\
&+g^{20}\Big(16445313024 - 13069615104\,\zeta_3 - 1509027840\,\zeta_3^2 + 578949120\,\zeta_3^3 \\
&\quad - 14929920\,\zeta_3^4 - 11247547392\,\zeta_5 + 1213581312\,\zeta_3\,\zeta_5 + 1234206720\,\zeta_3^2\,\zeta_5 \\
&\quad - 70170624\,\zeta_3^3\,\zeta_5 - 1390279680\,\zeta_5^2 - 654842880\,\zeta_3\,\zeta_5^2 + \frac{6966252288}{175}\,\zeta_5^3 \\
&\quad + 377212032\,\zeta_7 - 1610841600\,\zeta_3\,\zeta_7 + 154680192\,\zeta_3^2\,\zeta_7 + 222341760\,\zeta_5\,\zeta_7 \\
&\quad + 133788672\,\zeta_3\,\zeta_5\,\zeta_7 + 868662144\,\zeta_7^2 + 4915257984\,\zeta_9 - 332646912\,\zeta_3\,\zeta_9 \\
&\quad - 91072512\,\zeta_3^2\,\zeta_9 + 1099699200\,\zeta_5\,\zeta_9 + 2275620480\,\zeta_7\,\zeta_9 + \frac{9793211904}{5}\,\zeta_{11} \\
&\quad - 2334572928\,\zeta_3\,\zeta_{11} + 2713772160\,\zeta_5\,\zeta_{11} - \frac{787483944}{175}\,\zeta_{13} + 3372969600\,\zeta_3\,\zeta_{13} \\
&\quad - \frac{4308536566944}{875}\,\zeta_{15} - 21661960320\,\zeta_{17} + \frac{752219136}{5}\,Z_{11}^{(2)} - \frac{5070791808}{175}\,Z_{13}^{(2)} \\
&\quad - \frac{7159104}{7}\,Z_{13}^{(3)} + \frac{2716063488}{175}\,Z_{15}^{(2)} - \frac{17895168}{25}\,Z_{15}^{(3)} + 11943936\,\zeta_3\,Z_{11}^{(2)}\Big) + \mathcal{O}(g^{22})\,, \qquad (85)
\end{aligned}
$$

where $Z_a^{(n)}$ denote single-valued MZV's written in the basis [63]

$$
\begin{aligned}
Z_{11}^{(2)} &= -\zeta_{3,5,3} + \zeta_3\,\zeta_{3,5}\,, \\
Z_{13}^{(2)} &= -\zeta_{5,3,5} + 11\,\zeta_5\,\zeta_{3,5} + 5\,\zeta_5\,\zeta_8\,, \\
Z_{13}^{(3)} &= -\zeta_{3,7,3} + \zeta_3\,\zeta_{3,7} + 12\,\zeta_5\,\zeta_{3,5} + 6\,\zeta_5\,\zeta_8\,, \\
Z_{15}^{(2)} &= \zeta_{3,7,5} - \zeta_5\,\zeta_{3,7} - 3\,\zeta_5\,\zeta_{10} + 21\,\zeta_9\,\zeta_6 + \frac{175}{2}\,\zeta_{11}\,\zeta_4 + \frac{637}{2}\,\zeta_{13}\,\zeta_2\,, \\
Z_{15}^{(3)} &= -\zeta_{3,9,3} + \zeta_3\,\zeta_{3,9} + 12\,\zeta_5\,\zeta_{3,7} + 30\,\zeta_7\,\zeta_{3,5} + 6\,\zeta_5\,\zeta_{10} + 15\,\zeta_7\,\zeta_8\,. \qquad (86)
\end{aligned}
$$

$AdS_5 \times S^5$ superstring ($\mathcal{N}=4$ SYM)

- non-degenerate
- very complicated (no closed formula)

$AdS_3 \times S^3 \times T^4$ superstrings : 2 parameters

- string tension (g^2 in the paper I showed
  λ in $\sqrt{R^2 + \lambda(N+\bar{N})}$)

- ratio of RR / NSNS background fluxes.

If only NSNS : then the model is

level-k WZW model, can be solved

- very degenerate spectrum
- simple spectrum
- it has short + long strings $\to$ continuum

Turn on a bit of RR flux.

- can study @ 1st order from WZW picture
- long-string continuum disappears.

Generic mix of RR + NSNS

- expect complicated, non-degenerate spectrum.

Only RR

- complicated, non-deg spectrum
- qualitatively, like $AdS_5 \times S^5$

As a classical f.t., $AdS_3 \times S^3 \times T^4$ NLSM is <u>integrable</u>. (hugely constrained dynamics)

Want to use <u>integrability</u> to study the spectrum of string

In principle, quantize the classical structure.

Very difficult!

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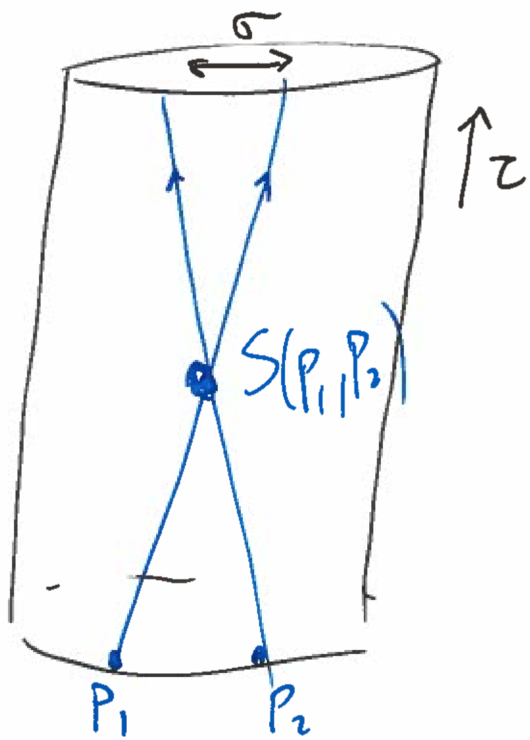
other approach: study the Hamiltonian of this 2D model. Also difficult.

Study the S-matrix of the theory (on worldsheet)



Introduce light-cone gauge

- remove unphysical d.o.f.
- relates $H_{w.s.}$ to the string energy

String energy is the eigenvalue of the cons. charge related to $t \to t + \delta t$ where t is AdS_3 time

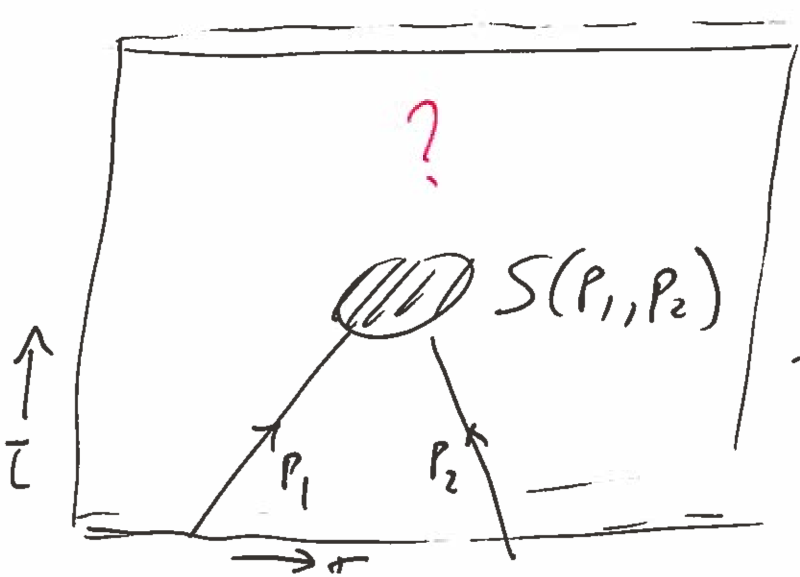
$H_{w.s.}$ is the charge related to $\tau \to \tau + \delta\tau$
(w. s. time)

Light-cone gauge

$$\underset{\uparrow \text{AdS time}}{t} \quad \tau = t + \phi \longleftarrow \text{equator on } S^3$$

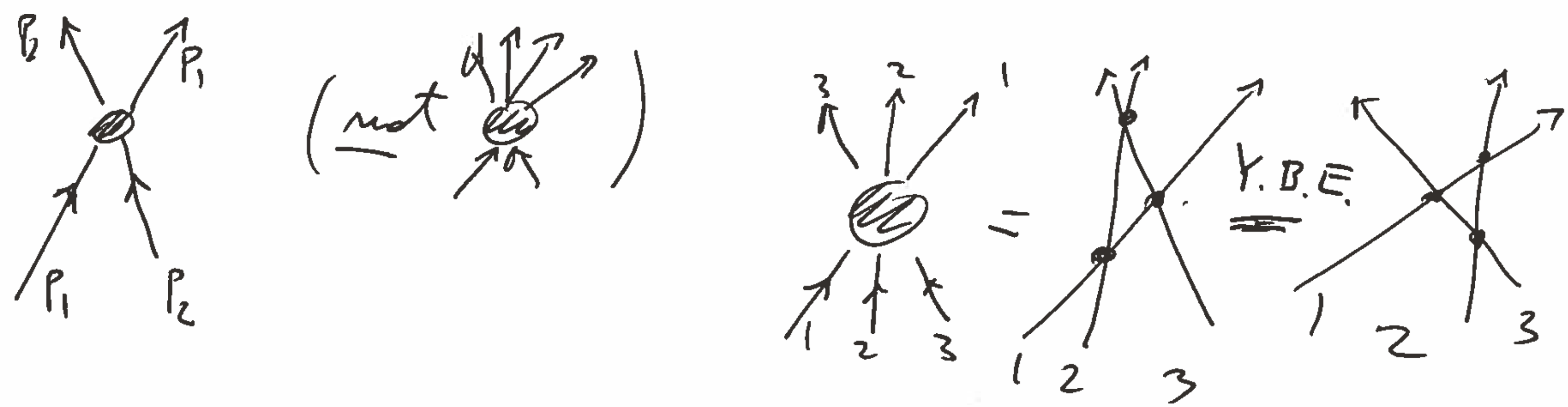
Also, radius of σ on w.s. is fixed
(related to J, the $u(1)$ charge of $\phi \to \phi + \delta\phi$)

I can consider $S(p_1, p_2)$.



Important assumption:
the theory is still integrable

If this is true, several
important properties of $S(p_1,p_2)$ follow.



Consequence: if I know $S_{ij}^{k\ell}(p_1, p_2)$

(which scatters $8b + 8f = 16$ particles, so it is a $16^2 \times 16^2$ matrix)

then I know <u>all</u> scattering. Solved the theory in infinite volume. But the spectrum arises in <u>finite</u> volume

$R = \int d\sigma\, p_\phi$ = R-charge.

But if I know $S(p_1, p_2)$ I can compute the finite-volume spectrum, by Bethe Ansatz.

If I have one particle on the plane, it has energy $E(p)$ (e.g. $\sqrt{p^2+m^2}$) and p is $p \in \mathbb{R}$.

If I have 1 part. on circle

$$e^{ipR} = 1 \qquad \text{and} \qquad E(p) = E\left(\frac{2\pi n}{R}\right).$$

If I have 2 particles, I get

$$e^{ip_1 R} S(p_1, p_2) = 1 \qquad e^{ip_2 R} S(p_2, p_1) = 1$$

$$E = E(p_1) + E(p_2)$$

How $S(p_1,p_2)$ and $E(p)$ follow from symmetry.

$AdS_3 \times S^3 \times T^4 \simeq \underbrace{psu(1,1|2) \oplus \widetilde{psu(1,1|2)}}$

$L_0, L_{\pm 1}$; $\tilde{L}_0, \tilde{L}_{\pm 1}$ $\leftarrow$ $su(1,1) \oplus su(1,1) \simeq so(2,2)$

$J^3, J^{\pm}$; $\tilde{J}^3, \tilde{J}^{\pm}$ $\leftarrow$ $su(2) \oplus su(2) \simeq so(4)$

and 4 Q's and 4 S's in $psu(1,1|2)$

4 $\tilde{Q}$'s and 4 $\tilde{S}$'s in $\widetilde{psu(1,1|2)}$

I only want what commutes with l.c. gauge fixing

$$\tau = t + \phi \Longrightarrow H_{w.s.} = \underbrace{L_0 + \tilde{L}_0}_{AdS_3\ \text{energy}} - \underbrace{J^3 - \tilde{J}^3}_{R\text{-charge}} \geq 0$$

because $L_0 - J^3 \geq 0$, $\tilde{L}_0 - \tilde{J}^3 \geq 0$ BPS bound.

I want all that commutes with $H_{w.s.}$

Half of susy's survive

$$\{Q^a, S_b\} = \delta^a_b \,(L_0 - J^3) \equiv \delta^a_b\, H \geq 0$$

$$\{\tilde{Q}_a, \tilde{S}^b\} = \delta_a^{\,b} \,(\tilde{L}_0 - \tilde{J}^3) \equiv \delta_a^{\,b}\, \tilde{H} \geq 0$$

$$\{Q^a, \tilde{Q}_b\} = \delta^a_b\, C \;; \quad \{S_a, \tilde{S}^b\} = \delta_a^{\,b}\, C^\dagger$$

$C, C^\dagger$ are central extensions that are 0 on physical states, satisfying $\sum p_i = 0 \mod 2\pi$

We have 8 bosons and 8 fermions. The short reps are 4 dimensional and obey

$$H \cdot \tilde{H} = C \cdot C^\dagger .$$ Note that $H + \tilde{H} = E = H_{w.s.}$

If I ~~know~~ know $C(p)$ $\quad C|p_1 \dots p_n\rangle = C(p_1 \dots p_n)|p_1 \dots p_n\rangle$

I can fix $E(p)$ and I find

$$E(p) = \sqrt{(\mu + \frac{k}{2\pi} p)^2 + 4h^2 \sin^2(p/2)}$$

WZW level (NSNS flux)

RR flux

$\mu \in \mathbb{Z}$ labels representations