$T\bar{T}$ AND NONLINEARLY REALIZED SYMMETRIES

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MOTIVATION

- $T\bar{T}$ OF FREE FERMION $\Rightarrow$
  $\Rightarrow$ NONLINEARLY REALIZED SUSY!
  (CRIBIORI, FARAKOS, VU, PRL 2019)
- $T\bar{T}$ OF FREE BOSON $\Rightarrow$
  $\Rightarrow$ NONLINEARLY REALIZED TRANSLATIONS!
- $T\bar{T}$ MOVING CFT INTO THE BULK
  (McGOUGH, MEZEI, VERLINDE, JHEP 2018)

NONLINEARLY REALIZED SYMMETRY

$G \longrightarrow H$ $\{$ generators $\hat{V}$ $\}$

G/H $\{$ generators $\hat{Z}$ $\}$

GENERIC ELT. OF BROKEN PART

$$g(\varphi) = e^{i\varphi \cdot \hat{Z}}$$

Goldstone bosons

ACTING WITH THE GROUP

$$\hat{g}(\varepsilon)\, g(\varphi) = g(\varphi')\, \underbrace{h(\varepsilon,\varphi)}_{\in H}$$

$\varphi \to \varphi'$ linear if $\hat{g}(\varepsilon) \in H$

$\varphi \to \varphi'$ non-linear if $\hat{g}(\varepsilon) \in G/H$

USEFUL TO DEFINE INVARIANT M.C. FORM

$$\Omega = g^{-1} dg = \Omega \cdot \hat{Z}$$

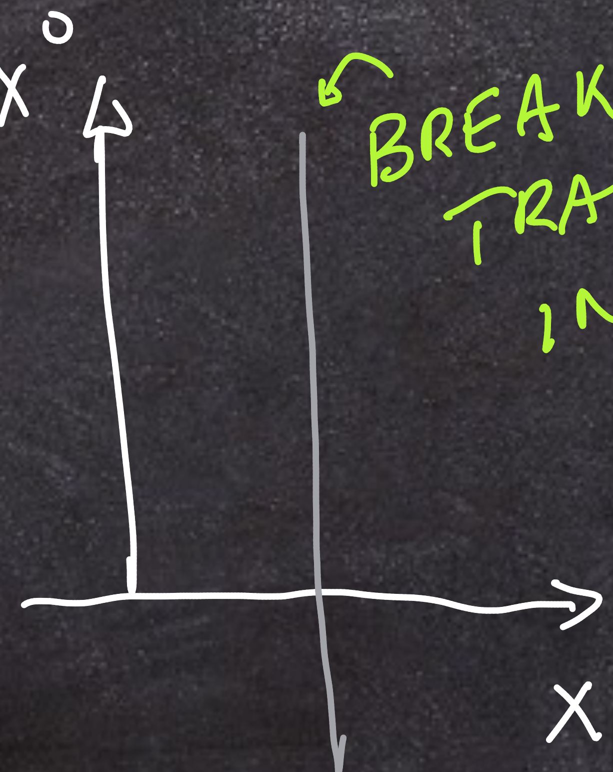
USE TO CONSTRUCT INVARIANT OBJECTS

NONLINEARLY REALIZED SHIFT SYMMETRY

(IAN MACARTHUR)

START IN 2D WITH POINCARÉ

$\hat{P}_0$, $\hat{P}_1$, $\hat{M}$ $\quad [\hat{M}, \hat{P}_0] = -i\hat{P}_1 \quad [\hat{M}, \hat{P}_1] = -i\hat{P}_0$



BREAKS TRANSLATION INVARIANCE

$\hat{P}_1$, $\hat{M}$ BROKEN

$\hat{P}_0$ UNBROKEN

$$g = e^{it\hat{P}_0 + i\,x(t)\,\hat{P}_1}\, e^{i\omega(t)\hat{M}}$$

GOLDSTONE BOSONS

$$g^{-1}dg = i\left(\Omega^0 \hat{P}_0 + \Omega^1 \hat{P}_1 + \Omega^M \hat{M}\right)$$

$$\begin{cases} \Omega^0 = \Omega^0{}_t\, dt = (\mathrm{ch}\,\omega - \dot{x}\,\mathrm{sh}\,\omega)\,dt \\ \Omega^1 = \Omega^1{}_t\, dt = (\dot{x}\,\mathrm{ch}\,\omega - \mathrm{sh}\,\omega)\,dt \\ \Omega^M = \Omega^M{}_t\, dt = \dot{\omega}\, dt \end{cases}$$

ONE FORMS ON THE PARICLE WORLD-LINE

TRANSFORMATIONS:

$$\hat{g}(\varepsilon) = e^{i\varepsilon \hat{P}_0} \quad \begin{cases} t \to t + \varepsilon \\ x \to x \\ w \to w \end{cases} \Rightarrow \begin{cases} \delta x = -\varepsilon \dot{x} \\ \delta w = -\varepsilon \dot{w} \end{cases}$$

$$\hat{g}(\varepsilon) = e^{i\varepsilon \hat{P}_1} \quad \begin{cases} t \to t \\ x \to x + \varepsilon \\ w \to w \end{cases} \Rightarrow \begin{cases} \delta x = \varepsilon \\ \delta w = 0 \end{cases}$$

NONLINEAR

$$\hat{g}(\varepsilon) = e^{i\varepsilon \hat{M}} \quad \begin{cases} t \to t + \varepsilon x \\ x \to x + \varepsilon t \\ w \to w + \varepsilon \end{cases} \Rightarrow \begin{cases} \delta x = \varepsilon(t - x\dot{x}) \\ \delta w = \varepsilon(1 - x\dot{w}) \end{cases}$$

IN THIS CASE Ω^1 IS INVARIANT

$$\Rightarrow \Omega^1 = (\operatorname{sh}\omega - \dot{x}\operatorname{ch}\omega)\,dt = 0$$

IS A GOOD CONSTRAINT

$$\omega = \operatorname{arctanh}\dot{x}$$

$$\operatorname{sh}\omega = \frac{\dot{x}}{\sqrt{1-\dot{x}^2}}$$

$$\operatorname{ch}\omega = \frac{1}{\sqrt{1-\dot{x}^2}}$$

WE MAY ELIMINATE ω!

$$\Omega^0 = \left(\frac{1}{\sqrt{1-\dot{x}^2}} - \frac{\dot{x}^2}{\sqrt{1-\dot{x}^2}}\right)dt = \sqrt{1-\dot{x}^2}\,dt$$

"NAMBU-GOTO" IN STATIC GAUGE

BY CONSTRUCTION, THIS IS INVARIANT UNDER ALL OF THE 2D POINCARÉ GRP.

- $\delta x = -\varepsilon \dot{x} \quad \Rightarrow \quad \delta \int dt \sqrt{1-\dot{x}^2} = -\varepsilon \int dt \frac{d}{dt} \sqrt{1-\dot{x}^2}$

- $\delta x = \varepsilon \quad \Rightarrow \quad \delta \int dt \sqrt{1-\dot{x}^2} = 0$

- $\delta x = \varepsilon (t - x\dot{x}) \quad \Rightarrow \quad \delta \int dt \sqrt{1-\dot{x}^2} = -\varepsilon \int dt \frac{d}{dt} \left[x \sqrt{1-\dot{x}^2} \right]$

QUESTION: DO $T\bar{T}$ DEFORMED THEORIES ALWAYS HAVE NONLINEARLY REALIZED SYMMETRIES?

USE FORMALISM OF GUICA, MONTEN KRAUS, LIU, MAROLF, CAPUTTA, JIANG TOLLEY, FREIDEL, MAZENC, SHYAM, SONI, ...

LET THE DEFORMATION BE ENCODED IN THE BACKGROUND GEOMETRY

VIELBEIN

$$S_\lambda = S(\lambda, E_a^{\;\alpha}, \text{"matter"})$$

TO DEFINE THE E.M. TENSOR VARY $E_a^{\;\alpha}$

$$\delta S_\lambda = \int E\, \delta E_a^{\;\alpha}\, T_\alpha^{\;a}$$

BUT WE MAY ALSO DIFFERENTIATE W.R.T. λ

$$\partial_\lambda S_\lambda = \frac{1}{2}\int E\, \Theta \qquad \boxed{\Theta = -2 \det T}$$

USE THAT $\delta \partial_\lambda S = \partial_\lambda \delta S$

$$\int \partial_\lambda (E T^a_\alpha) \delta E^\alpha_a + E T^a_\alpha \delta (\partial_\lambda E^\alpha_a) =$$

$$= \int \delta E^\alpha_a X^a_\alpha + E T^a_\alpha \delta Y^\alpha_a$$

$$\boxed{\begin{aligned} \partial_\lambda (E T^a_\alpha) &= X^a_\alpha \\ \partial_\lambda E^\alpha_a &= Y^\alpha_a \end{aligned}}$$

FROM THIS FOLLOWS:

$$\boxed{\partial_\lambda E_\alpha^{\ a} = -\hat{T}_\alpha^{\ a} \qquad \partial_\lambda \hat{T}_\alpha^{\ a} = 0}$$

WHERE $\hat{T}_\alpha^{\ a} = T_\alpha^{\ a} - E_\alpha^{\ a} E_b^{\ \beta} T_\beta^{\ b}$ (TRACE REVERSED)

- THESE EQUATIONS CAN BE SIMPLY SOLVED

$$E_\alpha^{\ a} = e_\alpha^{\ a} - \lambda \hat{T}_\alpha^{\ a}$$

WHERE $\hat{T}$ IS INDEPENDENT OF λ!

- FIND $\hat{T}_\alpha^{\ a}$ IN THE UNPERTURBED THY
- SOLVE FOR $E_\alpha^{\ a}$ ← "FLAT" ← "CURVED"
- CHANGE "FRAME" TO A SYSTEM WHERE THE BACKGROUND IS FLAT BUT THE THEORY IS PERTURBED
- THE COORDINATES NEED TO CHANGE ACCORDINGLY

EXAMPLE: FREE BOSON

EM TENSOR:

$$T_{uu} = \partial_u \phi \, \partial_u \phi \qquad T_{\alpha\beta} = \begin{pmatrix} (\partial_u \phi)^2 & 0 \\ 0 & (\partial_v \phi)^2 \end{pmatrix}$$

$$T_{vv} = \partial_v \phi \, \partial_v \phi$$

WHICH GIVES

$$\hat{T}_\alpha{}^\beta = \begin{pmatrix} 0 & (\partial_u \phi)^2 \\ (\partial_v \phi)^2 & 0 \end{pmatrix} \Longrightarrow E_\alpha{}^a = \begin{pmatrix} 1 & -\lambda (\partial_u \phi)^2 \\ -\lambda (\partial_v \phi)^2 & 1 \end{pmatrix}$$

$E_a{}^{\alpha}$ ITSELF GIVES A MAP

$$\begin{pmatrix} \partial_u \\ \partial_v \end{pmatrix} \xrightarrow{E} \begin{pmatrix} \partial_U \\ \partial_V \end{pmatrix} \quad \text{"FLAT FRAME"}$$

$$\partial_U \phi = E_U{}^u \partial_u \phi + E_U{}^v \partial_v \phi$$

$$\partial_V \phi = E_V{}^u \partial_u \phi + E_V{}^v \partial_v \phi$$

GIVES $\partial_u \phi$ and $\partial_v \phi$ EXPRESSED IN TERMS OF $\partial_U \phi$ and $\partial_V \phi$

LAGRANGIAN CAN BE FOUND FROM CALCULATING $\det T$ AS A FUNCTION OF $\partial_u \phi$, $\partial_v \phi$ AND INTEGRATING WITH RESPECT TO λ

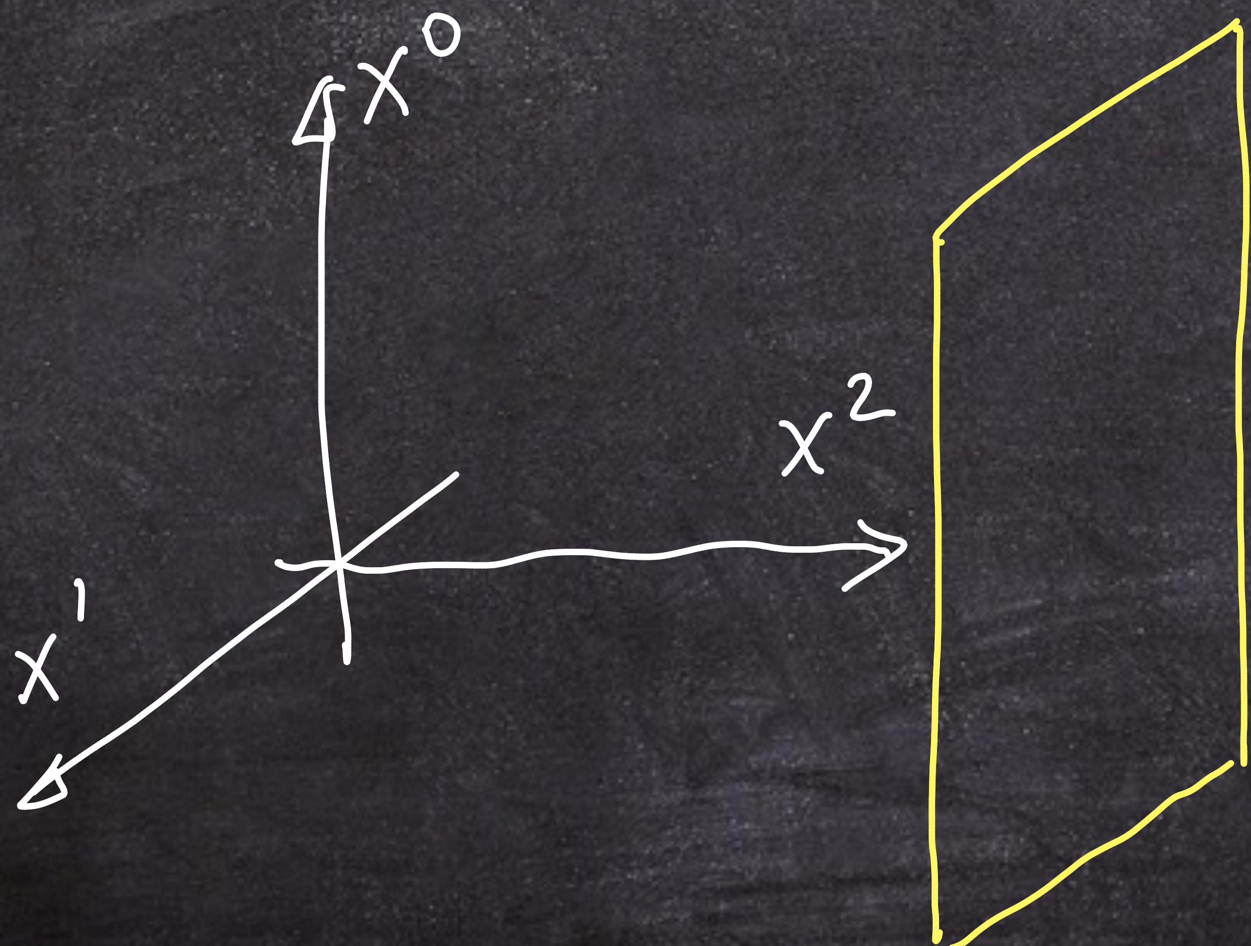
ALSO, EXPRESS E IN THE FLAT FRAME

$$E_\alpha{}^a = \begin{pmatrix} 1 & -\dfrac{4\lambda(\partial_u\phi)^2}{\left(1+\sqrt{1+4\lambda\partial_u\phi\,\partial_v\phi}\right)^2} \\ -\dfrac{4\lambda(\partial_v\phi)^2}{\left(1+\sqrt{1+4\partial_u\phi\,\partial_v\phi}\right)^2} & 1 \end{pmatrix}$$

THIS IS ALMOST THE VIELBEIN OF THE NONLINEARLY REALIZED SYMMETRY

3D POINCARÉ $\longrightarrow$ 2D POINCARÉ

WHERE



$\hat{P}^2, \hat{M}^{02}, \hat{M}^{12}$ BROKEN

$\hat{P}^0, \hat{P}^1, \hat{M}^{01}$ UNBROKEN

$$g^{-1}dg = E^{\#}\hat{P}_{\#} + E^{=}\hat{P}_{=} + \Omega^{3}\hat{P}_{3} + \Omega^{\#}\hat{M}_{\#3} + \Omega^{=}\hat{M}_{=3}$$

$\Omega^3 = 0$ IS A GOOD CONSTRAINT

$\Rightarrow$ ELIMINATE GOLDSTONE MODES ASSOCIATED WITH $\hat{M}_{\#3}$ & $\hat{M}_{=3}$

$$E^{\#} = \frac{1}{2}\left[1 + \sqrt{1 + 2\partial_{\#}\phi\partial_{=}\phi}\right]dx^{\#} + \frac{1}{2}\frac{\partial_{=}\phi}{\partial_{\#}\phi}\left[\sqrt{1+2\partial_{\#}\phi\partial_{=}\phi} - 1\right]dx^{=}$$

$$E^{=} = \frac{\partial_{\#}\phi}{2\partial_{=}\phi}\left[\sqrt{1+2\partial_{\#}\phi\partial_{=}\phi} - 1\right]dx^{\#} + \frac{1}{2}\left[1 + \sqrt{1+2\partial_{\#}\phi\partial_{=}\phi}\right]dx^{=}$$

CONCLUSIONS:

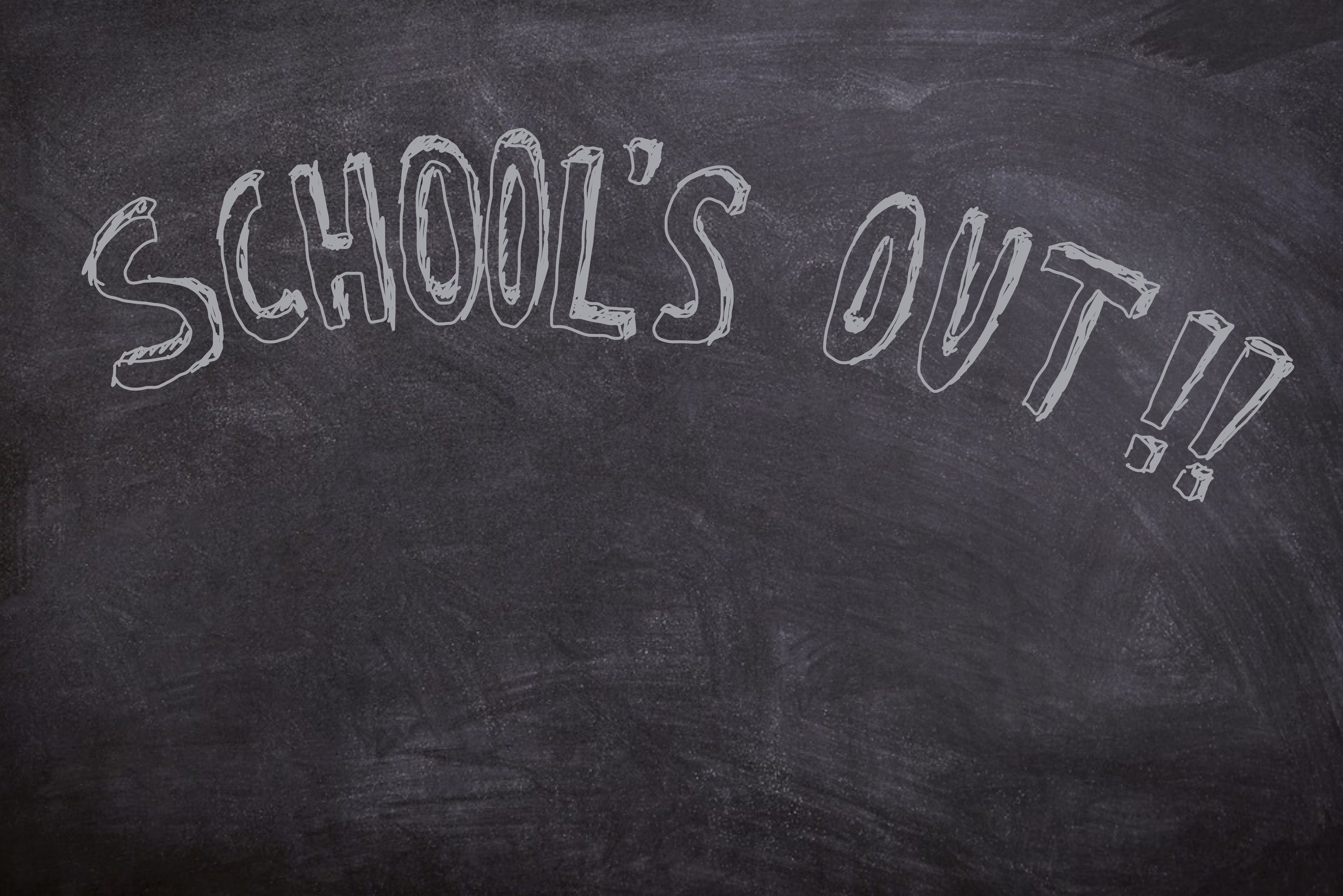
- $T\bar{T}$ DEFORMED THEORIES HAVE NONLINEARLY REALIZED SYMMETRIES
- USING THE VIELBEIN FORMALISM FOR $T\bar{T}$ WHERE THE BACKGROUND IS DEFORMED, THERE IS AN INTRIGUING SIMILARITY WITH THE VIELBEINS OF THE NONLINEAR REALIZATION

- IS THERE A CONNECTION WITH PUTTING THE CFT SURFACE AT FINITE r IN AdS_3 THEREBY BREAKING THE SYMMETRY

$$SO(2,2) \longrightarrow U(1) \times U(1)$$

WITH THE REST OF THE SYMMETRY NONLINEARLY REALIZED?

- WHAT IF THE ORIGINAL THEORY IS NOT CONFORMAL?
- OTHER TYPES OF DEFORMATIONS
- NO ANSWERS YET, ONLY QUESTIONS
- MORE WORK NEEDED


SCHOOL'S OUT!!