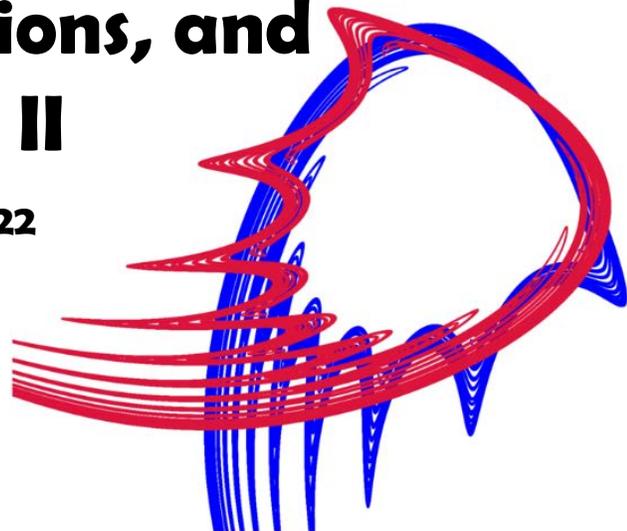# MATRIX Research Program:
Dynamics, Foliations, and Geometry II

31 Jan – 3 Feb 2022

Mini Course Abstracts

Speaker: Andy Hammerlindl (Monash)

Title: Branching Foliations

**Tentative Schedule: 10 - 11 am (local Australian time, AEDT, UTC+11)
on 31 January, 1 February, and 2 February**

Abstract: One of the most powerful tools used to study partially hyperbolic dynamical systems is the branching foliation theory developed by Brin, Burago, and Ivanov. A branching foliation is a collection of surfaces immersed in a three dimensional manifold which are allowed to intersect each other, but do not topologically cross through each other. In recent years, branching foliations have been used to achieve classification results for several families of partially hyperbolic dynamical systems and establish many other properties.

Despite its importance, the theorem showing the existence of these branching foliation has been treated as a "black box" and the details of the proof are not well known in the Dynamics community. In this mini-course, I will attempt to "open the black box" and give an explanation of the structure of the proof and the techniques and ideas it contains.

Speaker: Pierre Dehornoy (Grenoble)

Title: Birkhoff sections for 3-dimensional vector fields

**Tentative Schedule: 9:30 - 10:30 am (local German time, CET, UTC+1)
On 31 January, 1 February, and 2 February**

Abstract: A Birkhoff section for a vector field is transverse surface with boundary that intersects all orbits. It is a useful dynamical tool. In this series of lectures, I will present them together with several examples. In particular I will insist on the "lattice" structure on the set of Birkhoff sections.

(1) Birkhoff sections: examples and constructions (geodesic flows, Anosov flows):

We present global sections and Birkhoff sections for 3d flows. We provide examples for simple flows (Hopf flow, geodesic flow on hyperbolic surfaces). We present Fried's proof of the existence of Birkhoff sections for transitive Anosov flows.

(2) Asymptotic cycles and criteria for the existence of global sections (Schwartzman-Fried-Sullivan theory)

Schwartzman's asymptotic cycles give an elegant criterion to prove the existence of global sections for flows. We present it, together with Sullivan's proof, and sketch how to adapt it for Birkhoff sections.

(3) Flows with many Birkhoff sections: right-handed flows

Arnold-Vogel's asymptotic linking theory generalizes asymptotic cycles. Building on this, we present Ghys' right-handed flows, which correspond to those flow that have as many Birkhoff sections as one could hope. We give examples : the Hopf flow, and some geodesic flows in positive and in negative curvature.

(4) A classification result for geodesic flows on hyperbolic surfaces

If time permits, we explain Thurston's theory of fibered faces, what it says about Birkhoff sections of a given flow, and an explicit situation (the geodesic flow on a hyperbolic surface) where this information can be made elementary.