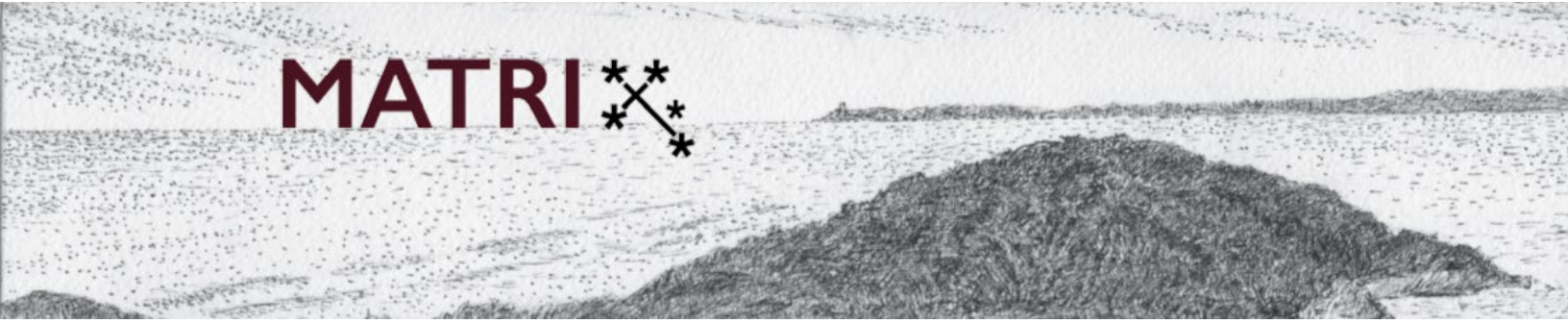

MATRIX Research Program:

Hyperbolic Differential Equations in Geometry and Physics

Talk Titles and Abstracts

4 – 8 April 2022

MATRIX, Creswick

Talk Schedule

Monday, 4 April 2022

- 14:00-15:00
  Speaker: Jan Sbierski
  Title: Instability of the Kerr Cauchy horizon under linearised gravity

- 15:00-16:00
  Speaker: Dean Baskin
  Title: Propagation of singularities for the Dirac--Coulomb system

Tuesday, 5 April 2022

- 9:00-10:00
  Speaker: Andrew Hassell
  Title: Time-dependent Schrodinger equation: Linear analysis

- 10:00-11:00
  Speaker: Sean Gomes
  Title: A Fredholm approach to the nonlinear Schrödinger equation

- 14:00-15:00
  Speaker: Shuang Miao (**Online talk)**
  Title: On the hard phase fluid with free boundary in relativity

Wednesday, 6 April 2022

- 9:00-10:00
  Speaker: Allen Fang
  Title: A new proof for the nonlinear stability of slowly-rotating Kerr-de Sitter

- 10:00-11:00
  Speaker: Kiril Datchev
  Title: Wave asymptotics and low-energy resolvent expansions in dimension two

Thursday, 7 April 2022

- 9:00-10:00
  Speaker: Grigorios Fournodavlos (**Online talk)**
  Title: Stable Big Bang formation

- 14:00-15:00
  Speaker: Arick Shao
  Title: Bulk-boundary correspondence for vacuum asymptotically Anti-de Sitter spacetimes

- 15:00-16:00
  Speaker: Xinliang An (**Online talk)**
  Title: Low regularity ill-posedness for 3D elastic waves and for 3D ideal compressible MHD driven by shock formation

Friday, 8 April 2022

- 9:00-10:00
  Speaker: Zihua Guo
  Title: Global well-posedness and scattering for the 2D cubic derivative hyperbolic Schrodinger equation

- 10:00-11:00
  Speaker: Florian Beyer (**Online talk)**
  Title: Fluids in the vicinity of Kasner big bang singularities

Titles and Abstracts

Speaker: Jan Sbierski

Title: Instability of the Kerr Cauchy horizon under linearised gravity

Abstract: Penrose's strong cosmic censorship conjecture in general relativity can be paraphrased in the language of partial differential equations as stating that global uniqueness holds generically for the initial value problem for the Einstein equations. An exception to globally unique time-evolution in general relativity is given by the Kerr solution, which models a rotating black hole. However, if Penrose's conjecture holds water, then generic perturbations of Kerr initial data should lead to a solution which is globally uniquely determined. This problem, and heuristics supporting Penrose's conjecture, go back to the 60s. As will be explained in the talk, it is intimately tied to the generic formation of a singularity at the Kerr Cauchy horizon. I will present a recent result which establishes the instability at the Kerr Cauchy horizon at the level of the linearised Einstein equations.

Speaker: Dean Baskin

Title: Propagation of singularities for the Dirac--Coulomb system

Abstract: The Dirac equation describes the relativistic evolution of electrons and positrons. We consider the (time-dependent!) Dirac equation in three dimensions coupled to a potential with Coulomb-type singularities. Motivated by a question from quantum field theory, we prove a propagation of singularities result for this equation and show that singularities are diffracted by the singularities of the potential. If time permits, I will describe how similar techniques can be used to characterize the asymptotic behavior of solutions of this system. This talk is based on joint work with Jared Wunsch as well as work with Bob Booth and Jesse Gell-Redman.

Speaker: Andrew Hassell
Title: Time-dependent Schrodinger equation: Linear analysis (joint work with Jesse Gell-Redman and Sean Gomes)
Abstract: We consider at the time-dependent Schrodinger operator P on $\mathbb{R}^{n+1}$, with fixed metric and potential that are flat/trivial outside a compact set in spacetime. Considering first the inhomogeneous equation
$$
Pu = f,
$$
in spacetime, we find Hilbert spaces of functions $P : X \to Y$ between which P maps invertibly. This is done by proving microlocal propagation estimates, following Melrose and Vasy, near the characteristic variety of P, and assembling them into a global Fredholm estimate. Using this, we can solve the “final state problem”, which is to find a global solution to $Pu = 0$ where $u(x,t)$ has the asymptotic
$$
u(x, t) \sim t^{-n/2} e^{i|x|^2/4t} f_+(x/t), \quad t \to +\infty,
$$
for a prescribed function f_+.

Our framework leads to some precise results in linear scattering, which seem to be new. More significantly it provides an entirely new approach to nonlinear scattering. This will be discussed more in Sean Gomes’ talk.

Speaker: **Sean Gomes**
Title: **A Fredholm approach to the nonlinear Schrödinger equation**
Abstract: (This talk is based on joint work with Hassell and Gell-Redman.)

Following up on Andrew's talk, we continue to consider the time-dependent Schrodinger operator P on $\mathbb{R}^{n+1}$, with fixed metric and compactly supported potential.

The Hilbert spaces between which P acts invertibly are refined using the notion of regularity with respect to a test module $\mathcal{M}\subset \Psi_{\mathrm{par}}$. The Fredholm approach adapted to these module regularity spaces yields a solution to the final state problem with precise mapping properties for the Poisson map $f_+\mapsto u$ as well as the scattering map $f_-\mapsto f_+$.

We shall then discuss how the preceding work generalises to nonlinear Schrodinger equations, provided that the incoming data f_- is small in a suitable norm.

Speaker: **Shuang Miao** (**Online talk**, shuang.m@whu.edu.cn)

Title: **On the hard phase fluid with free boundary in relativity**

Abstract: The hard phase model is an idealized model for a relativistic fluid when the sound speed approaches the speed of light. I will present some recent progress on the well-posedness for a free boundary problem of this model. This talk is based on joint works with Sohrab Shahshahani and Sijue Wu.

Speaker: **Allen Fang**

Title: **A new proof for the nonlinear stability of slowly-rotating Kerr-de Sitter**

Abstract: The nonlinear stability of the slowly-rotating Kerr-de Sitter family was first proven by Hintz and Vasy in 2016 using microlocal techniques. In my talk, I will present a novel proof of the nonlinear stability of slowly-rotating Kerr-de Sitter spacetimes that avoids frequency-space techniques outside of a neighborhood of the trapped set. The proof uses vectorfield techniques to uncover a spectral gap corresponding to exponential decay at the level of the linearized equation. The exponential decay of solutions to the linearized problem is then used in a bootstrap proof to conclude nonlinear stability.

Speaker: **Kiril Datchev**
Title: **Wave asymptotics and low-energy resolvent expansions in dimension two**
Abstract Consider the free wave equation in $\mathbb R^2$ with compactly supported initial data:
\[
(\partial_t^2 - \partial_{x_1}^2 - \partial_{x_2}^2) w(x,t) = 0 ,\qquad w(x,0) = f(x), \quad \partial_t w(x,0) = g(x).
\]
It is straightforward to check using Kirchoff's explicit solution formula that, as $t \to \infty$, the solution w has the following asymptotic behavior:
\[
w(x,t) = \sum_{j=0}^\infty w_j(x) t^{-1-j}, \qquad \qquad w_0(x) = \frac 1 {2\pi} \int g, \quad w_1(x) = \frac {-1} {2\pi}\int f, \quad \text{etc.}
\]
In this talk I will discuss work in progress on analogous long-time wave asymptotics for some compactly supported perturbations of the above two-dimensional Euclidean Laplacian $-\partial_{x_1}^2 - \partial _{x_2}^2$, including nontrapping obstacle problems and Schr\"odinger operators with nonnegative potentials. These are based on resolvent expansions near zero energy, which in turn are based on some identities due to Vodev. While more general resolvent expansions in dimension two can be very complicated, in our particular cases they simplify nicely.

This is joint work in progress with Tanya Christiansen.

Speaker: **Grigorios Fournodavlos** (**Online talk**, grigorios.fournodavlos@sorbonne-universite.fr)
Title: **Stable Big Bang formation**
Abstract: In this talk we will investigate the past dynamics of cosmological solutions to Einstein's equations, containing a Big Bang singularity. More precisely, we will focus on the classical generalised Kasner examples. The celebrated ``singularity'' theorem of Hawking tells us that the past of sufficiently small perturbations of such solutions are causally geodesically incomplete. However, it is not in general known whether such a degeneracy is related to the formation of a curvature singularity. In many cases, unstable dynamics are predicted, which add to the difficulty of the problem. I will present joint work with I. Rodnianski and J. Speck that classifies the behavior of perturbed solutions in the so-called subcritical regime.

Speaker: **Arick Shao**

Title: **Bulk-boundary correspondence for vacuum asymptotically Anti-de Sitter spacetimes**

Abstract: The AdS/CFT conjecture in physics posits the existence of a correspondence between gravitational theories in asymptotically Anti-de Sitter (aAdS) spacetimes and field theories on their conformal boundary. In this presentation, we prove rigorous mathematical statements toward this conjecture.

Speaker: **Xinliang An** (**Online talk**, matax@nus.edu.sg)

Title: **Low regularity ill-posedness for 3D elastic waves and for 3D ideal compressible MHD driven by shock formation**

Abstract: We construct counterexamples to the local existence of low-regularity solutions to elastic wave equations and to the ideal compressible magnetohydrodynamics (MHD) system in three spatial dimensions (3D). Inspired by the recent works of Christodoulou, we generalize Lindblad's classic results on the scalar wave equation by showing that the Cauchy problems for 3D elastic waves and for 3D MHD system are ill-posed in $H^3(R^3)$ and $H^2(R^3)$, respectively. Both elastic waves and MHD are physical systems with multiple wave-speeds. We further prove that the ill-posedness is caused by instantaneous shock formation, which is characterized by the vanishing of the inverse foliation density. In particular, when the magnetic field is absent in MHD, we also provide a desired low-regularity ill-posedness result for the 3D compressible Euler equations, and it is sharp with respect to the regularity of the fluid velocity. Our proofs for elastic waves and for MHD are based on a coalition of a carefully designed algebraic approach and a geometric approach. To trace the nonlinear interactions of various waves, we algebraically decompose the 3D elastic waves and the 3D ideal MHD equations into 6×6 and 7×7 non-strictly hyperbolic systems. Via detailed calculations, we reveal their hidden subtle structures. With them we give a complete description of solutions' dynamics up to the earliest singular event, when a shock forms. This talk is based on joint works with Haoyang Chen and Silu Yin.

Speaker: **Zihua Guo**

Title: **Global well-posedness and scattering for the 2D cubic derivative hyperbolic Schrodinger equation**

Abstract: We prove global well-posedness and scattering for the 2D cubic derivative hyperbolic Schrodinger equation with small data in the critical Besov space. This is a joint work with B. Wang and Y. Wang.

Speaker: **Florian Beyer** (**Online talk**, fbeyer@maths.otago.ac.nz)

Title: **Fluids in the vicinity of Kasner big bang singularities**

Abstract: In this talk I present recent results (in collaboration with Todd Oliynyk from Monash) about the asymptotic dynamics of perfect fluids with linear equations of state near Kasner and Kasner-scalar field big bang singularities. These are nonlinear stability results under the condition that the speed of sound of the fluid satisfies a lower bound determined by the Kasner exponents. I also provide evidence that this lower bound is sharp.