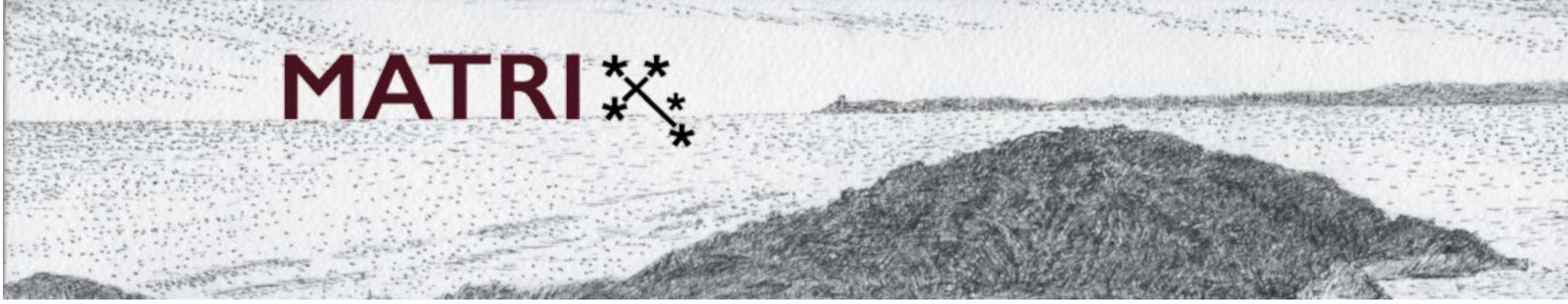
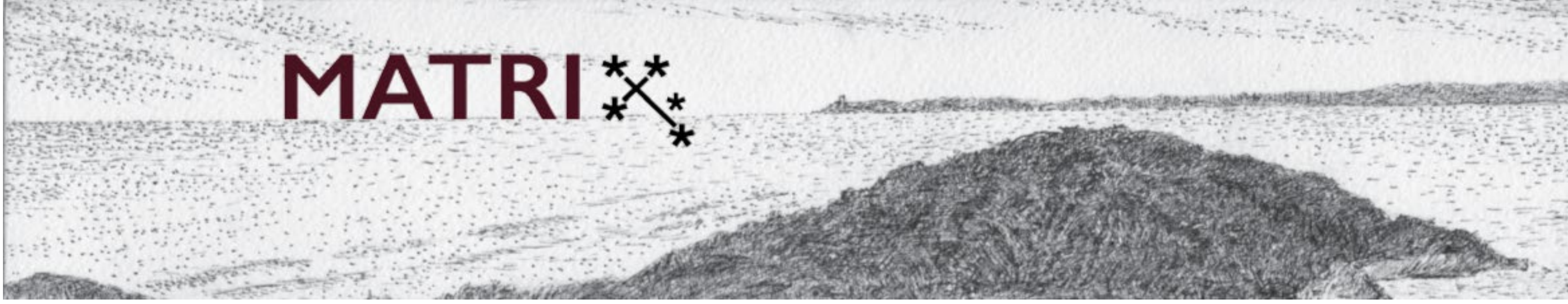


MATRIX Research Program:

THERMODYNAMIC FORMALISM FOR RANDOM DYNAMICAL SYSTEMS

Minicourses & Talks - Titles & Abstracts

9-20 January 2023
MATRIX, Creswick



Minicourse Schedule

Monday, 9 January 2023

- 14:00 – 15:30 Minicourse 1.1
  Speaker: Mariusz Urbanski
  Title: Thermodynamic Formalism for Symbolic Open Systems via Singular Perturbations.

Tuesday, 10 January 2023

- 9:00 – 10:30 Minicourse 1.2
  Speaker: Mariusz Urbanski
  Title: Thermodynamic Formalism for Symbolic Open Systems via Singular Perturbations.

- 14:00 – 15:30 Minicourse 2.1
  Speaker: Carlangelo Liverani
  Title: Some techniques useful to investigate statistical properties

Wednesday, 11 January 2023

- 9:00 – 10:30 Minicourse 1.3
  Speaker: Mariusz Urbanski
  Title: Thermodynamic Formalism for Symbolic Open Systems via Singular Perturbations.

- 14:00 – 15:30 Minicourse 3.1
  Speaker: Hongkun Zhang
  Title: Deep learning of chaotic systems.

Thursday, 12 January 2023

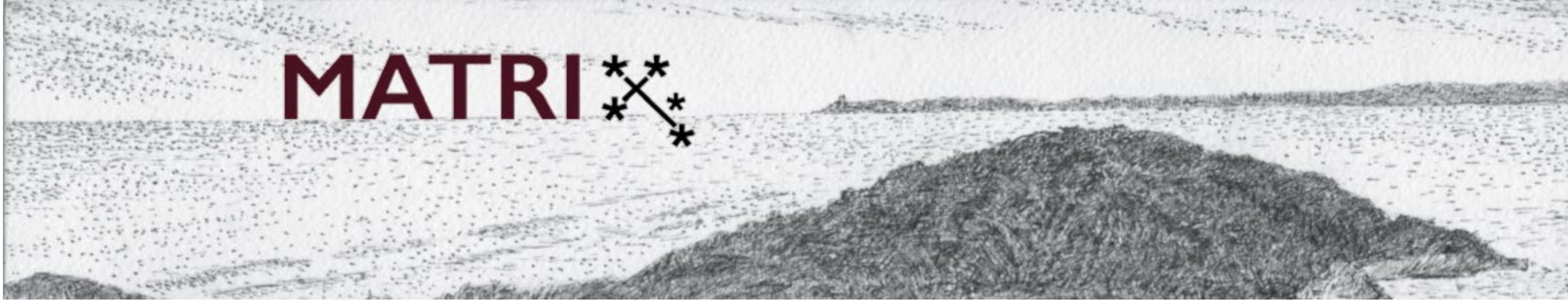
- 9:00 – 10:30 Minicourse 2.2
  Speaker: Carlangelo Liverani
  Title: Some techniques useful to investigate statistical properties

- 14:00 – 15:30 Minicourse 3.2
  Speaker: Hongkun Zhang
  Title: Deep learning of chaotic systems.

Friday, 13 January 2023

- 9:00 – 10:30 Minicourse 2.3
  Speaker: Carlangelo Liverani
  Title: Some techniques useful to investigate statistical properties.

- 14:00 – 15:30 Minicourse 3.3
  Speaker: Hongkun Zhang
  Title: Deep learning of chaotic systems



Minicourses

Minicourse 1

Speaker: Mariusz Urbanski

Title: Thermodynamic Formalism for Symbolic Open Systems via Singular Perturbations.

Abstract: This minicourse will be primarily based on my book "Open Dynamical Systems: Statistics, Geometry, and Thermodynamic Formalism" joint with Tushar Das, Giulio Tiozzo, and Anna Zdunik. The starting dynamical system will be given by any countable alphabet finitely primitive subshift of finite type and the invariant Gibbs state of a 1-cylinder summable Holder continuous potential φ. The holes defining open systems will be then formed by collections of open sets with appropriately defined thin boundaries. They will naturally lead to singularly perturbed transfer operators associated to the potential φ.

The course will develop conditionally invariant measures and surviving sets. The surviving variational principle, the existence and uniqueness of surviving equilibrium states, the asymptotic of perturbed leading eigenvalues, and escape rates when holes approach a given hole will be discussed. The logarithms of leading perturbed eigenvalues will turn out to be equal to topological pressures defined be means of surviving variational principle. Also stochastic properties of surviving equilibria such as exponential decay of correlation and the Central Limit Theorem will be discussed. All these objects will be produced and studied in our dynamical setting by means and applications of the Keller-Liverani Perturbation Theorem.

Applications to “real” systems will be also mentioned. In such a context, in Euclidean spaces, the holes giving rise to open systems can be take to be virtually any Euclidean balls; in particular not being dynamically defined like for example unions of cylinders of the same length. Examples will be given.

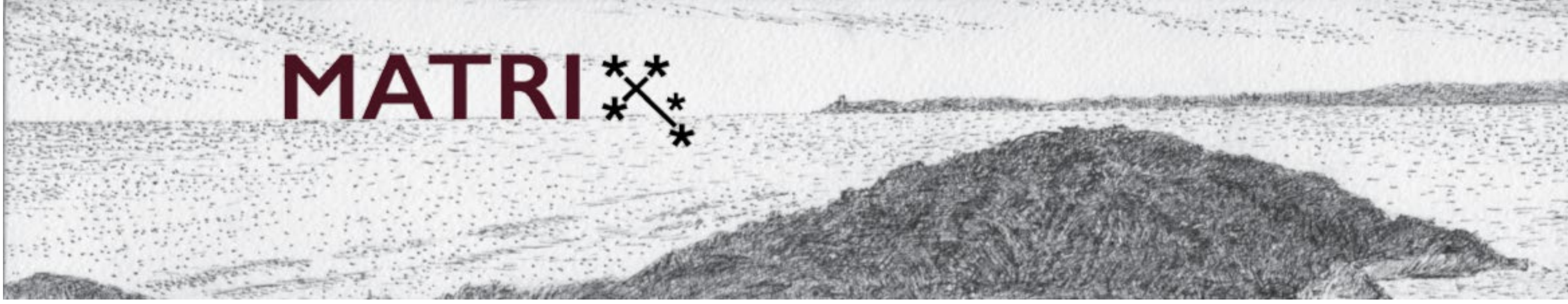
Minicourse 2

Speaker: Carlangelo Liverani

Titles: Some techniques useful to investigate statistical properties

Abstract: I will try to present three ideas (transfer operators, standard pairs, Hilbert metrics) relevant to the study of the asymptotic properties of Dynamical Systems. The emphasis will be on their application and how they can complement each other, rather than on technical details.

To this end, I will discuss three related problems relevant to the study of Random Dynamical Systems.



More precisely, the lectures will focus on the following:

lecture 1: limit theorems (CLT and its corrections for ergodic averages, Transfer Operators)

lecture 2: fast-slow systems (asymptotic properties, Standard Pairs)

lecture 3: deterministic walks in random environment (loss of memory, Hilber Metrics)

References:

1) Demers, Mark F.; Kiamari, Niloofar; Liverani, Carlangelo Transfer operators in hyperbolic dynamics—an introduction. 33 o Colóquio Brasileiro de Matemática. Instituto Nacional de Matemática Pura e Aplicada (IMPA), Rio de Janeiro, 2021. 238 pp

Minicourse 3

Speaker: Hongkun Zhang

Title: Some techniques useful to investigate statistical properties

Abstract: Chaotic systems are difficult to learn using neural networks. Mainly because of the existence of the positive Lyapunov exponents, and nontrivial entropy. We design a new data-driven network, which is efficient in deep learning of chaotic systems, including the Lorenz system, Standard map, etc.

1. Introduction to neural networks for dynamical systems

I will explain the concept of machine learning of dynamical systems using neural network, especially the Recurrent Neural Network (RNN), and its application to timeseries prediction, as well as deep learning of Lorenz system.

2. Koopman theory for dynamical systems

Recently, Koopman theory has become rather powerful in developing machine learning to complex dynamical systems. I will go through some theoretical properties of Koopman operator, especially the concept of Koopman Mode and its importance application to learn dynamical systems.

3. Dynamical mode decomposition (DMD) and deep Koopman neural network

Koopman operator plays an important role in deep learning of nonlinear system, especially it spectrum information. We will go through some machine learning methods - DMD and EDMD, to learn and predict the dynamical systems.

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Talk Titles and Abstracts

Speaker: Wael Bahsoun

Titles: Statistical properties of mean-filed coupled Anosov maps.

Abstract: We will talk about infinite systems of globally coupled Anosov diffeomorphisms with weak coupling strength. Using transfer operators acting on anisotropic Banach spaces, we prove that the coupled system admits a unique physical equilibrium. Moreover, we prove exponential convergence to equilibrium for a suitable class of distributions. This is joint work with C. Liverani and F. M. Sélley.

Speaker: Ayreena Bakhtawar

Titles: On the increasing rate of weighted products of multiple partial quotients in continued fractions

Abstract: In one-dimensional Diophantine approximation, by using the continued fractions, Khintchine's theorem and Jarnik's theorem are concerned with the size of the set of real numbers for which the partial quotients in their continued fraction expansions grow at a certain rate. Whereas it was observed that the improvability of Dirichlet's theorem is concerned with the growth of the product of consecutive partial quotients in the continued fraction expansion of a real number. In this talk, I will describe some metrical properties of the product of an arbitrary block of consecutive partial quotients raised to different powers in continued fractions, including the Lebesgue measure-theoretic result and the Hausdorff dimensional result.

Speaker: Julia Slipantschuk

Titles: Resonances for rational Anosov maps on the torus

Abstract: In this talk I will present a complete description of resonances for rational toral Anosov diffeomorphisms preserving certain Reinhardt domains. This allows us to show that every homotopy class of two-dimensional Anosov diffeomorphisms contains maps with the sequence of resonances decaying stretched-exponentially. Additionally, non-linear Anosov maps with trivial or exponentially decaying resonances are obtained from the same construction.

Speaker: Yushi Nakano

Titles: Finitude of physical measures for random maps and Markov operators

Abstract: V. Araujo established in 2000 finitude of time averages for any diffeomorphisms on a compact manifold under a certain type of noises. In the talk we give a generalization of Araujo's result in terms of constrictive Markov operators. In fact, for random compositions of independent and identically distributed diffeomorphisms, we provide a necessary and sufficient condition for the existence and finitude of absolutely continuous ergodic stationary probability measures whose basins of attraction cover the manifold Lebesgue almost everywhere. Furthermore, we hierarchize such random maps and show the difference between classes in the hierarchy by providing examples, including additive noise, multiplicative noise, and iterated function systems. This is based on a joint work with Pablo G. Barrientos, Fumihiko Nakamura and Hisayoshi Toyokawa.

Speaker: Renaud Leplaideur

Titles: Entropy for the Jacaranda tree

Abstract: I will introduce the notion of substreetution, which is a substitution on the set of colored binary trees. The Jacaranda tree is one fixed point for such a substreetution. Its orbits under the free semi-group action generates a minimal set that is not periodic.

This is the equivalent for trees of the classical substitutions as the Thue-Morse and the Fibonacci substitutions;

Following a paper of Bufetov, the computation of the entropy requires to exchange the free-semi-group action with an action of the set of natural numbers, by "randomnizing" the map. This new system is still not well understood, in particular we do not know if expansiveness holds. We remind that expansiveness is a sufficient condition to get upper-semi continuity for the entropy, which yields existence of equilibrium states.

Speaker: Sandro Vaienti

Titles: Limiting Entry and return times distribution for arbitrary null sets

Abstract: TBA

Speaker: Caroline Wormell

Titles: Conditional mixing and applications

Abstract: Under the evolution of a chaotic system, distributions which are sufficiently regular in a certain sense often converge rapidly to the system's SRB measure, a property which closely relates to the statistical behaviour of the system. In this talk we discuss the behaviour of other less regular measures, in particular slices of these physical measures along smooth submanifolds that are reasonably generic (e.g. not stable or unstable manifolds).

We give evidence that such conditional measures also have exponential convergence back to the full SRB measures, even though they lack the regularity usually required for this to occur (for example, they may be Cantor measures). Using Fourier dimension results, we will prove that so-called conditional mixing holds in a class of generalised baker's maps, and we will give rigorous numerical evidence in its favour for some non-Markovian piecewise hyperbolic maps. Conditional mixing naturally encodes the idea of long-term forecasting of systems using perfect partial observations, and appears key to a rigorous understanding of the emergence of linear response in high-dimensional systems. In light of random fractal theory, conditional mixing could potentially be easier to prove for quenched random maps.

Speaker: Matteo Tanzi

Titles: Random-like properties of chaotic forcing

Abstract: We prove that skew systems with a sufficiently expanding base have "approximate" statistical properties similar to random ergodic Markov chains. For example, they exhibit approximate exponential decay of correlations, meaning that the exponential rate is observed modulo a controlled error. The fiber maps are only assumed to be Lipschitz regular and to depend on the base in a way that guarantees diffusive behaviour on the vertical component. The assumptions do not imply a hyperbolic picture and one cannot rely on the spectral properties of the transfer operators involved. The approximate nature of the result is the inevitable price one pays for having so mild assumptions on the dynamics on the vertical component. The error in the approximation is shown to go to zero when the expansion of the base tends to infinity.

Speaker: Jason Atnip

Titles: Thermodynamic Formalism and Perturbation Formulae for Quenched Random Open Dynamical Systems

Abstract: In this talk we discuss recent work on thermodynamic formalism for random interval maps with holes. We establish a complete quenched thermodynamic formalism for our random open systems and prove an exponential decay of correlations for the associated equilibrium states. We then provide formulas for the escape rates and Hausdorff dimension of the surviving set via the expected pressure function as well as examples and applications to number theory. We then apply a perturbative approach based on a random version of Keller-Liverani perturbation theory to prove an extreme value law as well as stochastic limit laws for both the closed and open systems.

Speaker: Meagan Carney

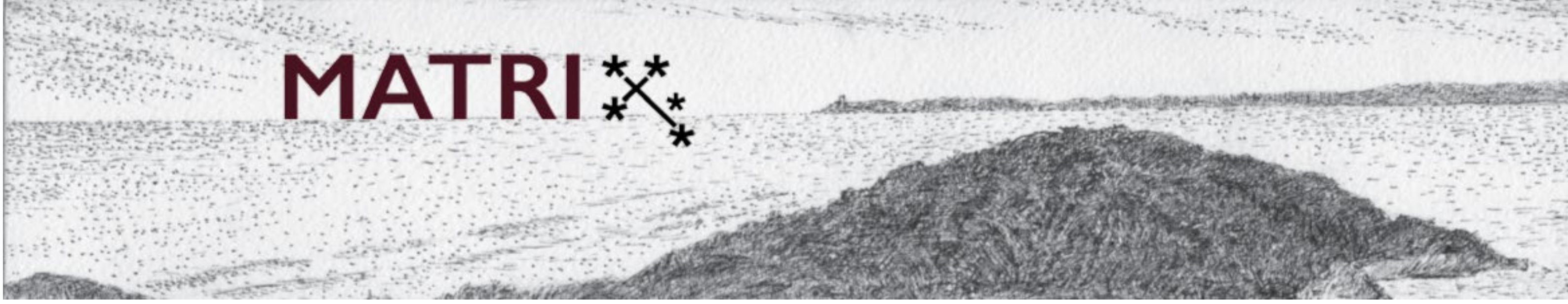
Titles: An exploration of extremes of classical and physical observables on hyperbolic systems

Abstract: We discuss extreme value theory in the classical setting and the dynamical framework for classical observables and then energy-like observables. We will see examples for how to prove an extreme value law exists with these observables on some common hyperbolic systems.

Speaker: Charlene Kalle

Titles: Stationary measures for random intermittent systems

Abstract: Intermittent dynamics, where systems irregularly alternate between long periods of different types of dynamical behaviour, has been studied since the work of Pomeau and Manneville in 1980. In this talk we will describe the intermittency of some families of random systems with a particular emphasis on how the intermittency of the random system depends on the intermittency of the underlying deterministic systems. This talk is based on joint works with Ale Jan Homburg, Tom Kempton, Valentin Matache, Marks Ruziboev, Masato Tsujii, Evgeny Verbitskiy and Benthen Zeegers.



Speaker: Carlangelo Liverani

Titles: CLT for sequential systems

Abstract: I'd like to illustrate a simple abstract idea to compute the characteristic function for the CLT of self-normed sequential systems. This is work in (early) progress with Mark Demers.

Speaker: Joshua Peters

Titles: Prevalence of Lyapunov Spectrum Stability for Finite Blaschke Product

Abstract: It is uncommon for dynamical systems models to exactly reflect reality. This limitation begs the question of `how and when do small errors in dynamical models drastically influence relevant outputs?'

In recent years, significant progress has been made in describing the robustness of autonomous and non-autonomous systems under perturbations, in both the finite and infinite dimensional settings, but many problems remain entirely open. In this work, we address this question using the theory of prevalence, and investigate stability properties of Lyapunov exponents from a measure-theoretic perspective. Our results focus on so-called second order random Blaschke product cocycles, a class of random dynamical systems amenable to theoretical and numerical analysis.