MATRIX-RIMS Tandem Workshop

Geometric Analysis in Harmonic Analysis and PDE

Talk Title and Abstracts

(27-31 March 2023)

MATRIX, Creswick, Australia

Titles and Abstracts

Yihong Du

Title: Propagation dynamics of the Fisher–KPP nonlocal diffusion equation with free boundary

Abstract: Propagation has been modelled by reaction-diffusion equations since the pioneering works of Fisher and Kolmogorov–Peterovski–Piskunov (KPP). Many new developments have been achieved in the past several decades on the modelling of propagation, with traveling wave and related solutions playing a central role. In this talk, I will report some recent results obtained with several collaborators on the Fisher–KPP equation with free boundary and "nonlocal diffusion", where the diffusion operator is given by a convolution integral instead of the traditional Laplace operator. A key feature of this nonlocal equation is that the propagation may or may not be determined by traveling wave solutions. There is a threshold condition on the kernel function in the diffusion operator which determines whether the propagation rate is linear or superlinear in time, also known as accelerated spreading in the latter case, where the rate of spreading is not determined by traveling waves. For some typical kernel functions, sharp spreading rates will be presented.

Xuan Thinh Duong

Title: Hardy spaces associated with operators and weak type estimates

Abstract: Let (X, d, μ) be a metric space with a metric d and a doubling measure μ. Assume that L is a non-negative self-adjoint operator on $L^2(X)$ whose heat kernel satisfies the Gaussian upper bound. In the last two decades, the Hardy space associated to the operator L, denote by $H^p_L(X)$ with $0 < p \leq 1$, has had an important role in the study of boundedness of singular integrals with rough kernels which do not belong to the class of standard Calderón-Zygmund operators. In this talk, we show that if a sublinear operator T is bounded from $L^2(X)$ to $L^2(X)$, and also bounded from $H^p_L(X)$ to $L^p(X)$ for some $0 < p < 1$, then T is of weak type $(1, 1)$. We then give some applications to estimates of singular integrals with rough kernels. This is joint work with The Anh Bui.

Zihua Guo

Title: Low-regularity well-posedness for the modified KdV equation

Abstract: We will talk about the low-regularity well-posedness for the modified KdV equation. We first review the previous results in Sobolev spaces, Fourier-Lebesgue space, and modulation spaces. Then we talk about our recent results on the generalized Fourier-Lebesgue space.

Daniel Hauer

Title: Regularity and Separation for p-Laplace equations on Sub-Riemannian manifolds.

Abstract: We introduce a Sub-Riemannian framework and define Sobolev spaces in this framework. Under an additional assumption on the given metric, we show that those spaces are reflexive and separable. With this, we can realize the p-Laplace operator on such a manifold as the negative sub-differential operator of the p-Dirichlet energy. In the case, the given metric induces the Grushin space, we present a separation phenomenon showing that weak solutions of the elliptic and parabolic equation governed by the p-Laplace operator always admit an L^q-L^r-regularization effect for $1 \leq q < r \leq \infty$, but there is a threshold, when they admit a simple jump discontinuity with respect to the special-variable provided the parameter are above the threshold, and positive weak solutions satisfy a Harnack inequality provided one remains strictly below the threshold.

This is a joint work with Adam Sikora (Macquarie University)

Eunhee Jeong

Title: Almost Everywhere Convergence of Bochner–Riesz Means for the Special Hermite Expansion

Abstract: In this talk, we are concerned with the Bochner–Riesz means $S_t^\delta(\mathcal{L})$ for the special Hermite expansion on C^d, $d \geq 1$, which is also known as the Bochner–Riesz mean associated to the twisted Laplacian $\mathcal{L}$. As in the celebrated work of Carbery, Rubio de Francia, and Vega for the classical Bochner–Riesz mean, we establish some weighted L^2 estimates for the maximal function $\sup_{t>0} |S_t^\delta(\mathcal{L})f|$. As a result, we determine the sharp range of the summability indices δ for which almost everywhere convergence of $S_t^\delta(\mathcal{L})$ holds for every $f \in L^p(\mathrm{C}^d)$, when $p \geq 2$. This talk is based on joint work with Sanghyuk Lee (Seoul National University) and Jaehyeon Ryu (KIAS).

Chongwei Liang

Title: Muckenhoupt-type weights and the intrinsic structure in the Bessel setting

Abstract: We study the Muckenhoupt-type weights which reveal the intrinsic structure in the Bessel setting along the lines of Muckenhoupt–Stein and Andersen–Kerman. We introduce a new class of weights such that the Hardy–Littlewood maximal function is bounded on the weighted Lebesgue L^p space if and only if the weight is in this new class. Moreover, along the lines of Coifman–Rochberg–Weiss, we investigate the commutator with the Bessel Riesz transform, and show that for the new Andersen—Kerman type weight, the commutator is bounded on weighted L^p if and only if the symbol is in the BMO space associated with Bessel operators.

Melissa Tacy

Title: Filament structure in random plane waves
Abstract: Numerical studies of random plane waves, functions

$$u = \sum_j c_j e^{\frac{i}{h}\langle x,\xi_j\rangle}$$

where the coefficients c_j are chosen "at random", have detected an apparent filament structure. The waves appear enhanced along straight lines. There has been significant difference of opinion as to whether this structure is indeed a failure to equidistribute, numerical artefact or an illusion created by the human desire to see patterns. In this talk I will present some recent results that go some way to answering the question. First we consider the behaviour of a random variable given by $F(x,\xi) = ||u||_{L^2(\gamma_{(x,\xi)})}$ where $\gamma_{(x,\xi)}$ is a unit ray from the point x in direction ξ. We will see that this random variable is uniformly equidistributed. That is, the probability that for any (x,ξ), $F(x,\xi)$ differs from its equidistributed value is small (in fact exponentially small). This result rules out a strong scarring of random waves. However, when we look at the full phase space picture and study a random variable $G(x,\xi) = \|P_{(x,\xi)}u\|_{L^2}$ where $P_{(x,\xi)}$ is a semiclassical localiser at Planck scale around (x,ξ) we do see a failure to equidistribute. This suggests that the observed filament structure is a configuration space reflection of the phase space concentrations.

Brett Wick

Title: Wavelet Representation of Singular Integral Operators

Abstract: In this talk, we'll discuss a novel approach to the representation of singular integral operators of Calderón-Zygmund type in terms of continuous model operators. The representation is realized as a finite sum of averages of wavelet projections of either cancellative or noncancellative type, which are themselves Calderón–Zygmund operators. Both properties are out of reach for the established dyadic-probabilistic technique. Unlike their dyadic counterparts, our representation reflects the additional kernel smoothness of the operator being analyzed. Our representation formulas lead naturally to a new family of $T1$ theorems on weighted Sobolev spaces whose smoothness index is naturally related to kernel smoothness. In the one parameter case, we obtain the Sobolev space analogue of the A_2 theorem; that is, sharp dependence of the Sobolev norm of T on the weight characteristic is obtained in the full range of exponents. As an additional application, it is possible to provide a proof of the commutator theorems of Calderón–Zygmund operators with BMO functions.

Po Lam Yung

Title: Techniques for proving decoupling theorems
Abstract: Many techniques have been developed to prove decoupling theorems, drawing from areas such as PDE, geometry, and number theory. We will try to explain some of the ideas involved. Partly based on joint work with Shaoming Guo, Zane Kun Li and Pavel Zorin-Kranich. Time permitting we will also make a connection to the Hardy spaces for Fourier Integral Operators (H^p_{FIO} for $p \geq 2$), introduced by Andrew Hassell, Pierre Portal and Jan Rozendaal.