

**MATRIX Research Program:
Spectrum and Symmetry for
Group Actions in Differential
Geometry II**



The hyperbolic disc in wycinanka lowicka style by Katarzyna Nurowska

Talk Titles and Abstracts
24 July – 4 August 2023

MATRIX, Creswick



MATRIX MEETING 24 JULY – 4 AUGUST 2023

SPECTRUM AND SYMMETRY FOR GROUP ACTIONS IN DIFFERENTIAL GEOMETRY

TITLES: WEEK 1

Monday 9:30am VICENTE CORTÉS

Generalized Einstein structures on Lie groups

Monday 11am HOLGER DULLIN

Quantum monodromy in Hamiltonian systems

Monday 2pm ROBERT BRYANT

Bonnet surfaces: Part 1, the Euclidean case

Tuesday 9am RENATO BETTIOL

A new Lie-theoretic approach to the Bochner technique: Part 1

Tuesday 11am CRAIG SUTTON

Can one hear the shape of various 3-dimensional geometries?

Tuesday 2pm ROBERT BRYANT

Bonnet surfaces: Part 2, the affine case

Wednesday 9am RENATO BETTIOL

A new Lie-theoretic approach to the Bochner technique: Part 2

Wednesday 11am FRAN BURSTALL

Informal discussion on eigenfunctions of the Laplacian

Wednesday 2pm UDO HERTRICH-JEROMIN

Symmetry breaking?

Thursday 9am RAMIRO LAFUENTE

New developments on Einstein manifolds with symmetry

Thursday 11am ARTEM PULEMOTOV

The Poisson equation for closed G_2 -structures

Friday 9am ROMINA ARROYO

SKT structures on solvmanifolds

Friday 11am YURI NIKOLAYEVSKI

Geodesic orbit pseudo-Riemannian nilmanifolds

Friday 2pm CRAIG SUTTON

Generic properties of Laplace eigenfunctions in the presence of symmetry



SPECTRUM AND SYMMETRY FOR GROUP ACTIONS
IN DIFFERENTIAL GEOMETRY

TITLES: WEEK 2

Monday 9am FRANZ PEDIT

Non-Abelian Hodge theory: Part 1

Monday 11am LYNN HELLER

Complete families of embedded high genus CMC surfaces in the 3-sphere

Tuesday 9am FRANZ PEDIT

Non-Abelian Hodge theory: Part 2

Tuesday 2pm TIM BUTTSWORTH

Inverting geometric operators: Part 1

Wednesday 11am FRAN BURSTALL

Isothermic submanifolds in symmetric R-spaces: Part 1

Wednesday 2pm TIM BUTTSWORTH

Inverting geometric operators: Part 2

Thursday 9am ANNA FINO

Geometry with torsion and symmetry

Thursday 11am FRAN BURSTALL

Isothermic submanifolds in symmetric R-spaces: Part 2

Friday 9am THOMAS LEISTNER

Conformal transformations of Cahen-Wallach spaces

SPECTRUM AND SYMMETRY FOR GROUP ACTIONS IN DIFFERENTIAL GEOMETRY

ABSTRACTS (if available): in order of appearance

VICENTE CORTÉS

Generalized Einstein structures on Lie groups

The structures considered are solutions to a generalization of Einstein's vacuum equation in the setting of Hitchin's generalized geometry. Restricting to left-invariant solutions on Lie groups one arrives at a purely algebraic problem.

HOLGER DULLIN

Quantum monodromy in Hamiltonian systems

What could possibly be said about the Kepler Problem that is new? It is well known that this superintegrable system can be separated in different coordinate systems, and each such separation defines a distinct Liouville integrable system. But it appears to be a new observation that for separation in prolate spheroidal coordinates the resulting integrable system has Hamiltonian monodromy. After symplectic reduction by the global S^1 action generated by the Hamiltonian a semi-toric system on $S^2 \times S^2$ is found. Analogous results are obtained for the corresponding quantum integrable system, for which the eigenfunctions are spheroidal harmonics. We show that the joint spectrum of this integrable quantum system has quantum monodromy.

ROBERT BRYANT

Minicourse on 'Bonnet surfaces'

The first talk on the Euclidean case is more classical and, in particular, will give the history. The second talk will be about the affine Bonnet problem. Since both talks are about surfaces in 3-space, they will be fairly visual and won't involve a lot of abstract machinery.

RENATO BETTIOL

Minicourse on ‘A new Lie-theoretic approach to the Bochner technique’

We give a general framework to use representation theory in the analysis of positivity for curvature terms in Bochner formulae, extending recent works of Petersen–Wink. As an application, we determine linear inequalities on the eigenvalues of curvature operators that imply vanishing of the twisted A-genus on a closed Riemannian spin manifold, where the twisting bundle is any prescribed parallel bundle of tensors. These inequalities yield surgery-stable curvature conditions tailored to annihilate further rational cobordism invariants, such as the Witten genus, elliptic genus, signature, and even the rational cobordism class itself.

CRAIG SUTTON

Can one hear the shape of various 3-dimensional geometries?

FRAN BURSTALL

Informal discussion on eigenfunctions of the Laplacian

Under what circumstances can one have a function f on a Riemannian manifold so that both f and f^2 are eigenfunctions of the Laplacian? This is a question for the audience but some motivation will be given.

UDO HERTRICH-JEROMIN

Symmetry breaking?

RAMIRO LAFUENTE

New developments on Einstein manifolds with symmetry

I will discuss the recent resolution of the 1975 Alekseevskii conjecture on homogeneous Einstein manifolds, in joint work with Christoph Böhm. I will also describe new results on the structure and rigidity of Einstein manifolds with a cocompact symmetry group, including the fact that a negative Einstein manifold with a cocompact unimodular group of isometries must split isometrically as a product of a compact Einstein manifold and a symmetric space of non-compact type.

ARTEM PULEMOTOV

The Poisson equation for closed G_2 -structures

We establish local solvability of the Poisson equation with a positive or negative right-hand side for closed G_2 -structures.

ROMINA ARROYO

SKT structures on solvmanifolds

YURI NIKOLAYEVSKI

Geodesic orbit pseudo-Riemannian nilmanifolds

We know that in the Riemannian case, (i) for every homogeneous space, there is a reductive decomposition at the level of Lie algebras, (ii) the isometry group of a simply connected nilmanifold is the semidirect product of isometric automorphisms and translations (Wolf/Wilson), and (iii) geodesic orbit nilmanifolds are necessarily two-step nilpotent or abelian (Gordon). Neither of this is true in pseudo-Riemannian signature. However, it turns out that in low signature, some results may still be “rescued”.

CRAIG SUTTON

Generic properties of Laplace eigenfunctions in the presence of symmetry

FRANZ PEDIT

Minicourse on ‘Non-Abelian Hodge theory’

The first lecture of the minicourse will give an introduction to non-abelian Hodge (or higher Teichmueller) theory relating self-duality equations, surface group representations and Higgs bundles. The second lecture will explain how to apply the non-abelian Hodge correspondence to construct countably many examples of special Lagrangian 3-torus fibrations of a Calabi-Yau 3-fold with a Y-vertex singularity motivated by the SYZ mirror symmetry picture. The basic ingredients will be hyperbolic affine spheres, cyclic Higgs bundles, and the character variety of the thrice punctured sphere.

LYNN HELLER

Complete families of embedded high genus CMC surfaces in the 3-sphere

TIM BUTTSWORTH

Minicourse on ‘Inverting geometric operators’

For every parabolic geometric evolution equation, there is typically an associated steady-state elliptic inverse problem governing the dynamics. For example, associated to the scalar heat equation is the problem of finding the inverse of the Laplacian operator. In this talk, I will describe the inverse problems associated to the Ricci flow, the cross curvature flow, and the G2 Laplacian flow, and will also elaborate on classical results, as well as some more recent results in the presence of symmetry.

FRAN BURSTALL

Minicourse on ‘Isothermic submanifolds in symmetric R-spaces’

Isothermic surfaces, which include surfaces of revolution, constant mean curvature surfaces and quadrics, constitute an integrable class in conformal surface geometry that was intensively studied by Christoffel, Darboux, Bianchi and others around the turn of the 20th century. In particular, they admit a beautiful and intricate transformation theory. In these lectures, I shall describe both the classical theory and how it carries over, with no loss of integrable structure, when the codomain is replaced by a symmetric R-space (thus G/P with G real semisimple and P parabolic with abelian nilradical).

ANNA FINO

Geometry with torsion and symmetry

The idea would be to talk about the geometry of metric connections with torsion and focus on homogeneous.

THOMAS LEISTNER

Conformal transformations of Cahen-Wallach spaces

An unsolved rigidity question in Lorentzian geometry is whether a compact Lorentzian manifold with an essential conformal transformation (i.e. one that is not contained in the isometry group of a metric in the conformal class) must be conformally flat. We look at this problem in the special case of indecomposable Lorentzian symmetric spaces of non-constant sectional curvature, the so-called Cahen-Wallach spaces, and their compact quotients. When a Cahen-Wallach space is conformally curved, its conformal transformations are homotheties. Using this we show that a conformal transformation of a conformally curved Cahen-Wallach space is essential if and only if it has a fixed point. Then we explore the possibility of properly discontinuous groups of conformal transformations acting with a compact orbit space. Here our result is that any such group cannot centralise an essential homothety and that for Cahen-Wallach spaces of imaginary type must be contained within the isometries. This is joint work with Stuart Teisseire (University of Auckland).