

MATRIX-RIMS Tandem Workshop: Evolutionary Partial Differential Equations and Applications

Titles and Abstracts

(Plenary talks at MATRIX and RIMS are indicated respectively by ♦ and ♠)

Michael Winkler (Paderborn University) ♦

Title: The role of cross-degeneracies in reaction-diffusion driven structure evolution

Abstract: Simple models for nutrient-oriented bacterial migration are compared. In particular, the potential of certain cross-degenerate diffusion mechanisms to adequately describe experimentally observed phenomena related to emergence and stabilization of structures are discussed. Resulting mathematical challenges are described and possible approaches outlined, both at levels of basic existence theories and the stage of qualitative analysis.

Yoshie Sugiyama (Osaka University) ♠

Title: Infinite speed of propagation of solutions to Keller-Segel systems

Abstract: In this talk, we consider the Keller-Segel system, which encompasses both the parabolic-parabolic and parabolic-elliptic types with a semi-linear form and demonstrate the uniqueness continuous theorem for its solutions. As the main theorem, based on the uniqueness continuous theorem, we establish the infinite propagation property, i.e., the immediate spreading of the support of solutions with nontrivial initial data to the entire space.

Danielle Hilhorst (Université Paris-Saclay) ♦

Title: Convergence of solutions of a one-phase Stefan problem with Neumann boundary data to a self-similar profile

Abstract: We study a one-dimensional one-phase Stefan problem with a Neumann boundary condition on the fixed part of the boundary. We construct a unique self-similar solution and show that for a large class of initial data, the solution of the time evolution problem converges to this self-similar solution as time tends to infinity. Similar results were already obtained by Bouguezzi, Hilhorst, Miyamoto, and Scheid in the case of Dirichlet data on the fixed boundary. However, they had to show that the space derivative of the solution uniformly converges to its limit. Here, our proof requires much less regularity, which should make our arguments easier to adapt to different settings.

Ki-Ahm Lee (Seoul National University) ◆

Title: Capacity Theory for nonlinear equations and its application

Abstract: In this talk we would like to discuss how capacity theory for nonlinear operators shows up in applications: Homogenization for the highly oscillating obstacles and Wiener Criterion for the boundary value problems. At Homogenization theory, we will consider the influence of the capacity of periodically or randomly oscillating obstacles. For various nonlinear operators to find the effective equation. For Wiener Criterion for the boundary value problems, we will measure the influence of the capacity of the complement of the domains in shrinking local regions to Weak Harnack Inequalities.

Hirokazu Ninomiya (Meiji University) ◆

Title: Turing's instability by equal diffusion

Abstract: In 1952, Turing proposed that a stable homogeneous equilibrium in a kinetic system could be destabilised by diffusion. This concept is central to the study of pattern formation, particularly in reaction-diffusion systems where Turing instability leads to the emergence of spatial patterns. Typically, Turing instability arises due to differences in the diffusion rates of the reacting species. Thus, when the diffusion coefficients are equal, the conditions for Turing instability may not be met. In a two-component reaction-diffusion system, Turing's instability requires the different diffusion coefficients. This leads to the question: Does Turing instability never occur when the diffusion coefficients are equal in a two-component reaction-diffusion system? To address this, consider a two-component kinetic system with an asymptotically stable equilibrium, while the corresponding reaction-diffusion system with equal diffusion coefficients exhibits a family of unstable stationary solutions that are arbitrarily close to the homogeneous stationary solution.

Xu-Jia Wang (The Australian National University) ◆

Title: A new topological method for the L_p -Minkowski problem

Abstract: The L_p -Minkowski problem has been extensively studied in the last two decades. According to the Blaschke-Santaló inequality, it can be divided into the subcritical, critical, and super-critical cases. The existence of solutions in the sub-critical case can be obtained by either the continuity method or variational method. In the critical case, a Kazdan-Warner type obstruction was discovered. In this talk, we will tell how a new topological method was found to prove the existence of solutions in the super-critical case.

Hideki Murakawa (Ryukoku University) ♣

Title: Relationship between haptotaxis and chemotaxis in cell sorting phenomena

Abstract: The phenomenon where cells with elongated protrusions, such as neurons, communicate by contacting other cells and arrange themselves appropriately is termed as cell sorting phenomena through haptotaxis. This phenomenon is described by partial differential equations involving non-local advection. In contrast, cell phenomena where cells communicate with other cells via chemical substances and arrange themselves appropriately are termed as cell sorting phenomena through chemotaxis. This phenomenon is described by chemotactic systems such as the Keller-Segel model. Although there are clear differences between haptaxis and chemotaxis phenomena, similar behaviors are also observed. In this study, we will consider the relationship between haptotaxis and chemotaxis in cell sorting phenomena.

Chang-Hong Wu (National Yang Ming Chiao Tung University)

Title: Spreading fronts arising from the singular limit of reaction-diffusion systems

Abstract: In this presentation, we will focus on the singular limit of reaction-diffusion systems to gain insight into the formation of spreading fronts of invasive species. We will derive several free boundary problems and provide interpretations for spreading fronts from a modelling perspective. Additionally, numerical examples will be presented to facilitate discussion on invasion speed. The talk is based on joint works with Hirofumi Izuohara and Harunori Monobe.

Sungrim Seirin-Lee (Kyoto University) ♣

Title: Cell fate and geometry

Abstract: In multicellular systems, cells communicate with adjacent cells to determine their positions and fates, an arrangement important for cellular development. Orientation of cell division, cell-cell interactions (i.e. attraction and repulsion) and geometric constraints are three major factors that define cell arrangement. In particular, geometric constraints are difficult to reveal in experiments, and the contribution of the local contour of the boundary has remained elusive. In this study, we developed a multicellular morphology model based on the phase-field method so that precise geometric constraints can be incorporated. We found that the amount of extra-embryonic space, the empty space within the eggshell that is not occupied by embryonic cells, affects cell arrangement. The prediction was validated experimentally by increasing the extra-embryonic space in the *Caenorhabditis elegans* embryo. Overall, our analyses characterized the roles of geometrical contributors, specifically the amount of extra-embryonic space and the local contour, on cell arrangements. These factors should be considered for multicellular systems.

Mary Vaughan (University of Western Australia)

Title: Multiple interphases for fractional Allen-Cahn equations

Abstract: In this talk, we will begin by reviewing the Peierls-Nabarro model for multiple loop dislocations in crystals. For the corresponding evolutionary problem, a phase parameter is used to rescale from the microscopic to the mesoscopic scale. This gives rise to a fractional Allen-Cahn equation. At the mesoscopic scale, we will show that the dislocation curves correspond to evolving interfaces that move according to their mean curvature. This is joint work with Stefania Patrizi (UT Austin).

Tatsuya Miura (Tokyo Institute of Technology) ♠

Title: Migrating elastic flows

Abstract: Huisken's problem asks whether there is an elastic flow of closed planar curves that is initially contained in the upper half-plane but migrates to the lower half-plane at a positive time. In this talk I will discuss variants of Huisken's problem for open curves under the natural boundary condition, both analytically and numerically. This talk is based on joint work with Tomoya Kemmochi (Nagoya University).

Ben Andrews (The Australian National University) ◆

Title: Sharp gradient estimates from two-point maximum principles

Abstract: I will describe a formulation of sharp gradient estimates for a natural family of elliptic and parabolic equations, using a two-point maximum principle which compares with a one-dimensional solution. This captures many natural examples including equations involving p -Laplacian operators. I will briefly describe an extension to non-local equations (joint with Sophie Chen).

Eiji Yanagida (The University of Tokyo / Meiji University) ♠

Title: On the heat equation with a moving singular potential

Abstract: We consider the heat equation with a time-dependent Hardy-type singular potential. In the subcritical case, it is shown that there exists a positive solution if the motion of the singularity is not so quick (at least, $1/2$ -Hölder continuous). On the other hand, when the singularity moves quickly like a fractional Brownian motion with the Hurst index smaller than $1/2$, it can be shown that there exists a positive solution for a wider range of parameters. We consider also the positivity of solutions in the case of a negative potential.

Jiakun Liu (University of Wollongong)

Title: How does the free boundary touch the fixed boundary?

Abstract: In this talk, we will discuss some recent regularity results of free boundary in optimal transportation. In particular, we show the free boundary touch the fixed boundary in a nice way, namely no cusp exists, which leads to a global regularity of the free boundary up to the fixed boundary.

Jian Fang (Harbin Institute of Technology)

Title: A delay-induced population model with seasonal succession

Abstract: In this talk, we focus on a time-delayed and nonlocal reaction-diffusion equation by modelling the population invasion of alien species with stage structures. For this model, we present some studies on its spatial dynamics in different scenarios.