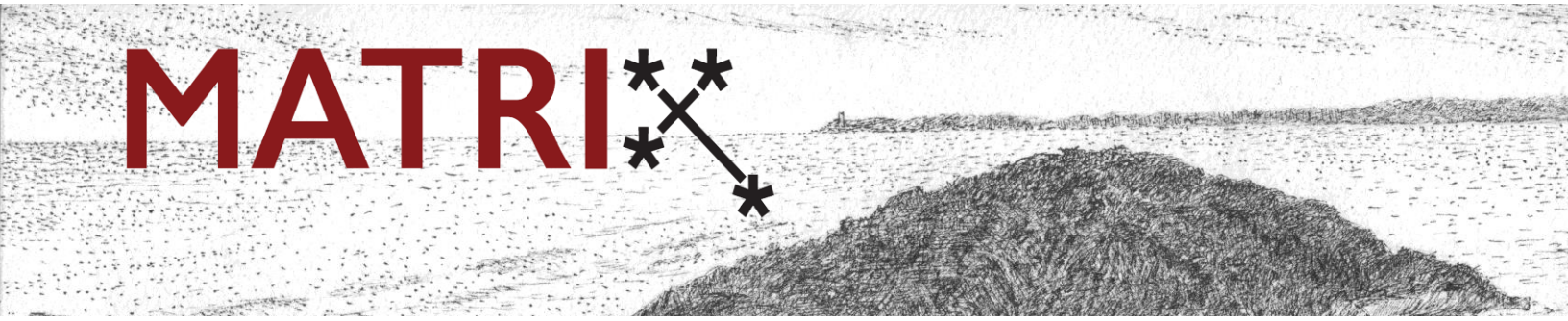

MATRIX RESEARCH PROGRAM

HARMONIC AND MICROLOCAL ANALYSIS IN PARTIAL DIFFERENTIAL EQUATIONS

16 - 20 DECEMBER 2024

CRESWICK, AUSTRALIA

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Monday 9:00-10:00 AM:

Global dynamics and weighted energy estimates for the nonlinear Klein-Gordon equation

Kenji Nakanishi

Abstract: This talk is based on joint work with Gong Chen. We consider the focusing cubic Klein-Gordon equation in 3D. To study global behavior of solutions staying around or getting away from multi-solitons, crucial analytic tools are weighted energy estimates for the linearized equation around multi-solitons. I will explain rough ideas how they are proven and used for the nonlinear problem.

Monday 2:00-3:00 PM:

The Calogero-Sutherland derivative NLS equation

Rana Badreddine

Abstract: We consider a type of nonlocal nonlinear derivative Schrödinger equation on the torus, called the Calogero-Sutherland DNLS equation.

We derive an explicit formula to the solution of this nonlinear PDE. Moreover, using the integrability tools, we establish the global well-posedness of this equation in all the Hardy-Sobolev spaces $H^s_+(\mathbb{T}), s \geq 0$ down to the critical regularity space, and under a mass assumption on the initial data for the focusing equation, and for arbitrary initial data for the defocusing equation.

Finally, a sketch of the proof for extending the flow to the critical regularity $L^2_+(\mathbb{T})$ will be presented.

Tuesday 9:00-10:00 AM:

Transport of Gaussian measures under the flow of nonlinear Hamiltonian PDEs

Justin Forlano

Abstract: In the past decades, there has been dramatic progress in the understanding of invariant Gibbs measures under the flow of nonlinear Hamiltonian PDEs. Outside of the completely integrable case, invariant measures typically exist only in very specific and rough regularity regimes. In contrast, the study of the transport of Gaussian measures under the flow of nonlinear Hamiltonian PDEs has been initiated in part to probe a continuum of regularities and to understand how randomness is propagated outside of an invariant state. In this talk, I will discuss aspects of this development including a joint work with L. Tolomeo (University of Edinburgh) which provides the first instance of the transport property of Gaussian measures being used to globalise local-in-time solutions in a regime which is beyond the deterministic well-posedness theory.

Tuesday 2:00-3:00 PM:

Unique continuation principles in dispersive equations

Xueying Yu

This talk focuses on a fundamental concept in the field of partial differential equations — unique continuation principles. This principle describes the propagation of the zeros of solutions to PDEs. Specifically, it answers the question: what condition is required to guarantee that if a solution to a PDE vanishes on a certain subset of the spatial domain, then it must also vanish on a larger subset of the domain. Motivated by Hardy's uncertainty principle, Escauriaza, Kenig, Ponce, and Vega were able to show in a series of papers that if a linear Schrödinger solution decays sufficiently fast at two different times, the solution must be trivial. In this talk, we will discuss unique continuation properties of solutions to higher-order Schrödinger equations and variable-coefficient Schrödinger equations, and extend the classical Escauriaza-Kenig-Ponce-Vega type of result to these models. This is based on joint works with S. Federico-Z. Li, and Z. Lee.

Wednesday 9:00-10:00 AM:

The Feynman propagator for the Klein-Gordon equation

Jesse Gell-Redman

Abstract: We construct the Feynman propagator for Klein-Gordon (KG) equation on Minkowski space perturbed by a decaying spatial potential. In particular, we construct global in time solutions to the inhomogeneous KG equation, each of whose wavefront sets is contained in the flowout of the wavefront set of the source in the direction of the Hamilton flow. Such solutions were shown to exist locally by Duistermaat-Hormander. To accomplish this, we prove a global Fredholm estimate. The persistence of the potential in time means that estimates for KG can be obtained using positive commutator estimates with operators in the three-body calculus of Vasy. This is joint work with Dean Baskin and Moritz Doll.

Wednesday 2:00-3:00 PM:

Microlocal Methods for The Elastic Travel Time Tomography Problem for Transversely Isotropic Media

Yuzhou Joey Zou

Abstract: I will discuss the travel time tomography problem for elastic media in the transversely isotropic setting. The mathematical study of this problem relates to X-ray tomography and boundary rigidity problems studied by de Hoop, Stefanov, Uhlmann, Vasy, et al., where the inverse problems are reduced to the microlocal analysis of certain operators obtained from a pseudolinearization argument. In the previous works, the authors made strong use of the scattering pseudodifferential calculus, particularly using the inversion theory of elliptic scattering operators. However, in the current setting the analysis is more subtle, as the operators obtained are somewhat degenerate; they instead resemble parabolic operators in terms of how the ellipticity degenerates. In this talk, I will discuss the qualitative features of the operators obtained in the pseudolinearization argument, as well as the construction of a hypoelliptic calculus, whose symbols satisfy iterated regularity with respect to a suitable blowup of the original cotangent bundle, which accommodates the inversion of these degenerate operators.

Thursday 9:00-10:00 AM:

Recovering the nonlinearity in nonlinear Schrödinger equations

Jason Murphy

Abstract: We discuss the problem of recovering an unknown nonlinearity from the scattering behavior of solutions in the setting of nonlinear Schrödinger equations. We will discuss both scattering and modified scattering, as well as the question of stability. After presenting some approaches to address the case of gauge-invariant nonlinearities, we will describe some new difficulties arising in the case of non-gauge-invariant nonlinearities. This talk will cover some joint work with R. Killip, M. Visan, and G. Chen.

Thursday 10:45-11:45 AM

Deep- and shallow-water limits of statistical equilibria for the intermediate long wave equation

Andreia Chapouto

The intermediate long wave equation (ILW) models the internal wave propagation of the interface in a stratified fluid of finite depth, providing a natural connection between the deep-water regime (= the BO regime) and the shallow-water regime (= the KdV regime). Exploiting the complete integrability of ILW, I will discuss the statistical convergence of ILW to both BO and KdV, namely the convergence of the huger order conservation laws for ILW and their associated invariant measures. In particular, as KdV possesses only half as many conservation laws as ILW and BO, we observe a novel 2-to-1 collapse of ILW conservation laws to those of KdV, which yields alternative modes of convergence for the associated measures in the shallow-water regime. This talk is based on joint work with Guopeng Li and Tadahiro Oh.

Thursday 2:00-3:00 PM:

Local decay estimate: a new proof and its application

Xiaoxu Wu

Abstract: The primary objective of quantum scattering theory is to classify the possible large-time behaviors of solutions to Schrödinger equations and to prove that all solutions converge to a superposition of these asymptotic states as time goes to plus or minus infinity. This is known as asymptotic completeness (AC).

Historically, local decay estimates (LD) have played a fundamental role in establishing AC. In this talk, I will introduce how to capture all free flows without using LD and in the case of five or more spatial dimensions, how AC can be used to establish LD for the scattering part of the solution. As a direct application, we obtain global-in-time Strichartz estimates.

Today's discussion will focus on the application of this LD approach to Schrödinger equations with interactions that are spatially localized and quasi-periodic in time, as well as to the purely nonlinear Schödinger equations with the radial symmetry. This is based on joint works with Avy Soffer.

Friday 9:00-10:00 AM:

Asymptotic stability of the sine-Gordon kink outside symmetry

Gong Chen

Abstract: We consider scalar field theories on the line with Ginzburg-Landau (double-well) self-interaction potentials. Prime examples include the $\phi 4$ model and the sine-Gordon model. These models feature simple examples of topological solitons called kinks. The study of their asymptotic stability leads to a rich class of problems owing to the combination of weak dispersion in one space dimension, low power nonlinearities, and intriguing spectral features of the linearized operators such as threshold resonances or internal modes.

We present a perturbative proof of the full asymptotic stability of the sine-Gordon kink outside symmetry under small perturbations in weighted Sobolev norms. The strategy of our proof combines a space-time resonances approach based on the distorted Fourier transform to capture modified scattering effects with modulation techniques to take into account the invariance under Lorentz transformations and under spatial translations. A major difficulty is the slow local decay of the radiation term caused by the threshold resonances of the non-selfadjoint linearized matrix operator around the modulated kink. Our analysis hinges on two remarkable null structures that we uncover in the quadratic nonlinearities of the evolution equation for the radiation term as well as of the modulation equations.

The entire framework of our proof, including the systematic development of the distorted Fourier theory, is general and not specific to the sine-Gordon model. We conclude with a discussion of potential applications in the generic setting (no threshold resonances) and with a discussion of the outstanding challenges posed by internal modes such as in the well-known $\phi 4$ model.

Friday 10:45-11:45 AM:

The Benjamin–Ono Equation

Rowan Killip

Abstract: I will first explain the origin of the Benjamin–Ono equation as an effective model for interfacial waves in fluids of great total depth. Like its shallow-water cousin, the Korteweg–de Vries equation, this model is completely integrable. After describing the integrable structures, I will describe the sharp well-posedness theory and a slew of new virial-type identities. This talk is based on joint work with Thierry Laurens and Monica Visan.