Book of Abstracts

Numerical Analysis of Interface and Multiphysics Problems

MATRIX Research Program | 5 - 16 May 2025 | Creswick, Australia

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Program | Week 1

MATRIX Research Program: Numerical Analysis of Interface and Multiphysics Problems - Week 1

	Sunday 04/05	Monday 05/05	Tuesday 06/05	Wednesday 07/05	Thursday 08/05	Friday 09/05	Saturday 10/05
08:00 - 09:00		**Breakfast** Venue: Dining Hall	**Breakfast** Venue: Dining Hall	**Breakfast** Venue: Dining Hall	**Breakfast** Venue: Dining Hall	**Breakfast** Venue: Dining Hall	**Breakfast** Venue: Dining Hall
09:00 - 9:30		**Welcome/Creswick Campus Orientation, MATRIX & Creswick Staff** Venue: Lecture Theatre	**Alexandre Ern** Venue: Lecture Theatre Chair: Ricardo Ruiz Baier	**Mats Larson** Venue: Lecture Theatre Chair: Santiago Badia	**Silvia Bertoluzza** Venue: Lecture Theatre Chair: Martina Bukač	**Daniele Boffi** Venue: Lecture Theatre Chair: Victor Calo	
9:30 - 10:30	Program participants are required to make their own way to Ballarat (Train) Station and arrive before 20:55 (local time). MATRIX staff will arrange transfers from Ballarat Train Station to MATRIX meeting at **Bus Bay 3** the following times: **10:15, 14:15, 16:45, 18:45 & 20:50**	**Lucia Gastaldi** Venue: Lecture Theatre Chair: Santiago Badia	**Nitesh Verma** Venue: Lecture Theatre Chair: Ricardo Ruiz Baier	**Rekha Khot** Venue: Lecture Theatre Chair: Santiago Badia	**Progress Discussion** Venue: Lecture Theatre	**Antoine Marteau** Venue: Lecture Theatre Chair: Victor Calo	
10:30 - 12:00		**Discussion/Collaboration** Venue: MATRIX House	**Discussion/Collaboration** Venue: MATRIX House	**Discussion/Collaboration** Venue: MATRIX House	**Collaboration** Venue: MATRIX House	**Discussion/Collaboration** Venue: MATRIX House	
12:00 - 14:00		**Lunch** Creswick	**Lunch** Creswick	**Lunch** Creswick	**Lunch** Creswick	**Lunch** Creswick	
14:00 - 14:30		**Jai Tushar** Venue: Lecture Theatre Chair: Victor Calo	**Bishnu Lamichhane** Venue: Lecture Theatre Chair: Anne Boschman	**Eric Neiva** Venue: Lecture Theatre Chair: Ricardo Ruiz Baier	**Afternoon Walk**	**Alexandre Magueresse** Venue: Lecture Theatre Chair: Martina Bukač	
14:30 - 15:00		**Jordi Manyer** Venue: Lecture Theatre Chair: Victor Calo	**Sorin Pop** Venue: Lecture Theatre Chair: Anne Boschman	**Andres Rubiano Martinez** Venue: Lecture Theatre Chair: Ricardo Ruiz Baier		**Sergio Carrasco Hildago** Venue: Lecture Theatre Chair: Martina Bukač	
15:00 - 17:00		**Discussion/Collaboration** Venue: MATRIX House	**Discussion/Collaboration** Venue: MATRIX House	**Discussion/Collaboration** Venue: MATRIX House		**Discussion/Collaboration** Venue: MATRIX House	
17:00 - 18:00			**MATRIX cheese and wine afternoon** Venue: Recreational Room				
18:00 - 19:00	Pub dinner in Farmers' Arms Creswick (18:30 - 21:00)	**Dinner** Venue: Dining Hall	**Dinner** Venue: Dining Hall	**Dinner** Venue: Dining Hall	**Dinner** Venue: Dining Hall	**Dinner** Venue: Dining Hall	

Coffee is available at all times in MATRIX House

MATRIX

Program | Week 2

MATRIX Research Program: Numerical Analysis of Interface and Multiphysics Problems - Week 2

	Sunday 11/05	Monday 12/05	Tuesday 13/05	Wednesday 14/05	Thursday 15/05	Friday 16/05
08:00 - 09:00	**Breakfast** Venue: Dining Hall	**Breakfast** Venue: Dining Hall	**Breakfast** Venue: Dining Hall	**Breakfast** Venue: Dining Hall	**Breakfast** Venue: Dining Hall	**Breakfast** Venue: Dining Hall
						Check out - please vacate rooms and accommodation building by 9am
09:00 - 09:30		**Maxim Olshanskii** Venue: Lecture Theatre Chair: Santiago Badia	**Sara Zahedi** Venue: Lecture Theatre Chair: Victor Calo	**Martina Bukač** Venue: Lecture Theatre Chair: Anne Boschman	**Ricardo Ruiz Baeier** Venue: Lecture Theatre Chair: Santiago Badia	**Discussion/Collaboration (09:00 - 12:30)** Venue: MATRIX House
09:30 - 10:00				**Sergio Rojas Hernandez** Venue: Lecture Theatre Chair: Anne Boschman	**Discussion/Collaboration** Venue: MATRIX House	
10:00 - 12:00		**Discussion/Collaboration** Venue: MATRIX House	**Discussion/Collaboration** Venue: MATRIX House	**Discussion/Collaboration (10:00 - 12:30)** Venue: MATRIX House		
12:00 - 14:00		**Lunch** Creswick	**Lunch** Creswick	**Lunch (12:30 - 13:30)** Venue: Dining Hall	**Lunch** Creswick	**Lunch (12:30 - 13:30)** Venue: Dining Hall
14:00 - 14:30		**Anne Boschman** Venue: Lecture Theatre Chair: Martina Bukač	**Victor Calo** Venue: Lecture Theatre Chair: Ricardo Ruiz Baier	**Afternoon Walk**	**Santiago Badia** Venue: Lecture Theatre Chair: Anne Boschman	**Transportation to Melbourne. 14:00 departure from Reception. ~15:30 arrival at Melbourne Tullamarine Airport ~16:15 arrival at Southern Cross Station, Melbourne CBD.**
14:30 - 15:00		**Miguel Fernández** Venue: Lecture Theatre Chair: Martina Bukač	**Alberto Martin** Venue: Lecture Theatre Chair: Ricardo Ruiz Baier		**Concluding remarks** Venue: Lecture Theatre	
15:00 - 17:00		**Discussion/Collaboration** Venue: MATRIX House	**Discussion/Collaboration** Venue: MATRIX House		**Discussion/Collaboration** Venue: MATRIX House	
17:00 - 18:00			**MATRIX cheese and wine afternoon** Venue: Recreational Room			
18:00 - 19:00		**Dinner** Venue: Dining Hall	**Dinner** Venue: Dining Hall	**Dinner** Venue: Dining Hall	**Dinner** Venue: Dining Hall	

Coffee is available at all times in MATRIX House

Week 1 | Monday 5 May | 09:30 - 10:30

Lucia Gastaldi (Università degli studi di Brescia)

Lagrange multiplier approaches for the finite element approximation of interface problems

This talk is devoted to the finite element approximation of boundary value problems with interfaces. We shall consider a simple second order elliptic equation with discontinuous coefficients and present the most common approaches for its finite element discretization. One main feature of these methods consists in the construction of the mesh which can be *fitted* or *unfitted*. In the first case the mesh is constructed such that the elements are not cut by the interface, so that the resulting approximation has optimal rate of convergence according to the regularity of the solution.

Unfitted meshes are independent of the position of the interface, leading possibly to non optimal rate of convergence of the approximation. In particular, we shall present methods based on the introduction of a Lagrange multiplier to enforce weakly the transmission conditions across the interface using unfitted meshes. These approaches can be efficiently extended to the finite element approximation of fluid-structure interaction systems.

Coauthors are Daniele Boffi and Luca Heltai.

Week 1 | Monday 5 May | 14:00 - 14:30

Jai Tushar (Monash University)

BDDC preconditioners for non-conforming polytopal hybrid discretisation methods.
Part I - A discrete trace theory

The analysis of non-overlapping domain decomposition method-based solvers like BDDC relies on the exchange of information across intersubdomain boundaries. It requires three main ingredients: a *trace inequality*, which implies that the restriction of functions to the subdomain interface is stable; a *lifting result*, which lifts this restriction to the interior of the neighboring subdomain; and continuity of a *face truncation operator* on piecewise polynomial functions. The bound on this operator leads to a mesh-dependent logarithmic estimate.

For conforming finite element methods, this is realized with the help of continuous trace theory. For non-conforming methods, such as polytopal methods, the continuous trace theory fails, since the trace of piecewise polynomial functions in $L^2(\Omega)$ does not possess $H^{1/2}(\partial\Omega)$ regularity. The current state of the art to address this involves constructing an interpolant of a function on the interface (intersubdomain boundary) onto a conforming finite element space and then applying the continuous trace theory. As a result, all the analysis for non-conforming spaces so far has been carried out on conforming simplicial/tetrahedral or quadrilateral/hexahedral meshes.

In this talk, we will present a discrete trace theory for non-conforming polytopal methods. This theory is based entirely on the fully discrete hybrid spaces appearing in these methods. It hinges on the design of a novel discrete trace seminorm. For this seminorm, we establish discrete trace and lifting inequalities that are independent of mesh size and hold on quasi-uniform polytopal meshes. We also derive a truncation estimate in this discrete trace seminorm for piecewise polynomials in a hybrid setting. Finally, we compute the proposed discrete trace operator and show that its spectrum is equivalent to that of the discrete energy operator (a consequence of the discrete trace and lifting inequalities). This talk is based on [1].

The tools presented in this talk will be used in the talk by *Jordi Manyer* titled "*BDDC preconditioners for non-conforming polytopal hybrid discretisation methods. Part II: The preconditioner*" to prove the condition number bounds of the BDDC preconditioner for non-conforming polytopal methods [2].

References
[1] S. Badia, J. Droniou, and J. Tushar, “A discrete trace theory for non-conforming polytopal hybrid discretisation methods,” arXiv: 2409.15863, 2024.
[2] S. Badia, J. Droniou, J. Manyer, and J. Tushar, “BDDC preconditioners for non-conforming polytopal hybrid discretisation methods,” Under preparation, 2025.

Week 1 | Monday 5 May | 14:30 - 15:00

Jordi Manyer (Monash University)

BDDC preconditioners for non-conforming polytopal hybrid discretisation methods.
Part II: The preconditioner.

Polytopal methods are a class of finite element methods that can be applied on polytopal meshes, i.e, meshes that are composed of general polygons or polyhedra. Polytopal methods have become increasingly popular since they can deal with complex geometries without the need for complex meshing tools. These often involve hybrid spaces, where degrees of freedom are attached to mesh entities of different dimensions.

The design of efficient, robust, scalable solvers for linear systems arising from these kind of discretizations is important to make them competitive with traditional methods on real world applications. One family of such scalable preconditioners are non-overlapping domain decomposition methods.

The analysis of these preconditioners relies on bounds for the so-called trace and lifting operators. For conforming methods, these bounds are based on continuous trace theory and are well understood. This is not the case for non-conforming methods, such as polytopal methods, since the trace of piecewise polynomial functions in $L^2(\Omega)$ does not possess $H^{1/2}(\partial\Omega)$ regularity. A novel discrete trace theory for polytopal methods [1] has been introduced by *Jai Tushar* in the companion talk “*BDDC preconditioners for non-conforming polytopal hybrid discretisation methods. Part I: A discrete trace theory*”. The theory provides a robust framework to analyse non-overlapping domain decomposition preconditioners for polytopal methods.

In this talk, we will use this theory to prove optimal condition number estimates for the Balancing Domain Decomposition by Constraints (BDDC) preconditioner. We numerically validate our claims for the HDG and HHO schemes, on both structured and fully polytopal meshes, in 2D and 3D. We show robustnes and scalability of our preconditioner for up to several hundreds of processors. All experiments are performed using the open-source finite-element library Gridap, and run on the GADI supercomputer using resources provided by the Australian Government through NCI under the NCMAS Merit Allocation Scheme. This talk is based on [2].

Coauthors are Santiago Badia (Monash University), Jérôme Droniou (CNRS & University of Montpellier, France) and Jai Tushar (Monash University).

References
[1] S. Badia, J. Droniou, and J. Tushar, “A discrete trace theory for non-conforming polytopal hybrid discretisation methods,” arXiv: 2409.15863, 2024.
[2] S. Badia, J. Droniou, J. Manyer, and J. Tushar, “BDDC preconditioners for non-conforming polytopal hybrid discretisation methods,” Under preparation, 2025.

Week 1 | Tuesday 6 May | 09:00 - 10:00

Alexandre Ern (ENPC, IP Paris and INRIA Paris)

Title TBA

Abstract TBA

Week 1 | Tuesday 6 May | 10:00 - 10:30

Nitesh Verma (Universidad del Bio-Bio)

Virtual element approximations for Biot-elasticity problem

In this talk, we will discuss on a virtual element discretisation for an interfacial poroelasticity/elasticity consolidation problem. The formulation of the time−dependent poroelasticity equations uses displacement, fluid pressure and total pressure, and the elasticity equations in displacement-pressure formulation. The construction of the virtual element scheme does not require Lagrange multipliers to impose the transmission conditions on the interface. We show the stability and convergence of the virtual element method for different polynomial degrees. Some simple numerical examples to illustrate the properties of the scheme.

Week 1 | Tuesday 6 May | 14:00 - 14:30

Bishnu Lamichhane (University of Newcastle)

Finite Element Computations Using Biorthogonal Systems

In this talk, I will briefly discuss my research contributions. Then, some applications of biorthogonal systems will be highlighted. In particular, applications to non-conforming triangulations, elasticity, biharmonic equations, thin plate splines and Reissner-Mindlin please equations will be discussed. Finally, some open problems will be discussed.

Week 1 | Tuesday 6 May | 14:30 - 15:00

Sorin Pop (Hasselt University)

Stable linear iterative schemes for nonlinear doubly-degenerate diffusion equations

Doubly degenerate parabolic equations appear as mathematical models for various real-life processes. Examples in this sense include unsaturated flow through porous media (e.g. in the subsurface), chemical or biological processes, or phase change (the Stefan problem). The main feature of such problems is that the diffusion term may vanish, or blow up, or even multi-valued for certain values of the solution. This leads to a change of the type in the equation, from parabolic to hyperbolic or elliptic, in time-space sub-domains that are not known a priori, and which are separated by free-boundaries.
For stability reasons, (semi-)implicit time stepping methods are popular for such problems. Among these, and motivated by the lacking regularity of the solution, the backward Euler method is often used for such types of problems. This leads to a sequence of nonlinear, time-discrete elliptic equations, for which the diffusion may still vanish or blow up.

To find an approximation of the solutions, linear iterative schemes are needed. Widely used is the Newton scheme, which converges quadratically, but under severe constraints for the initial guess. In the present context, when the equation to be solved is the time-discrete counterpart of the original model, since the initial guess is the solution at the previous time step, having a sufficiently good initial guess induces a severe restriction on the time step. This restriction may depend on the chosen spatial discretization, and can become more severe as the mesh is refined. Therefore, alternative approaches are considered, one being the L-scheme, which has a robust, but only linear convergence behaviour. A modified variant of it combines the features of the two schemes mentioned before, with a robust, yet linear convergence behaviour, but, wth the convergence rate decreasing with the time step size. It can be seen as an interpolation between the two schemes, as, depending on a parameter, one the scheme is closer to either the L-scheme, or to Newton.

In this presentation we discuss different linear iterative schemes for the time-discrete equations stemming from nonlinear, doubly-degenerate parabolic equations. The starting point is a reformulation in terms of a new unknown, which reduces the complexity of the nonlinearities involved and leads to a formulation that is more suitable for dealing with the degeneracies. Also, a splitting approach is used, in which the nonlinear dependencies are written as algebraic equations, leaving the original equation linear. After performing an Euler implicit time-stepping for the transformed equations, different iterative linearization strategies are considered, namely the Newton scheme, the L-scheme, and the modified L-scheme. The convergence of the schemes is proved even for the doubly-degenerate case. In the non-degenerate case, we prove that the scheme is contractive, and the contraction rate is proportional to a non-negative exponent of the time-step size. Moreover, we propose an a posteriori estimator-based adaptive algorithm to select the optimal parameters for the modified L-scheme, which accelerates its convergence.

Numerical results are presented, showing that the modified L-scheme and its adaptive variant are more stable than the Newton scheme, as they converge irrespective of the mesh. This makes them relevant also for adaptive mesh refinement, and for problems involving multiple scales. Moreover, the adaptive scheme consistently out-competes the Newton scheme, showing a quadratic convergence behaviour.

This is a joint work with Ayesha Javed (Hasselt) and Koondaninbha Mitra (Eindhoven).

Week 1 | Wednesday 7 May | 09:00 - 10:00

Mats Larson (Umea University)

Finite Element Approximation of Constrained and Contact Problems

We review recent advances in applying the augmented Lagrangian method as a general approach for generating stabilized methods with and without explicit multipliers. The augmented Lagrangian method enhances the standard Lagrange multiplier approach by incorporating a penalty term that enforces the constraint equations. Traditionally known as the basis for iterative algorithms in constrained optimization, its application as a stabilization method in computational mechanics has recently gained significant interest. We examine equality and inequality constraints, showcasing the framework through various examples in computational mechanics, including contact problems. For contact problems, we also introduce a hybridized formulation where the hybrid space can facilitate the transfer of information between the two bodies in contact or represent physical phenomena at the interface. Several numerical examples are presented.

Week 1 | Wednesday 7 May | 10:00 - 10:30

Rekha Khot (Inria)

Hybrid high-order methods for the wave equation in first-order form

In this talk, we will discuss the approximation of the acoustic wave equation in its first-order Friedrichs formulation using hybrid high-order (HHO) methods, proposed and numerically investigated in [Burman-Duran-Ern, 2022]. We first look at energy-error estimates in the time-continuous setting and give several examples of interpolation operators: the classical one in
the HHO literature based on L2-orthogonal projections and others from, or inspired from, the hybridizable discontinuous Galerkin (HDG) literature giving improved convergence rates on simplices. The time-discrete setting is based on explicit Runge-Kutta (ERK) schemes in time combined with HHO methods in space. In the fully discrete analysis, the key observation is that it becomes crucial to bound the consistency error in space by means of the stabilization seminorm only. We formulate three abstract properties (A1)-(A3) to lead the analysis. Our main result proves that, under suitable CFL conditions for second- and third-order ERK schemes, the energy error converges optimally in time and quasi-optimally in space, with optimal rates recovered on simplicial meshes. The abstract foundations of our analysis should facilitate its application to other nonconforming hybrid methods such as HDG and weak Galerkin (WG) methods. We will also employ the tools developed to analyze the time-continuous HHO setting for the coupling between the acoustic and elastic wave equations.

Week 1 | Wednesday 7 May | 14:00 - 14:30

Eric Neiva (CNRS)

Unfitted FE modelling of coupled surface-bulk viscous flows in animal cells

This talk presents a novel unfitted finite element framework to model coupled surface-bulk viscous flows in animal cells, focusing on the interaction between the active actomyosin cortex and the viscous cytoplasm. The cortex generates contractile forces through molecular motors, driving cell shape changes and inducing intracellular flows. To simulate this complex system, we combine the trace finite element method for the surface flows with the aggregated finite element method for the bulk flows, enabling sharp-interface modelling on fixed grids without the need for remeshing. The model incorporates mechanochemical feedback via the surface transport of a molecular species that regulates active tension. This leads to a mixed-dimensional PDE system solved in a fixed Cartesian grid using level-set-based surface tracking. Numerical experiments confirm the method's accuracy and stability, capturing phenomena such as self-organized pattern formation, curvature-driven relaxation, and cell cleavage. The framework offers a robust and extendable tool for exploring increasingly complex morphogenetic processes in animal cells.

Week 1 | Wednesday 7 May | 14:30 - 15:00

Andres Rubiano Martinez (Monash University)

A posteriori error analysis of a robust virtual element method for stress-assisted diffusion problems

We develop and analyse residual-based a posteriori error estimates for the virtual element discretisation of a nonlinear stress-assisted diffusion problem in two and three dimensions. The model problem involves a two-way coupling between elasticity and diffusion equations in perturbed saddle-point form. A robust global inf-sup condition and Helmholtz decomposition for $\mathbf{H}(\mathrm{div},\Omega)$ lead to a reliable and efficient error estimator based on appropriately weighted norms that ensure parameter robustness. The a posteriori error analysis uses quasi-interpolation operators for Stokes and edge virtual element spaces, and we include the proofs of such operators with estimates in 3D for completeness. Finally, we present numerical experiments in both 2D and 3D to demonstrate the optimal performance of the proposed error estimator.

Week 1 | Thursday 8 May | 09:00 - 10:00

Silvia Bertoluzza (IMATI)

An abstract framework for heterogeneous coupling: stability, approximation and preconditioning

We introduce an abstract coupling framework reminiscent of FETI, for which we establish conditions for stability, minimal requirements for well-posedness both on the continuous and on the discrete level, as well as a stabilization strategly, acting only on the multiplier, that can be used to achieve stability of the discrete problem under very mild conditions. A general abstract recipe for the design of preconditioners is proposed, also inspired by FETI. Various applications are presented. Particularly, the natural application in the domain decomposition framework is shown to result in both some known and some novel domain decomposition preconditioners.

Week 1 | Friday 9 May | 09:00 - 10:00

Daniele Boffi (King Abdullah University of Science and Technology)

Stability and conditioning of a fictitious domain approach for fluid structure interactions

In this talk I review our approach to the approximation of fluid structure interactions, based on a fictitious domain technique with distributed Lagrange multiplier. Our analysis includes the well-posedness of a linearization of the continuous problem, unconditional stability in time for a semi-explicit scheme, stability in space and convergence for the discrete steady state solution.

Unlike other unfitted approaches, our method does not need any stabilization in presence of small cut cells as confirmed by our theoretical results and by extensive numerical tests. Recent results discuss the conditioning of the matrix arising from the monolithic discretization and study rigorously the quadrature errors of the coupling terms.

This is a joint work with Fabio Credali and Lucia Gastaldi.

Week 1 | Friday 9 May | 10:00 - 10:30

Antoine Marteau (Monash University)

Multiscale models for nonlinear transient electromagnetic problems with confined eddy currents

Heterogeneous materials are widespread in electrical engineering, but they lead to unaffordable costs for the computation of electromagnetic fields using the Finite Element Method (FEM). This presentation is mainly about the multiscale modeling of electromagnetic fields on heterogeneous materials with periodic geometries. The numerical method used is the Heterogeneous Multiscale Method (HMM), also known as FE2 method, for solving homogenized Maxwell's equations. The main contribution of this work is to introduce new homogenization formulas for the magnetic field, making the homogenization robust to the presence of strong locally confined induced currents, responsible for hysteresis in the homogenized material law.

Week 1 | Friday 9 May | 14:00 - 14:30

Alexandre Magueresse (Monash University)

Energy minimisation using tensor-product free-knot B-splines and sparse grids

We introduce a general framework for minimising energy functionals in parametric functional spaces. This approach offers an alternative to standard h- and p-adaptive methods for solving PDEs with solutions that exhibit localised features, such as singularities, sharp fronts, and multi-scale phenomena. Our method relies on parametric spaces constructed from free-knot patches, formed via tensor products of one-dimensional bases (e.g., B-splines, Hermite, or Bézier). These patches can move freely across the domain, enabling adaptive resolution of solution features while preserving computational efficiency through their tensor-product structure. A key advantage of our framework is its versatility. It supports a wide range of functional spaces and can be combined with sparse grids for improved efficiency. We provide elements of numerical analysis regarding the smoothness of the discrete energy functional and show the convergence of a projected gradient descent scheme. We present several representative numerical experiments, illustrating the method's effectiveness and applicability.

Week 1 | Friday 9 May | 14:30 - 15:00

Sergio Carrasco Hidalgo (Monash University)

A mixed FEM approach in Banach spaces for fluid flow in porous media with heterogeneous permeability

In this talk, we analyse a version of the steady-state model arising from the filtration of an incompressible fluid through a non-deformable, saturated porous medium with heterogeneous permeability, separated by an interface. In the mathematical setting, a fully mixed FEM formulation leads to a two-fold perturbed saddle point problem within a Banach space framework, where the variables at the interface are introduced via Lagrange multipliers in appropriate trace spaces. We will also discuss some aspects of the discrete scheme and present some numerical results.

Week 2 | Monday 12 May | 09:00 - 10:00

Maxim Olshanskii (University of Houston)

Analysis of a finite element method for PDEs in evolving domains with topological changes

The talk presents the first rigorous error analysis of an unfitted finite element method for linear parabolic problems posed on time-dependent domains that may undergo topological changes. The domain evolution is assumed to be smooth away from a critical time, at which the topology may change instantaneously. To accommodate such transitions in the error analysis, we introduce several structural assumptions on the evolution of the domain in the vicinity of the critical time. These assumptions guarantee a specific control over the variation of a solution norm in time, even across singularities, and form the foundation for the numerical analysis. We demonstrate the applicability of our assumptions with examples of level-set domains undergoing topological transitions and discuss cases where analysis fails. The theoretical error estimate is supported by the results of a numerical experiment. Questions that remain open will be outlined.

Week 2 | Monday 12 May | 14:00 - 14:30

Anne Boschman (Monash University)

A Divergence-Preserving Unfitted Finite Element Method for the Darcy Problem

Geometrically unfitted finite element methods are powerful discretization tools for the numerical approximation of partial differential equations on complex or evolving geometries. These methods, such as CutFEM, XFEM, or AgFEM, allow the computational mesh to remain independent of the domain's geometry, thereby providing greater flexibility and efficiency. However, without appropriate stabilization techniques, unfitted methods can lead to unstable and severely ill-conditioned discrete problems, typically due to the presence of small cut cells. To address this, stabilization techniques, such as ghost penalty stabilization and cell aggregation methods, are employed to ensure stability regardless of the cut configuration.

In this contribution, we present a stable unfitted finite element method for the Darcy problem on a complex geometry. In the context of body-fitted methods, div-conforming finite element pairs, such as Raviart–Thomas elements and discontinuous piecewise polynomials, are typically used for the Darcy problem to preserve its underlying structure (i.e. achieve mass conservation) at the discrete level. Here, we propose stabilization terms specifically designed for the ill-posed cut cells, ensuring the stability of the mixed Darcy problem while maintaining its div-conforming properties. We will discuss the stability and convergence properties of the proposed unfitted method, and demonstrate these in numerical experiments.

This is joint work with Santiago Badia, Alberto F. Martin and Ricardo Ruiz Baier.

Week 2 | Monday 12 May | 14:30 - 15:00

Miguel Fernández (Inria)

Immersed Mixed-Dimensional Fluid-Structure Interaction Problems

This talk focuses on some modeling and numerical difficulties that arise in simulating immersed fluid-structure interactions with a dimensional gap of one or two, i.e, in bulk-surface and bulk-curve settings, discretized with unifitted meshes and low-order finite element methods.

Week 2 | Tuesday 13 May | 09:00 - 10:00

Sara Zahedi (KTH Royal Institute of Technology)

Cut Finite Element Methods for Interface Problems

In this talk, I will introduce Cut Finite Element Methods (CutFEM) for interface problems and present our recent developments within this class of unfitted finite element techniques, with a focus on achieving mass conservation. While finite element methods (FEM) are widely used to approximate solutions to partial differential equations (PDEs) in interface problems, they typically rely on computational meshes that conform to the interface. CutFEM relaxes this constraint, allowing interfaces and boundaries to intersect the mesh arbitrarily, while aiming to preserve key properties of standard FEM. I will begin with stationary problems, then extend the discussion to time-dependent PDEs in evolving domains, and demonstrate our method's application to simulating surfactant dynamics in incompressible two-phase flow systems.

Week 2 | Tuesday 13 May | 14:00 - 14:30

Victor Calo (Curtin University)

Adaptive Stabilized Finite Element Methods: Reinterpretation as a Variational Multiscale Method

Convection-dominated diffusion problems remain among computational science's greatest challenges, creating sharp boundary layers at high Péclet numbers that elude standard numerical methods.

Stabilization techniques evolved from Brooks and Hughes' streamline-upwind Petrov-Galerkin (SUPG) method to the more general Galerkin/least-squares approach. These early methods, while effective, required problem-specific parameter tuning.

Hughes' variational multiscale method (VMS) transformed the field by decomposing solutions into coupled coarse and fine scales, reframing residual-based methods as sub-grid scale models. Concurrently, the Discontinuous Galerkin (dG) method emerged, offering robust stabilization through interface fluxes.

We combined residual minimization with dG stability when proposing the Adaptive Stabilized Finite Element Method (AS-FEM), which uses residual minimization on dual dG to refine the mesh automatically. AS-FEM computes continuous coarse-scale approximations while generating error estimates for adaptivity, deriving fine-scale components through VMS reinterpretation. This reinterpretation of the residual minimization creates a comprehensive framework that decomposes spaces, defines optimal inter-scale operators, derives stable fine-scale contributions, generates reliable error indicators, and performs robustly across numerical regimes. Testing confirms that AS-FEM provides stable solutions with optimal convergence rates for challenging convection-dominated problems.

Week 2 | Tuesday 13 May | 14:30 - 15:00

Alberto F. Martin (Australian National University)

Unfitted finite element interpolated neural networks

We present a novel approach that integrates unfitted finite element methods and neural networks to approximate partial differential equations on complex geometries. Easy-to-generate background meshes (e.g., a simple Cartesian mesh) that cut the domain boundary (i.e., they do not conform to it) are used to build suitable trial and test finite element spaces. The method seeks a neural network that, when interpolated onto the trial space, minimises a discrete dual norm (defined over the test space) of the weak residual functional associated to the equation. As with unfitted finite elements, essential boundary conditions are weakly imposed by Nitsche's method. The method is robust to variations in Nitsche coefficient values, and to small cut cells. We experimentally demonstrate the method's effectiveness in solving both forward and inverse problems across various 2D and 3D complex geometries, including those defined by implicit level-set functions and explicit stereolithography meshes. For forward problems with smooth analytical solutions, the trained neural networks achieve several orders of magnitude smaller H1 errors compared to their interpolation counterparts. These interpolations also maintain expected h- and p-convergence rates. Moreover, using preconditioners (achieved by stabilisations) remarkably accelerates neural network training and further enhances robustness to the choice of Nitsche coefficient values. The experiments also show the method's high accuracy and reliability in solving inverse problems, even with incomplete observations.

Week 2 | Wednesday 14 May | 09:00 - 09:30

Martina Bukač (University of Notre Dame)

The diffuse interface method for fluid-structure interaction problems

Fluid-structure (elastic or poroelastic) interaction problems arise in many applications, such as geomechanics, aerodynamics, and blood flow dynamics (hemodynamics). Many existing numerical methods for such problems are based on a sharp interface approach, in the sense that the interface between the two regions is parametrized using an exact specification of its geometry and location, and the nodes in the computational mesh align with the interface. However, when the interface displacement is large, classical methods may require expensive remeshing techniques. In this talk, we present a diffuse interface method, which tracks each region using a phase-field function. The phase field function which transitions from 1 in one region to -1 in the other region. We will talk about the diffuse interface approximation of fluid-(poro)elastic structure interaction and discuss current challenges in this area.

Week 2 | Wednesday 14 May | 09:30 - 10:00

Sergio Rojas Hernandez (Monash University)

Recent Advances in Robust Variational Physics-Informed Neural Networks

In early 2024, we introduced RVPINNs, a robust extension of the Variational Physics-Informed Neural Networks (VPINNs) method. In RVPINNs, the loss functional is formulated using a Petrov-Galerkin-type variational approach, where the trial space consists of a (Deep) Neural Network and the test space is a finite-dimensional vector space. Unlike standard VPINNs, RVPINNs minimise a loss based on the discrete dual norm of the residual, providing a reliable estimator of the approximation error in the energy norm under the assumption of a local Fortin operator. This ensures more accurate approximations of the partial differential equations governing experimental data. However, a key challenge of RVPINNs is the need to invert a Gram matrix at each Neural Network nonlinear solver step, making the method computationally expensive if the variational formulation and discrete test space are not carefully selected.

In this talk, we will present recent advances in RVPINNs, focusing on adaptive strategies for test space selection and variational formulations that enable the construction of block-diagonal Gram matrices, significantly accelerating training while maintaining accuracy. We will also demonstrate the effectiveness and robustness of our approach through tailored numerical experiments, confirming theoretical error estimates and highlighting substantial improvements in computational efficiency.

MATRIX

Week 2 | Thursday 15 May | 09:00 - 09:30

Ricardo Ruiz Baier (Monash University)

Fully mixed FEM formulations of poroelasticity with stress-dependent permeability

We develop a family of mixed finite element methods for a model of nonlinear poroelasticity where, thanks to a rewriting of the constitutive equations, the permeability depends on the total poroelastic stress and on the fluid pressure and therefore we can use the Hellinger–Reissner principle with weakly imposed stress symmetry for Biot's equations. The problem is adequately structured into a coupled system consisting of one saddle-point formulation, one linearised perturbed saddle-point formulation, and two off-diagonal perturbations. This system's unique solvability requires assumptions on regularity and Lipschitz continuity of the inverse permeability, and the analysis follows fixed-point arguments and the Babuska–Brezzi theory. The discrete problem is shown uniquely solvable by applying similar fixed-point and saddle-point techniques as for the continuous case. The method is based on the classical PEERS elements, it is exactly equilibrium and mass conservative, and it is robust with respect to the nearly incompressible as well as vanishing storativity limits. We derive a priori error estimates, we also propose fully computable residual-based a posteriori error indicators, and show that they are reliable and efficient with respect to the natural norms, and robust in the limit of near incompressibility. These a posteriori error estimates are used to drive adaptive mesh refinement. The theoretical analysis is supported and illustrated by several numerical examples in 2D and 3D.

Week 2 | Thursday 15 May | 14:00 - 14:30

Santiago Badia (Monash University)

Level-set optimisation with unfitted finite elements and automatic shape differentiation

Shape and topology optimisation are important computational techniques for designing structures that minimize a certain objective function, generally related to the structure properties. Within level-set methods, the design domain boundary is implicitly represented via a level-set function and updated using an evolution equation. The evolution of the level-set requires the derivation of the so-called shape derivatives. Typically, this involves manual, problem-dependent, and tedious computations. To this end, automatic shape differentiation, that is the application of automatic differentiation techniques to compute shape derivatives, has been proposed in the literature to help reduce this burden.

In conventional level-set methods, the underlying partial differential equations are solved over a domain that is immersed in a static background domain by extending it using a smooth Heaviside function. This allows integration to be relaxed over the whole computational domain. While efficient, this can lead to undesirable computational artifacts particularly in nonlinear and solid-fluid interaction problems. Furthermore, differentiation with respect to the level-set function in a smoothed boundary regime may not fully capture the shape derivative of a functional under boundary perturbations. Unfitted finite element methods embed a complex computational domain into a fixed background domain while providing a precise description of the boundary as a (D-1)-dimensional manifold. This makes them natural candidates to address the aforementioned issues.

In this work, we extend recent existing results in analytic shape calculus for unfitted methods. Furthermore, we develop automatic shape differentiation techniques for unfitted methods, and show we can recover the analytic derivatives to machine precision regardless of the mesh size.

We present an implementation of these techniques for both serial and distributed computing frameworks in the Julia package GridapTopOpt [2] and the wider Gridap ecosystem. As part of this implementation we propose a novel graph-based approach for the detection of isolated volumes. We demonstrate the applicability of the unfitted automatic shape differentiation framework and our implementation by considering the three-dimensional minimum compliance topology optimisation of a linear elastic wheel and of a linear elastic structure in a fluid-structure interaction problem with Stokes flow. This talk is based on [1].

Coauthors are Zachary J. Wegert (Queensland University of Technology), Jordi Manyer (Monash University), Connor Mallon (Monash University) and Vivien J. Challis (Queensland University of Technology).

References

[1] Z. J. Wegert, J. Manyer, C. Mallon, S. Badia, and V. J. Challis. Level-set topology optimisation with unfitted finite elements and automatic shape differentiation. 2025. arXiv: 2504.09748
[math.OC].

[2] Z. J. Wegert, J. Manyer, C. N. Mallon, S. Badia, and V. J. Challis. “GridapTopOpt.jl: a scalable Julia toolbox for level set-based topology optimisation”. en. In: Structural and Multidisciplinary Optimization 68.1 (Jan. 2025), p. 22. doi: 10.1007/s00158-024-03927-3.